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HARMONIC INDEX AND ECCENTRIC HARMONIC INDEX FOR SOME SPECIAL TYPES OF GRAPHS

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Abstract. The harmonic index $H(G)$ of a graph G is defined as $\frac{2}{d(u)+d(v)}$ for all edges uv, where $d(u)$ and $d(v)$ be the degree of the corresponding vertices u and v. Likewise, the eccentric harmonic index $H_e(G)$ of a graph G is defined as $\frac{2}{e(u)+e(v)}$ for edges uv, where $e(u)$ and $e(v)$ be the eccentricity of the corresponding vertices u and v of graph G. In this paper, we deduce $H(G)$ and $H_e(G)$ for star starbell graph, wheel starbell graph, double star starbell graph and petersenstarbell graph.

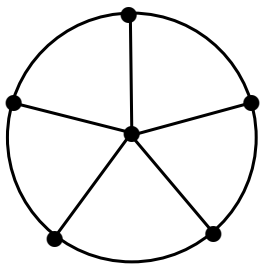
Keywords: harmonic index, eccentric harmonic index.

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1. Introduction

In this paper, $G = (V, E)$ be a simple, finite, undirected, connected graph with vertex set V and edge set E. Let $|V(G)| = \nu$ and $|E(G)| = \epsilon$, be the order and size of the graph G. For every vertex $v \in V$, the open neighbourhood $N(v)$, is the set of all vertices that are adjacent to v. The degree of vertex v in a graph G is $d(v) = |N(v)|$. A vertex of degree 1 is called pendant vertex.

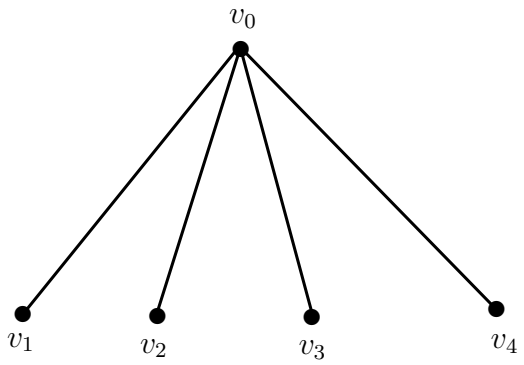
The eccentricity of a vertex u in a graph G is $e(u) = max\{d(u, v) : v \in V(G)\}$. A $W_m,\ m \geq 4$ denote the wheel on m vertices, in which a vertex is joined to all the vertices of a cycle C_{m-1}, see Figure 1.1.



W_6 – *Wheel on* 6 *vertices.*

Figure 1.1

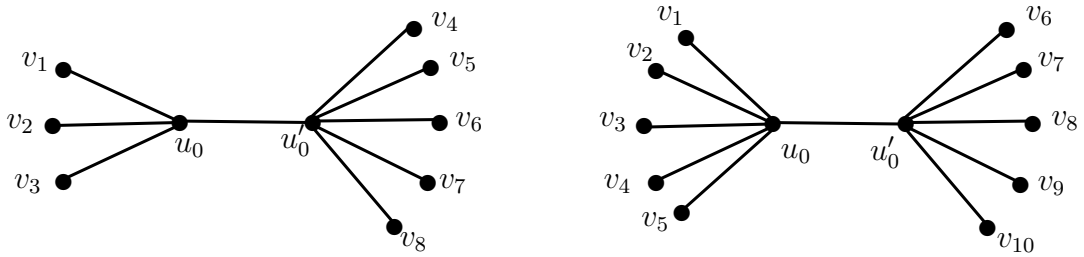
For $m \geq 2$, S_m denotes the star on m vertices in which one vertex is adjacent to all the other vertices. Also $S_m \cong K_{1,m-1}$, see Figure 1.2.



