Journal of Indian Acad. Math. ISSN: 0970-5120
Vol. 48, No. 1 (2026) pp. 121–138.

FIXED POINT RESULTS FOR (α-ψ)-RATIONAL TYPE CONTRACTIVE CONDITIONS IN PARTIAL METRIC SPACES

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Abstract. In this paper, we present the concept of (α, ψ)-rational type contractive mapping and provide sufficient condition for the existence and uniqueness of fixed point in partial metric space. The concept of partial metric spaces, introduced by Matthews in 1994, has proven to be a valuable generalization of metric spaces, allowing for a broader range of applications in various fields of mathematics and its applications. We also deduce several results as consequences and examples are also provided to validate the results obtained. Our investigation contributes to the ongoing research on fixed point theory in partial metric spaces by introducing and analyzing (α, ψ)-rational type contractive conditions. The developed theory not only enriches our understanding of fixed point phenomena in non-standard metric spaces but also opens avenues for future research in related areas of mathematics and its applications. Our results extend and generalize existing fixed point theorems in the literature, providing a deeper understanding of the applicability of such contractive conditions in the context of partial metric spaces.

Keywords: Rational type contraction, partial metric space, fixed point theorem, $\alpha-$admissible mapping.

2010 AMS Subject Classification: 47H10, 54H25.

1. Introduction

The study of fixed points and fixed point theorems have been an active area of research for the last five decades for many researchers as fixed points and their related results have always been a major theoretical tools in fileds as widely apart as Differential Equation, Topology, Economics, Game theory, Dynamics, Optical control, Analysis and Functional Analysis. In 1994, the notion of partial metric space was introduced by Matthews to study the denotationnal semantics of dataflow nertworks. The partial metric spaces are acknowledged for their significant role in the development of models in the theory of computation. Matthews[22] also obtained the partial metric version of Banach contraction theorem. The concept of α-admissible mappings and (α, ψ)- rational type contraction mappings are introduced by samet et. el.[32] and this concept is extended by different reseachers (see [5, 6, 7, 8, 9, 10, 11, 13]). In 2015, Hamid et.al [13] used (α, ψ)-rational type contraction mappings to establish the fixed point theorem in generalized metric spaces. This work examines a further generalization of a metric spaces, known as a partial metric spaces, which extends the concept of a normal metric space introduced by Frchet in 1906. This concept was presented by Mathews[21]. Mathews[21] established the partial

metric contraction theorem, which renders the partial metric function significant in fixed point theory. Oltra and Valero [24] extended the results of Mathews in the setting of complete partial metric spaces. Subsequently, significant findings in partial metric space were developed as enhancements and generalizations of previously established results in the literature (see [5, 6, 7, 8, 9, 11, 12, 18, 30, 32]). In this work, we establish some theorems for the existence and uniqueness of a fixed point in partial metric space along with the concept of (α, ψ)- rational type contractive mapping using auxiliary functions.

2. Preliminaries

We start with the following definition. Let X be a nonempty set and $\xi : X \times X \to [0, \infty)$ satisfy the following conditions:

(i) $\xi(u, v) = 0$ if and only if $u = v$ for all $u, v \in X$.
(ii) $\xi(u, v) = d(v, u)$ for all $u, v \in X$.
(iii) $\xi(u, v) \leq d(u, w) + d(w, v)$ for all $u, v, w \in X$.

Then the map ξ is called metric on X and the pair (X, ξ) is called a metric space.

Definition 2.1. [21] *Let X be a nonempty set and let $\xi : X \times X \to [0, \infty)$ satisfies:*

(i) $\xi(u, v) \geq 0 \ \forall \ u, v \in X$ *and* $\xi(u, v) = \xi(u, u) = \xi(v, v)$ *if and only if* $u = v$.
(ii) $\xi(u, u) \leq \xi(u, v) \ \forall \ u, v \in X$.
(iii) $\xi(u, v) = \xi(v, u) \ \forall \ u, v \in X$.
(iv) $\xi(u, v) \leq \xi(u, w) + \xi(w, v) - \xi(w, w) \ \forall \ u, v, w \in X$.

Consequently, the map ξ is referred to as a partial metric on X, and the pair (X, ξ) is designated as a partial metric space.

Example 2.2. [21] *Let $X = [0, \infty)$ and $\xi : X \times X \to [0, \infty)$ given by*

$$\xi(u, v) = \max\{u, v\}, \ \forall \ u, v \in X.$$

Then (X, ξ) is a partial metric space.

It is reasonable to introduce fundamental topological notions-such as convergence of a sequence, the Cauchy sequence criterion, continuity of functions, and the completeness of a topological space-within the context of partial metric spaces.

Definition 2.3. [19, 21, 31, 32] *Let (X, ξ) be a partial metric space. We say that*

(i) A sequence $\{u_n\}$ converges to the limit u if $\xi(u, u) = \lim_{n \to \infty} \xi(u, u_n)$.
(ii) A sequence $\{u_n\}$ is Cauchy if $\lim_{n,m \to \infty} \xi(u_n, u_m)$ exists and finite.
(iii) A partial metric space (X, ξ) is complete if each Cauchy sequence $\{u_n\}$ converges to a point $u \in X$ such that $\xi(u, u) = \lim_{n,m \to \infty} \xi(u_n, u_m)$.
(iv) A mapping $F : X \to X$ is continuous at a point $u_0 \in X$ if for each $\epsilon > 0$, there exists $\delta > 0$ such that $F(B_\xi(u_0, \delta)) \subseteq B_\xi(Fu_0, \epsilon)$.

