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ON SOME LOCAL NONUNIQUE FIXED POINT THEOREMS FOR CERTAIN MAPS IN THE RELATIONAL METRIC SPACES

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Abstract. In this paper, we establish a couple of local nonunique fixed point results for the orbital relationally continuous selfmappings of a orbital relationally complete metric space which include some interesting nonunique local fixed point results for for certain partially contraction mappings on partially ordered metric spaces as the special cases.

Keywords: Relational complete metric space; Relational contraction mapping; Local nonunique fixed point theorem.

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1. Introduction

It is well-known that the local fixed point theory is very much useful for finding the existence of local solution of nonlinear differential and integral equations in the sense of Dhage and Dhage [11, 12] and Dhage *et al.* [13]. See for example, Gramas and Dugundji [16], Dhage [7, 8] and references therein. The local version of Banach fixed point theorem for Banach contraction mappings in a metric space is useful for fining the unique local solution of nonlinear equations. Since then several authors have obtained the local versions of some notable fixed point theorems involving Kaannan, Riech, Chaterjea, and Hardy-Rogers type contraction mapping in a metric space. See Trusca [19] and references therein. Similarly Ciric [4] initiated the study of local fixed point theorems for contraction mappings in a metric space with a nonunique fixed point under weaker form of continuity of mappings and completeness of the metric space which is further developed and generalized by Achari [1, 2], Pachpatte [17] and Dhage [6]. Similarly, Alam and Imdad [3] proved a basic relation theoretic fixed point theorem for relational contraction mappings in relational complete metric spaces. Later, this result is generalized to a wider classes of mappings by Dubey *et al.* [15] and Samet and Turinici [18] with fixed point. All the above fixed point results are about the existence of a unique fixed point and to the best of our knowledge there is no result on nonunique local fixed point. Very recently, the present authors in [12] initiated the study of certain contraction mappings in a relational metric space with nonunique fixed point. Therefore, it is of interest to prove some relational theoretic results concerning the existence of nonunique local fixed point of such maps on a relational metric space under weaker form of continuity and completeness conditions. Before going to the main results we give some preliminary definitions.

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Let (X, d) be a metric space and let $\mathcal{T} : X \to X$ be a mapping. Then $\mathcal{T}$-orbit of $\mathcal{T}$ at a point $x \in X$ is a set $O(x, \mathcal{T})$ in X defined by

$$O(x, \mathcal{T}) = \{x, \mathcal{T}x, \mathcal{T}^2x, \cdots, \mathcal{T}^nx, \cdots\} = \{\mathcal{T}^nx\}_{n=0}^{\infty}. \tag{1.1}$$

The mapping $\mathcal{T}$ is called $\mathcal{T}$-orbitally continuous if and only if a sequence $\{x_n\} \subseteq O(x, \mathcal{T})$ converges to x implies $\mathcal{T}x_n \to \mathcal{T}x$. X is called $\mathcal{T}$-orbitally complete if every Cauchy sequence $\{x_n\}$ in $O(x, \mathcal{T})$ converges to a point in X. It is known that every continuous mapping is $\mathcal{T}$-orbitally continuous and every complete metric space X is $\mathcal{T}$-orbitally complete, but the reverse implication need not hold. See Dhage *et al.* [14] and references therein.