S_5 – *Star on* 5 *vertices.*

Figure 1.2

In [9] double star and bistar is defined. A double star is a graph obtained by joining an edge to center of $K_{1,n}$ and $K_{1,m}$, for $m > n \geq 2$ and it is denoted by $B_{m,n}$ (see Figure 1.3 (a) for $B_{3,5}$). If $n = m$ in the double star it is called as bistar and is denoted by $B_{n,n}$ (see Figure 1.3 (b) for $B_{5,5}$). For the graph $B_{m,n}$, the number of vertices is $\nu(B_{m,n}) = m+n+2$ and the number of edges is $\epsilon(B_{m,n}) = m+n+1$.



Double star $B_{3,5}$ *Bistar* $B_{5,5}$

(a). (b).

Figure 1.3

A topological index is a numerical quantity related to a graph. In a theoretical chemistry, topological indices is called as molecular descriptor. It is used to analyse and investigate some physicochemical properties of a molecule. Molecular descriptors play a significant role in Quantitative Structure-Property Relationship (QSPR). Also in Quantitative Structure-Activity Relationship (QSAR) investigations. At present, there are numerous topological indices which are classified by the structural properties of the graphs. In many cases harmonic index serve as one of the useful descriptor in QSPR model. It relates one or more molecular descriptors (boiling point, solubility, toxicity, etc.). Furthermore, the most widely used topological indice is the Wiener index [19] which was used by Harold Wiener in 1947, to compare the boiling points of some of the alkane isomers. For a graph G the Wiener index is defined by

$$W(G) = \sum_{u,v \in V(G)} d(u,v),$$

where $d(u, v)$ is the distance between u and v in G and the sum goes over all the unordered pairs of vertices. In literature Randić connectivity index and the Zagreb group indices are calculated using the degree of vertices. In [16] Randić connectivity index, $R(G)$ introduced by Milan Randić given by

$$R(G) = \sum_{uv \in E(G)} (d(u)d(v))^{-\frac{1}{2}}.$$

Based on this, a numerous topological indices have been proposed. In [4, 5] the first Zagreb index M_1 and the second Zagerb index M_2 are introduced as an successor of R, given by

$$M_1(G) = \sum_{i=1}^{n} d(v_i)^2 \text{ and } M_2(G) = \sum_{uv \in E(G)} d(u)d(v).$$

In [6], Fajtlowicz introduced an another topological index named harmonic index, given by

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d(u)+d(v)}.$$

In [18] Sowaity et.al., had introduced the eccentric harmonic index as similar to harmonic index, by considering the eccentricity of the vertices instead of degree of the vertices, which is given by,

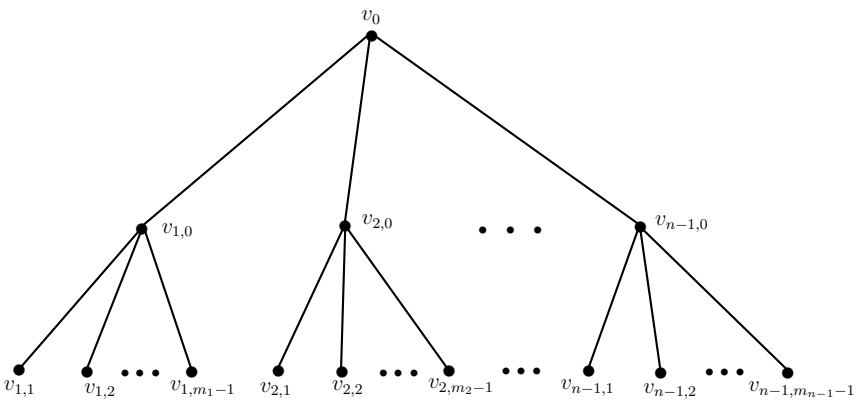
$$H_e(G) = \sum_{uv \in E(G)} \frac{2}{e(u)+e(v)}.$$

In literature various topological indices are studied by several authors in [1, 2, 8, 7, 10, 11, 12, 13, 14, 15, 17, 20, 21].

2. Star Starbell Graph and Wheel Starbell Graph

In this section we deduce the harmonic index and the eccentric harmonic index of star starbell graph and wheel starbell graph.

In [2] the star starbell graph $SS_{m_1,m_2,\ldots,m_{n-1}}$ is introduced and it is obtained from augmenting each star S_{m_i}, $i = 1, 2, \ldots, n-1$ to each pendant vertex of the star S_n, for $n \geq 3$ and $m_i \geq 2$. Here the augmented stars, S_{m_i} are called as starbell (see Figure 2.1).



Star starbell graph $SS_{m_1,m_2,\ldots,m_{n-1}}$

Figure 2.1

Here we discuss about the star starbell graph $SS_{m_1,m_2,\ldots,m_{n-1}}$ with the starbells of uniform size, that is, $m_i = m,\ 1 \le i \le n-1$.

Theorem 2.1. *For $n \ge 3$ and $m \ge 2$, the star starbell graph of uniform starbell size, $G = SS\underbrace{m, m, \ldots, m}_{(n-1)times}$ has*

(i). $H(G) = 2(n-1)\left(\frac{m-1}{m+1} + \frac{1}{m+n-1}\right)$.

(ii). $H_e(G) = 2(n-1)\left(\frac{m-1}{7} + \frac{1}{5}\right)$.

Proof. The star starbell graph $G = SS\underbrace{m, m, \ldots, m}_{(n-1)times}$ with $n \ge 3$ and $m \ge 2$, has $(n-1)m+1$ vertices and $(n-1)m$ edges. Let $V(G) = \{v_0\} \cup V_1 \cup V_2 \cup \cdots \cup V_{n-1}$ where $V_i = \{v_{i,0}, v_{i,1}, \ldots, v_{i,m-1}\}$, for $i = 1, 2, \ldots, n-1$.

(i). The harmonic index of star starbell graph G is

$$\begin{aligned} H(G) &= \sum_{i=1}^{n-1}\sum_{j=1}^{m-1} \frac{2}{d(v_{i,0}) + d(v_{i,j})} + \sum_{i=1}^{n-1} \frac{2}{d(v_0) + d(v_{i,0})}. \\ &= (n-1)(m-1)\left(\frac{2}{1+m}\right) + (n-1)\left(\frac{2}{m+n-1}\right). \end{aligned}$$

Therefore,

$$H(G) = 2(n-1)\left(\frac{m-1}{m+1} + \frac{1}{m+n-1}\right).$$

(ii). The eccentricity of the vertices in a graph G is given by

$e(v_0) = 2$,
$e(v_{i,0}) = 3$ for $i = 1, 2, \ldots, n-1$ and
$e(v_{i,j}) = 4$ for $i = 1, 2, \ldots, n-1$ and $j = 1, 2, \ldots, m-1$.

The eccentric harmonic index of G is given by

$$\begin{aligned} H_e(G) &= \sum_{i=1}^{n-1}\sum_{j=1}^{m-1} \frac{2}{e(v_{i,0}) + e(v_{i,j})} + \sum_{i=1}^{n-1} \frac{2}{e(v_0) + e(v_{i,0})}. \\ &= (n-1)(m-1)\left(\frac{2}{3+4}\right) + (n-1)\left(\frac{2}{2+3}\right). \end{aligned}$$