A binary relation or simply relation $\mathbf{R}$ in X is a subset of the Cartesian product $X \times X$ which is full if $\mathbf{R} = X \times X$ and which is empty if $\mathbf{R} = \emptyset$. $\mathbf{R}$ is called partial relation or proper relation if $\emptyset \neq \mathbf{R} \neq X \times X$. We introduce a partial relation $\mathbf{R}$ in X and two elements x and y in X are said to be relatable if they are related to each other under the relation $\mathbf{R}$. In this case we write $x\,\mathbf{R}\,y$ or $(x, y) \in \mathbf{R}$. A metric space X together wth a partial relation $\mathbf{R}$ is called a relational metric space and it is denoted by $(X, d, \mathbf{R})$. A sequence $\{x_n\}$ of points in X is called **relatonic** or **term-wise related** if $(x_n, x_{n+1}) \in \mathbf{R}$ and **inverse relatonic** if $(x_{n+1}, x_n) \in \mathbf{R}$ for each $n \in \mathbb{N}$. We say the metric space X is $\mathbf{R}$-regular or PR-closed or simply closed with respect to the relation $\mathbf{R}$, if $\{x_n\}$ is a relatonic (resp. inverse relatonic) sequence of points in X converging to a point x^*, then $(x_n, x^*) \in \mathbf{R}$ (resp. $(x^*, x_n) \in \mathbf{R}$) for all $n \in \mathbb{N}$. A relatonic sequence $\{x_n\}$ in $(X, d, \mathbf{R})$ is called $\mathbf{R}$-Cauchy if $\lim_{m,n\to\infty} d(x_m, x_n) = 0$. Again, $(X, d, \mathbf{R})$ is called $\mathbf{R}$-complete if every $\mathbf{R}$-Cauchy sequence converges to a point in X. A mapping $\mathcal{T} : X \to X$ is called is called relational $\mathbf{R}$-continuous at a point $x \in \mathbb{R}^n$ if and only if $x_n \to x$ implies $\mathcal{T}x_n \to \mathcal{T}x$ for every sequence $\{x_n\}$ of points relatable to x in $\mathbb{R}^n$. A few details of above notions appear in Anam and Imdad [3] and Dhage [11].

2. Main Results

Before going to the main nonunique fixed point theorems we state a couple of mathematical concepts whih we need in what follows.

Definition 2.1. *A mapping $\mathcal{T} : X \to X$ is called is called $\mathcal{T}$-orbital $\mathbf{R}$-continuous at a point $x \in X$ if and only if it is $\mathbf{R}$-continuous on $O(x, \mathcal{T})$ for each $x \in X$. Again, $(X, d, \mathbf{R})$ is called $\mathcal{T}$-orbital $\mathbf{R}$-complete if $O(x, \mathcal{T})$ is $\mathbf{R}$-complete for each $x \in X$.*

Remark 2.2. *It is clear that*

$$\textit{Completeness of } X \Rightarrow \mathcal{T}\textit{-orbital completeness of } X \Rightarrow \mathcal{T}\textit{-orbital } \mathbf{R}\textit{-completeness of } X$$

and

$$\textit{Continuity of } \mathcal{T} \Rightarrow \mathcal{T}\textit{-orbital continuity of } \mathcal{T} \Rightarrow \mathcal{T}\textit{-orbital } \mathbf{R}\textit{-continuity of } \mathcal{T},$$

however the reverse implications may not hold.

Definition 2.3. *A mapping $\mathcal{T} : X \to X$ is called a* **relational R-relatone** *if it preserves the binary relation* $\mathbf{R}$ *in* X*, that is, if* $x, y \in X$ *with* $(x, y) \in \mathbf{R}$*, then* $(\mathcal{T}x, \mathcal{T}y) \in \mathbf{R}$*.*

We consider the following notations in what follows. We denote

(i) $F_{\mathcal{T}}(X) = \{x \in X : \mathcal{T}x = x\}$,
(ii) $L_{\mathcal{T}}(X) = \{x \in X : (x, \mathcal{T}x) \in \mathbf{R}\}$, and
(iii) $U_{\mathcal{T}}(X) = \{x \in X : (\mathcal{T}x, x) \in \mathbf{R}\}$.

A closed ball $B[x, r]$ in a metric spce (X, d) centered at a point $x \in X$ of radius r is defined by $B[x, r] = \{y \in X \mid d(x, y) \leq r\}$. Is it clear that a closed ball in a relationaly $\mathbf{R}$-complete metric space is $\mathbf{R}$-complete.