Therefore,

$$H_e(G) = 2(n-1)\left(\frac{m-1}{7} + \frac{1}{5}\right).$$

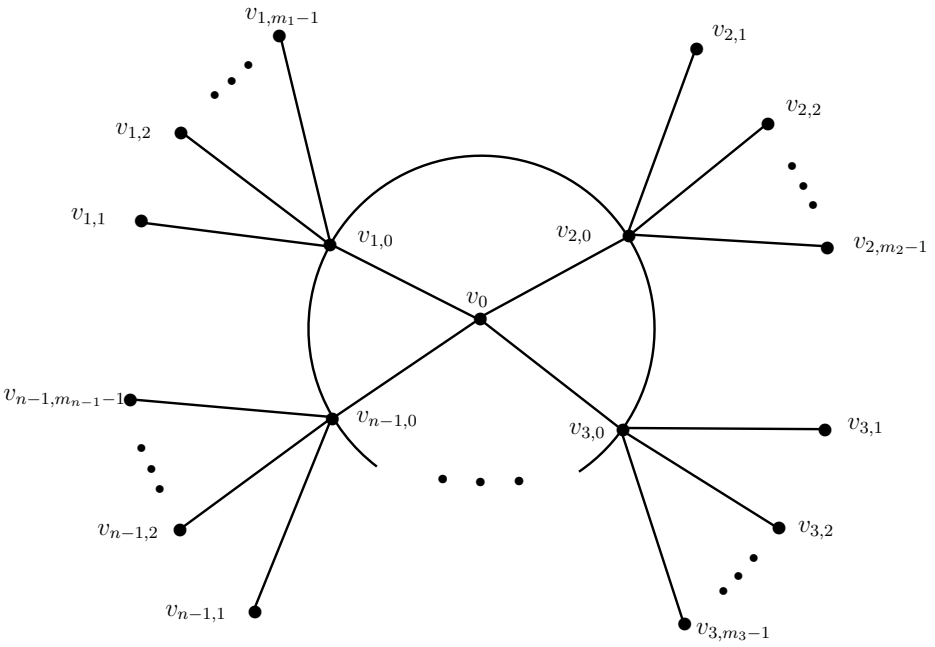
□

Note 2.1. *In Theorem 2.1 if we take $n = 2$, then $G \cong S_{m+1}$. From Theorem 2.1,*

$$H(G) = 2(n-1)\left(\frac{m-1}{m+1} + \frac{1}{m+n-1}\right).$$

If we put $n = 2$, *we have* $H(G) = \frac{2m}{m+1}$. *In* [20] *separately they have proved* $H(S_{m+1}) = \frac{2m}{m+1}$. □

In [2] the wheel starbell graph $WS_{m_1,m_2,\ldots,m_{n-1}}$ is introduced and it is obtained from wheel graph W_n and $(n-1)$ star graph S_{m_i} by merging root vertex from each S_{m_i} to the i^{th} vertex of W_n, where $m_i \geq 2,\ 1 \leq i \leq n-1$ and $n \geq 4$ (see Figure 2.2).



Wheel starbell graph $WS_{m_1,m_2,\ldots,m_{n-1}}$

Figure 2.2

Here we discuss about the wheel starbell graph $WS_{m_1,m_2,\ldots,m_{n-1}}$ with uniform starbell size, that is, $m_1 = m_2 = \cdots = m_{n-1} = m$.

Theorem 2.2. *For* $n \geq 4$ *and* $m \geq 2$, *the wheel starbell graph* $G = WS\underbrace{m, m, \ldots, m}_{(n-1)times}$ *has*

$(i).\ H(G) = (n-1)\left[\frac{2(m-1)}{m+3} + \frac{2}{n+m+1} + \frac{1}{m+2}\right].$

$$(ii).\ H_e(G) = \begin{cases} \frac{6}{5}(m-1)+3 & : n = 4 \\ (n-1)\left[\frac{2}{7}(m-1) + \frac{11}{15}\right] & : n \geq 5 \end{cases}$$

Proof. For $n \geq 4$ and $m \geq 2$, the wheel starbell graph $G = WS\underbrace{m, m, \ldots, m}_{(n-1)times}$ has $(n-1)m+1$ vertices and $(n-1)(m+1)$ edges.

(i). The harmonic index of wheel starbell graph G is

$$H(G) = \sum_{i=1}^{n-1}\sum_{j=1}^{m-1} \frac{2}{d(v_{i,0}) + d(v_{i,j})} + \sum_{i=1}^{n-1} \frac{2}{d(v_0) + d(v_{i,0})} + \sum_{i=1}^{n-1} \frac{2}{d(v_{i,0}) + d(v_{i+1,0})},$$

where the addition $i+1$ in suffix of v_{i+1} is taken modulo $n-1$.

$$= (n-1)(m-1)\left(\frac{2}{m+3}\right) + (n-1)\left(\frac{2}{n+m+1}\right) + (n-1)\left(\frac{1}{m+2}\right).$$

Therefore,

$$H(G) = (n-1)\left[\frac{2(m-1)}{m+3} + \frac{2}{n+m+1} + \frac{1}{m+2}\right].$$

(ii). **Case 1:** When $n = 4$

The eccentricity of the vertices in a graph G is

$e(v_0) = 2$,
$e(v_{i,0}) = 2$ for $1 \leq i \leq 3$ and
$e(v_{i,j}) = 3$ for $1 \leq i \leq 3$ and $j = 1, 2, \ldots, m-1$.

Then the eccentric harmonic index of G is given by

$$H_e(G) = \sum_{i=1}^{3}\sum_{j=1}^{m-1}\frac{2}{e(v_{i,0}) + e(v_{i,j})} + \sum_{i=1}^{3}\frac{2}{e(v_0) + e(v_{i,0})} + \sum_{i=1}^{3}\frac{2}{e(v_{i,0}) + e(v_{i+1,0})},$$

where the addition $i+1$ in suffix of v_{i+1} is taken modulo 3.

$$= 3(m-1)\left(\frac{2}{2+3}\right) + 3\left(\frac{2}{2+2}\right) + 3\left(\frac{2}{2+2}\right).$$

$$= \frac{6}{5}(m-1) + 3.$$

Case 2: When $n \geq 5$

The eccentricity of the vertices in a graph G is

$e(v_0) = 2$,
$e(v_{i,0}) = 3$ for $i = 1, 2, \ldots, n-1$ and
$e(v_{i,j}) = 4$ for $i = 1, 2, \ldots, n-1$ and $j = 1, 2, \ldots, m-1$.

Then the eccentric harmonic index of G is given by

$$H_e(G) = \sum_{i=1}^{n-1}\sum_{j=1}^{m-1}\frac{2}{e(v_{i,0}) + e(v_{i,j})} + \sum_{i=1}^{n-1}\frac{2}{e(v_0) + e(v_{i,0})} + \sum_{i=1}^{n-1}\frac{2}{e(v_{i,0}) + e(v_{i+1,0})},$$

where the addition $i+1$ in suffix of v_{i+1} is taken modulo $n-1$.

$$= (n-1)(m-1)\left(\frac{2}{3+4}\right) + (n-1)\left(\frac{2}{2+3}\right) + (n-1)\left(\frac{2}{3+3}\right).$$

$$= (n-1)\left[\frac{2}{7}(m-1) + \frac{11}{15}\right].$$

Therefore,

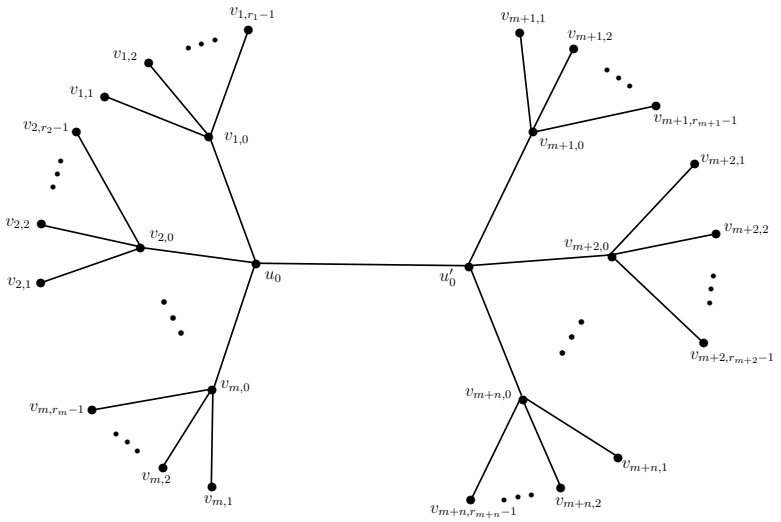
$$H_e(G) = \begin{cases} \frac{6}{5}(m-1) + 3 & : n = 4 \\ (n-1)\left[\frac{2}{7}(m-1) + \frac{11}{15}\right] & : n \geq 5 \end{cases}$$