Theorem 2.4. *Let $\mathcal{T}$ be a $\mathcal{T}$-orbital $\mathbf{R}$-continuous and $\mathbf{R}$-relatone self-mapping of a $\mathbf{R}$-regular and $\mathcal{T}$-orbital $\mathbf{R}$-complete metric space X. Suppose that there exists a real number b such that*

$$\begin{aligned} 0 \leq \min\{d(\mathcal{T}x, \mathcal{T}y), d(x, \mathcal{T}x), d(x, \mathcal{T}x)\} \\ + b \min\{d(x, \mathcal{T}y), d(y, \mathcal{T}x)\} \leq q\, d(x, y) \end{aligned} \tag{2.1}$$

for all relatable elements $x, y \in X$, where $0 \leq q < 1$. If there is an element $x_0 \in L_{\mathcal{T}}(X)$ or $x_0 \in U_{\mathcal{T}}(X)$ such that

$$d(x_0, \mathcal{T}x_0) \leq (1 - q)r, \;\; r > 0, \tag{2.2}$$

then $\mathcal{T}$ has a local fixed point $x^* \in B[x_0, r]$ which is unique relatable provided $b > q$.

Proof. Suppose first that there exists an element $x_0 \in L_{\mathcal{T}}(X)$, then $(x_0, \mathcal{T}x_0) \in \mathbf{R}$. Define a sequence $\{x_n\}_{n=0}^{\infty} = \{\mathcal{T}^n x_0\}_{n=0}^{\infty}$ of iterations of $\mathcal{T}$ at $x_0 \in X$. Since $\mathcal{T}$ is relatone, we have $(x_n, x_{n+1}) \in \mathbf{R}$ for each n, $n = 0, 1, \ldots$. If $x_r = x_{r+1}$ for some $r \in \mathbb{N}$, then the conclusion of the theorem follows immediately. Therefore, we assume that $x_n \neq x_{n+1}$ for each $n \in \mathbb{N}$. We show that $\{x_n\}$ is a R-Cauchy sequence. Let $x = x_{n-1}$ and $y = x_n$ in the inequality (2.1), we obtain

$$d(x_n, x_{n+1}) = d(Tx_{n-1}, Tx_n) \leq q d(x_{n-1}, x_n) \tag{2.3}$$

for all $n \in \mathbb{N}$. By repeated application of the above inequality yields

$$d(x_n, x_{n+1}) \leq q^n d(x_0, x_1) \tag{2.4}$$

for each $n \in \mathbb{N}$. Next for any positive integer p, by triangle inequality, we set

$$\begin{aligned} d(x_n, x_{n+p}) &\leq d(x_n, x_{n+1}) + \ldots + d(x_{n+p-1}, x_{n+p}) \\ &\leq (q^n + \ldots + q^{n+p-1}) d(x_0, x_1) \\ &= q^n \left(\frac{1 - q^{p-1}}{1 - q} \right) d(x_0, x_1) \\ &\leq \left(\frac{q^n}{1 - q} \right) d(x_0, x_1) \\ &\to 0 \quad \text{as} \quad n \to \infty. \end{aligned} \tag{2.5}$$

This shows that $\{x_n\}$ is a **R**-Cauchy sequence in $O(x, \mathcal{T})$. Next, we show that $O(x, \mathcal{T}) \subset B[x_0, r]$. Now form inequality (2.2) we obtain

$$d(x_0, x_1) = d(x_0, \mathcal{T}x_0) \leq (1-q)r \leq r$$

and so $x_1 \in B[x_0, r]$. Similarly, by induction, we have

$$\begin{aligned} d(x_0, x_n) &\leq d(x_0, x_1) + d(x_1, x_2) + \cdots + + d(x_{n-1}, x_n) \\ &\leq d(x_0, x_1) + qd(x_0, x_1) + \cdots + q^{n-1} d(x_0, x_1) \\ &= (1 + q + q^2 + \cdots + q^{n-1}) q(x_0, x_1) \\ &= \frac{1-q^{n-1}}{1-q}(1-q)r \\ &\leq \frac{1}{1-q} \cdot (1-q)r \\ &= r \end{aligned}$$

showing that $x_n \in B[x_0, r]$ for all $n \in \mathbb{N}$. As a result we have that $O(x, \mathcal{T}) \subset B[x_0, r]$. Since $B[x_0, r]$ is a closed subset of X, it is $\mathcal{T}$-orbital **R**-complete. Hence $\{x_n\}$ converges to a point u in $B[x_0, r]$. From $\mathcal{T}$-orbital **R**-continuity of $\mathcal{T}$, it follows that

$$u = \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} \mathcal{T}x_n = \mathcal{T}\left(\lim_{n\to\infty} x_n\right) = \mathcal{T}u.$$

Thus u is a fixed point of $\mathcal{T}$. Smilarly, if there is an element $x_0 \in U_{\mathcal{T}}(X)$ satisfying (2.2), then following the similar arguments, it can be shown that $\mathcal{T}$ has a local fixed point. Further assume that $b > q$ and let $v(\neq u)$ be another relatable fixed point of $\mathcal{T}$. Then form condition (2.1) we get a contradiction whence $\mathcal{T}$ has a unique relatable fixed point. This completes the proof. □

On taking $b = 0$ in contraction condition (2.1) we obtain the following fixed point result.

Corollary 2.5. *Let $\mathcal{T}$ be a $\mathcal{T}$-orbital **R**-continuous and **R**-relatone self-mapping of a **R**-regular and $\mathcal{T}$-orbital **R**-complete metric space X. Suppose that $\mathcal{T}$ satisfies the contraction condition*

$$\min\{d(\mathcal{T}x, \mathcal{T}y), d(x, \mathcal{T}x), d(x, \mathcal{T}x)\} \leq q\, d(x, y) \tag{2.6}$$

for all relatable elements $x, y \in X$, where $0 \leq q < 1$. If there is an element $x_0 \in L_{\mathcal{T}}(X)$ or $x_0 \in U_{\mathcal{T}}(X)$ such that the condition (2.2) holds, then $\mathcal{T}$ has a local fixed point $x^* \in B[x_0, r]$.

Again taking $b = -1$ in contraction condition (2.1), we obtain the following fixed point result Ciric [4] type mappings in metric spaces.

Corollary 2.6. *Let $\mathcal{T}$ be a $\mathcal{T}$-orbital **R**-continuous and **R**-relatone self-mapping of a **R**-regular and $\mathcal{T}$-orbital **R**-complete metric space X. Suppose that $\mathcal{T}$ satisfies the contraction condition*

$$\begin{aligned} 0 \leq \min\{d(\mathcal{T}x, \mathcal{T}y), d(x, \mathcal{T}x), d(x, \mathcal{T}x)\} \\ - \min\{d(x, \mathcal{T}y), d(y, \mathcal{T}x)\} \leq q\, d(x, y) \end{aligned} \tag{2.7}$$

for all relatable elements $x, y \in X$, where $0 \leq q < 1$. If there is an element $x_0 \in L_{\mathcal{T}}(X)$ or $x_0 \in U_{\mathcal{T}}(X)$ such that the condition (2.2) holds, then $\mathcal{T}$ has a local fixed point $x^* \in B[x_0, r]$.

Remark 2.7. *It is shown in Dhage and Dhage* [12] *that the contraction condition (2.7) is an improvement upon the Ciric's sufficient contraction condition*

$$\min\{d(\mathcal{T}x,\mathcal{T}y),d(x,\mathcal{T}x),d(x,\mathcal{T}x)\} - \min\{d(x,\mathcal{T}y),d(y,\mathcal{T}x)\} \leq q\, d(x,y)$$

for the existence of a nonunique fixed point of the mapping $\mathcal{T}$.