□

3. Double Star Starbell Graph and Petersenstarbell Graph

In this section we deduce the harmonic index and eccentric harmonic index of a double star starbell graph and petersenstarbell graph.

In [3] the double star starbell graph $BSS_{r_1,r_2,\ldots,r_{m+n}}$ is introduced and it is obtained from double star $B_{m,n}$ and $(m+n)$ star graph S_{r_i} by merging root vertex of each S_{r_i} to the i^{th} leaf of $B_{m,n}$, where $r_i \geq 2,\ 1 \leq i \leq m+n$ and $m, n \geq 2$ (see Figure 3.1).



Double star starbell graph $BSS_{r_1,r_2,\ldots,r_{m+n}}$

Figure 3.1

Here we discuss about the double star starbell graph $BSS_{r_1,r_2,\ldots,r_{m+n}}$ with uniform size of star graph, that is, $r_1 = r_2 = \cdots = r_{m+n} = r$.

Theorem 3.1. *For $m, n \geq 2$ and $r \geq 2$, the double star starbell graph $G = BSS_{\underbrace{r, r, \ldots, r}_{(m+n)times}}$ has*

$(i).\ H(G) = \frac{2(m+n)(r-1)}{r+1} + \frac{2m}{m+r+1} + \frac{2n}{n+r+1} + \frac{2}{m+n+2}$.

$(ii).\ H_e(G) = \frac{2(m+n)(r-1)}{9} + \frac{2(m+n)}{7} + \frac{1}{3}$.

Proof. The double star starbell graph $G = BSS_{\underbrace{r, r, \ldots, r}_{(m+n)times}}$ has $|V(G)| = (m+n)r + 2$ and $|E(G)| = (m+n)r + 1$.

(i). The harmonic index of double star starbell graph G is

$$\begin{aligned} H(G) &= \sum_{i=1}^{m+n}\sum_{j=1}^{r-1}\frac{2}{d(v_{i,0})+d(v_{i,j})} + \sum_{i=1}^{m}\frac{2}{d(u_0)+d(v_{i,0})} \\ &\quad + \sum_{i=m+1}^{m+n}\frac{2}{d(u_0')+d(v_{i,0})} + \frac{2}{d(u_0)+d(u_0')}. \\ &= (m+n)(r-1)\left(\frac{2}{r+1}\right) + m\left(\frac{2}{m+1+r}\right) \\ &\quad + n\left(\frac{2}{n+1+r}\right) + \frac{2}{m+1+n+1}. \end{aligned}$$

Therefore,

$$H(G) = \frac{2(m+n)(r-1)}{r+1} + \frac{2m}{m+r+1} + \frac{2n}{n+r+1} + \frac{2}{m+n+2}.$$

(ii). The eccentricity of the vertices in a graph G is

$$\begin{aligned} e(u_0) &= e(u_0') = 3, \\ e(v_{i,0}) &= 4 \text{ for } i = 1,2,\ldots,m+n \text{ and} \\ e(v_{i,j}) &= 5 \text{ for } i = 1,2,\ldots,m+n \text{ and } j = 1,2,\ldots,r-1. \end{aligned}$$