We introduce a partial order $\preceq$ in X which is a reflexive, anti-symmetric and transitive relation, so that non-empty set X now becomes a **partially ordered metric space** and it is denoted by $(X,d,\preceq)$. A mapping $\mathcal{T}$ is called **partially continuous** at a point $x \in X$ if and only if $x_n \to x$ implies $\mathcal{T}x_n \to \mathcal{T}x$ for every sequence $\{x_n\}$ of points comparable to x in X. $\mathcal{T}$ is called partial continuous on X if it is partial continuous at each point $x \in X$. A **partially ordered complete metric space** X is one in which every monotonic Cauchy sequence converges to a point in X. The details of these partial notions appear in Dhage [8] and Dhage *et al.* [11] and references therein. We consider the following definitions in what follows.

Definition 2.8. *A mapping* $\mathcal{T} : X \to X$ *is called is called* $\mathcal{T}$*-orbital partially continuous at a point* $x \in X$ *if and only if it is partial continuous on* $O(x,\mathcal{T})$ *for each* $x \in X$. *Again,* $(X,d,\preceq)$ *is called* $\mathcal{T}$*-orbital partially complete if* $O(x,\mathcal{T})$ *is partial complete for each* $x \in X$.

Definition 2.9. *A mapping* $\mathcal{T} : X \to X$ *is called a* **monotone nondecreasing** *if it preserves the partial order* $\preceq$ *in* X, *that is, if* $x, y \in X$ *with* $x \preceq y$, *then* $\mathcal{T}x \preceq \mathcal{T}y$.

Now as a consequence of Theorem 2.4 we obtain the following corollary which is also new to the literature.

Corollary 2.10. *Let* $\mathcal{T}$ *be a* $\mathcal{T}$*-orbital partial continuous and monotone nondecreasing self-mapping of a regular and* $\mathcal{T}$*-orbital partial complete metric space* X. *Suppose that there exists a real number* b *such that*

$$0 \leq \min\{d(\mathcal{T}x,\mathcal{T}y),d(x,\mathcal{T}x),d(x,\mathcal{T}x)\} + b\min\{d(x,\mathcal{T}y),d(y,\mathcal{T}x)\} \leq q\, d(x,y) \tag{2.8}$$

for all comparable elements $x, y \in X$, *where* $0 \leq q < 1$. *If there is an element* $x_0 \in L_{\mathcal{T}}(X)$ *or* $x_0 \in U_{\mathcal{T}}(X)$ *such that the condition (2.2) holds, then* $\mathcal{T}$ *has a local fixed point* $x^* \in B[x_0, r]$. *Moreover,* $\mathcal{T}$ *has a unique comparable fixed point provided* $b > q$.

Remark 2.11. *We note that Corollary 2.10 includes two more nonunique fixed point results for the mapping satisfying the contraction conditions (2.5) and (2.6) in the partially ordered metric spaces.*

Next, we generalize Corollary 2.5 by the improvement of the contraction condition (2.5).

Theorem 2.12. *Let $\mathcal{T}$ be a $\mathcal{T}$-orbital $\mathbf{R}$-continuous and $\mathbf{R}$-relatone self-mapping of a $\mathbf{R}$-regular and $\mathcal{T}$-orbital $\mathbf{R}$-complete metric space X. Suppose that $\mathcal{T}$ satisfies the contraction condition*

$$\begin{aligned}\min\big\{d(\mathcal{T}x,\mathcal{T}y),&d(x,\mathcal{T}x),d(x,\mathcal{T}x)\big\}\\ &\leq q\ \max\big\{d(x,y)\bigcup\big[\min\{d(x,\mathcal{T}y),d(y,\mathcal{T}x)\}\big]\big\}\end{aligned}\tag{2.9}$$

for all relatable elements $x,y\in X$, where $0\leq q<1$. If there is an element $x_0\in L_{\mathcal{T}}(X)$ or $x_0\in U_{\mathcal{T}}(X)$ such that the condition (2.2) holds, then $\mathcal{T}$ has a local fixed point $x^*\in B[x_0,r]$.