Then the eccentric harmonic index of G is given by

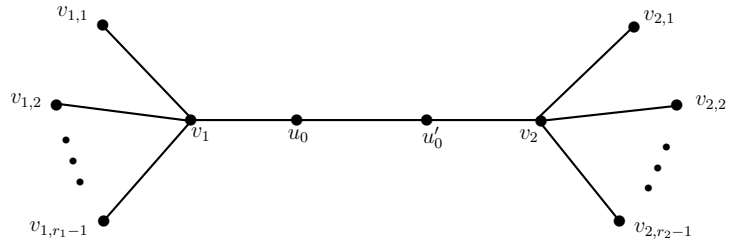
$$\begin{aligned} H_e(G) &= \sum_{i=1}^{m+n}\sum_{j=1}^{r-1}\frac{2}{e(v_{i,0})+e(v_{i,j})} + \sum_{i=1}^{m}\frac{2}{e(u_0)+e(v_{i,0})} \\ &\quad + \sum_{i=m+1}^{m+n}\frac{2}{e(u_0')+e(v_{i,0})} + \frac{2}{e(u_0)+e(u_0')}. \\ &= (m+n)(r-1)\left(\frac{2}{4+5}\right) + m\left(\frac{2}{3+4}\right) + n\left(\frac{2}{3+4}\right) + \frac{2}{3+3}. \end{aligned}$$

Therefore,

$$H_e(G) = \frac{2(m+n)(r-1)}{9} + \frac{2(m+n)}{7} + \frac{1}{3}.$$

□

Remark 3.2. *In Theorem* 3.1*, if we replace* $m, n = 1$*, we get a particular graph* G*, as shown in the Figure* 3.2.



G

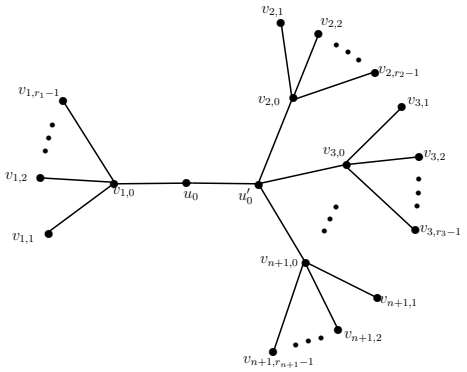
Figure 3.2

For the above graph G, the same proof of Theorem 3.1 *works by assigning the values $m, n = 1$. We have,*

$$H(G) = \frac{4(r-1)}{r+1} + \frac{4}{r+2} + \frac{1}{2} \text{ and}$$
$$H_e(G) = \frac{4(r-1)}{9} + \frac{19}{21}.$$

□

Remark 3.3. *In Theorem* 3.1*, if we replace any one of the values of m, n as* 1*, without loss of generality we take $m = 1$ and $n \geq 2$ then we get a particular graph G as shown in Figure* 3.3



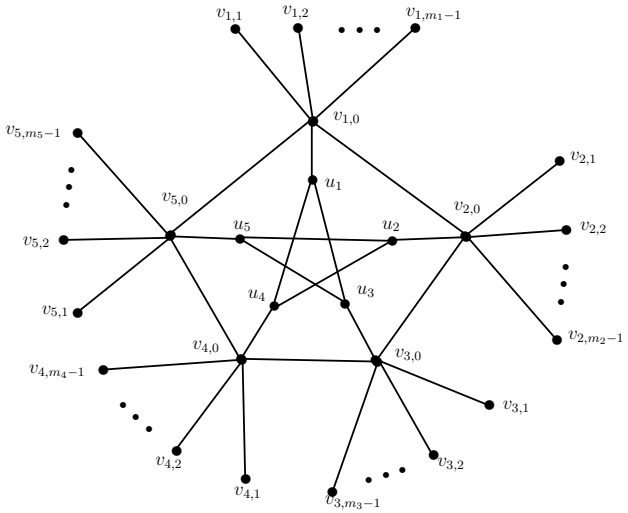
G

Figure 3.3

As similar to above Remark 3.2*, if we replace the value of $m = 1$ and $n \geq 2$ in Theorem* 3.1 *it gives the $H(G)$ and $H_e(G)$.*

□

In [3] the petersenstarbell graph $PS_{m_1,m_2,\ldots,m_5}$ is introduced and it is obtained from Petersen graph and star graph S_{m_i} by merging root of each S_{m_i} to each of outer 5 vertices of Petersen graph, as shown in the Figure 3.4, where $m_i \geq 2$ and $1 \leq i \leq 5$.



Petersenstarbell graph $PS_{m_1,m_2,\ldots,m_5}$

Figure 3.4

Here we discuss the petersenstarbell graph $PS_{m_1,m_2,\ldots,m_5}$ with uniform size starbell, that is, $m_1 = m_2 = \cdots = m_5 = m$.

Theorem 3.4. *For $m \geq 2$, the petersenstarbell graph $G = PS_{\underbrace{m, m, \ldots, m}_{5 \text{ times}}}$ have*

(i). $H(G) = 10\left(\frac{m-1}{m+3} + \frac{1}{m+5}\right) + 5\left(\frac{1}{m+2} + \frac{1}{3}\right)$.

(ii). $H_e(G) = \frac{10(m-1)}{7} + 5$.

Proof. The petersenstarbell graph $G = PS_{\underbrace{m, m, \ldots, m}_{5 \text{ times}}}$ has $5(m+1)$ vertices and $5(m+2)$ edges.

(i). The harmonic index of petersenstarbell graph G is

$$H(G) = \sum_{i=1}^{5}\sum_{j=1}^{m-1} \frac{2}{d(v_{i,0}) + d(v_{i,j})} + \sum_{i=1}^{5} \frac{2}{d(v_{i,0}) + d(v_{i+1,0})} + \sum_{i=1}^{5} \frac{2}{d(u_i) + d(v_{i,0})} + \sum_{i=1}^{5} \frac{2}{d(u_i) + d(u_{i+2})},$$

where the addition $i+1$ and $i+2$ in the suffices are taken modulo 5.

$$= 5(m-1)\left(\frac{2}{m+2+1}\right) + 5\left(\frac{2}{m+2+m+2}\right) + 5\left(\frac{2}{3+m+2}\right) + 5\left(\frac{2}{3+3}\right).$$

Therefore,

$$H(G) = 10\left(\frac{m-1}{m+3} + \frac{1}{m+5}\right) + 5\left(\frac{1}{m+2} + \frac{1}{3}\right).$$

(ii). The eccentricity of the vertices in a graph G is

$e(u_i) = e(v_{i,0}) = 3$ for $i = 1, 2, \ldots, 5$ and

$e(v_{i,j}) = 4$ for $i = 1, 2, \ldots, 5$.

Then the eccentric harmonic index of G is given by

$$H_e(G) = \sum_{i=1}^{5}\sum_{j=1}^{m-1} \frac{2}{e(v_{i,0}) + e(v_{i,j})} + \sum_{i=1}^{5} \frac{2}{e(v_{i,0}) + e(v_{i+1,0})} + \sum_{i=1}^{5} \frac{2}{e(u_i) + e(v_{i,0})} + \sum_{i=1}^{5} \frac{2}{e(u_i) + e(u_{i+2})},$$

where the addition $i+1$ and $i+2$ in the suffices are taken modulo 5.

$$= 5(m-1)\left(\frac{2}{3+4}\right) + 5\left(\frac{2}{3+3}\right) + 5\left(\frac{2}{3+3}\right) + 5\left(\frac{2}{3+3}\right).$$

Therefore,

$$H_e(G) = \frac{10(m-1)}{7} + 5.$$

□

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