Proof. Suppose first that there exists an element $x_0\in L_{\mathcal{T}}(S)$ satisfying the condition (2.2). Define a sequence $\{x_n\}_{n=0}^{\infty}=\{\mathcal{T}^n x_0\}_{n=0}^{\infty}$ of iterations of $\mathcal{T}$ at $x_0\in S$. Since $\mathcal{T}$ is relatone, we have $(x_n,x_{n+1})\in\mathbf{R}$ for each n, $n=0,1,\ldots$. If $x_r=x_{r+1}$ for some $r\in\mathbb{N}$, then the conclusion of the theorem follows immediately. Therefore, we assume that $x_n\neq x_{n+1}$ for each $n\in\mathbb{N}$. We show that $\{x_n\}$ is a R-Cauchy sequence. Let $x=x_{n-1}$ and $y=x_n$ in the inequality (2.1), we obtain

$$\begin{aligned}\min\big\{d(\mathcal{T}x_{n-1},\mathcal{T}x_n),&d(x_{n-1},\mathcal{T}x_{n-1}),d(x_n,\mathcal{T}x_n)\big\}\\ &\leq q\ \max\big\{d(x_{n-1},x_n)\bigcup\big[\min\{d(x_{n-1},\mathcal{T}x_n),d(x_n,\mathcal{T}x_{n-1})\}\big]\big\},\end{aligned}$$

$$\begin{aligned}\text{i.e.}\quad \min\big\{d(x_n,x_{n+}),&d(x_{n-1},x_n),d(x_n,x_{n+1})\big\}\\ &\leq q\ \max\big\{d(x_{n-1},x_n)\bigcup\big[\min\{d(x_{n-1},x_{n+1}),d(x_n,x_n)\}\big]\big\},\end{aligned}$$

which implies

$$\min\big\{d(x_n,x_{n+}),d(x_{n-1},x_n)\big\}\leq q\ d(x_{n-1},x_n).$$

Since $d(x_{n-1},x_n)\}\leq q\ d(x_{n-1},x_n)$ is not possible because $q<1$, we have that

$$d(x_n,x_{n+1})\leq q\ d(x_{n-1},x_n)\leq\cdots\leq q^n d(x_0,x_1)$$

for all $n\in\mathbb{N}$. Now from the above inequality it can be shown that the sequence $\{x_n\}$ is $\mathbf{R}$-Cauchy and $\{x_n\}\subset B[x_0,r]$. The rest of the proof is similar to the proof of Theorem 2.4. Hence we omit the details. □

As a consequence of Theorem 2.12, we obtain the following result which is also new the metric fixed point theory.

Corollary 2.13. *Let $\mathcal{T}$ be a $\mathcal{T}$-orbital partial continuous and monotone nondecreasing self-mapping of a regular and $\mathcal{T}$-orbital partial complete metric space X. Suppose that $\mathcal{T}$ satisfies the contraction condition*

$$\begin{aligned}\min\big\{d(\mathcal{T}x,\mathcal{T}y),&d(x,\mathcal{T}x),d(x,\mathcal{T}x)\big\}\\ &\leq q\ \max\big\{d(x,y)\bigcup\big[\min\{d(x,\mathcal{T}y),d(y,\mathcal{T}x)\}\big]\big\}\end{aligned}\tag{2.10}$$

for all comparable elements $x,y\in X$, where $0\leq q<1$. If there is an element $x_0\in L_{\mathcal{T}}(X)$ or $x_0\in U_{\mathcal{T}}(X)$ such that the condition (2.2) holds, then $\mathcal{T}$ has a local fixed point $x^*\in B[x_0,r]$.

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