For what follows, we shall recall the following lemma that can be derived easily (see [21]).

Lemma 2.4. *Let ξ be a partial metric on a non-empty set X and ρ_ξ be the corresponding standard metric space on the same set X. Then*

(a) A sequence $\{u_n\}$ is fundamental in the framework of a partial metric (X, ξ) if and only if it is a fundamental sequence in the setting of the corresponding standard metric space (X, ρ_ξ).

(b) A partial metric space (X, ξ) is complete if and only if the corresponding standard metric space (X, ρ_ξ) is complete. Moreover

$$\lim_{n\to\infty \rho_\xi(u,u_n)} = 0 \Leftrightarrow \xi(u,u) = \lim_{n\to\infty} \xi(u,u_n) = \lim_{n,m\to\infty} \xi(u_n,u_m)$$

(c) If $u_n \to u$ as $n \to \infty$ in a partial metric space (X, ξ) with $\xi(u, u) = 0$, then we have

$$\lim_{n\to\infty} \xi(u_n, v) = \xi(u, v), for\ everey\ v \in X$$

Definition 2.5. [30] *Consider a nonempty set X and let $\tau : X \to X$ and $\alpha : X \times X \to [0, \infty)$ be two mappings. We define τ as an α-admissible mapping if $\alpha(u, v) \geq 1$ implies $\alpha(\tau u, \tau v) \geq 1$ for all $u, v \in X$.*

Definition 2.6. [13] *Let (X, ξ) be a partial metric space and $\alpha : X \times X \to [0, \infty)$.$X$ is defined as a α-regular partial metric space if, for a sequence $\{u_n\}$ in X where $\{u_n \to u\}$ and $\alpha(u_n, u_{n+1}) \geq 1$, there exists a subsequence $\{u_{n_k}\}$ of $\{u_n\}$ such that $\alpha(u_{n_k}, x) \geq 1$ for all $k \in \mathbb{N}$.*

3. Fixed point results

In this section, we employ (α, ψ)-rational type contraction mappings to establish a unique fixed point theorem in the context of partial metric spaces. The current study shall analyze the auxiliary function ψ as defined by Alsulami et al. [3].

Consider Ψ denote the family of functions $\psi : [0, \infty) \to [0, \infty)$ that satisfy the following properties:

(i) ψ is both upper semi-continuous and strictly increasing.
(ii) Any value of $t > 0$ will lead to $[\psi^n(t)]_{n\in N}$coverges to 0 as $n \to \infty$.
(iii) For all $t > 0$, $\psi(t) < t$.

Definition 3.1. *Let (X, ξ) be a partial metric space and $\alpha : X \times X \to [0, \infty)$. A self-mapping $\tau : X \to X$, is characterized as (α, ψ)-rational type-I contractive mapping, is defined by the existence of a function $\psi \in \Psi$ such that for all $u, v \in X$, the following condition holds:*

$$\alpha(u, v)\xi(\tau u, \tau v) \leq \psi(\Omega(u, v))$$

where

$$\begin{aligned}\Omega(u,v) &= \max\Big\{\xi(u,v),\xi(u,\tau u),\xi(v,Tv),\\ &\quad \frac{\xi(u,v)+p(u,\tau u)}{2},\frac{\xi(u,\tau u)+\xi(v,\tau v)}{2}\\ &\quad \frac{\xi(u,\tau u)\xi(v,\tau v)}{1+\xi(u,v)},\frac{\xi(u,\tau u)\xi(v,\tau v)}{1+\xi(\tau u,\tau v)}\Big\}\end{aligned} \tag{1}$$

Theorem 3.2. *Consider a complete partial metric space $(X,\xi)$and let $\tau : X \to X$ be a self mapping and*

$$\alpha : X \times X \to [0,\infty)$$

be a given function. Assume that each of the following conditions holds:

(i) The mapping τ is α-admissible.
(ii) τ is an (α,ψ)-rational type-I contractive mapping.
(iii) there exists $u_0 \in X$ such that $\alpha(u_0,\tau u_0) \geq 1$, $\alpha(u_0,\tau^2 u_0) \geq 1$.
(iv) Either X exhibits α-regular or τ is continuous.

Consequently, τ possesses a fixed point $u^* \in X$, and the sequence $\{\tau^n u_0\}$ converges to u^*. Moreover, if for any $u,v \in F(\tau)$, it holds that $\alpha(u,v) \geq 1$. Then, τ possesses a unique fixed point in X.

Proof. Let $u_0 \in X$ such that $\alpha(u_0,\tau u_0) \geq 1$ and $\alpha(u_0,\tau^2 u_0) \geq 1$. We formulate the sequence, $\{u_n\} \in X$ as $u_n = \tau^n u_0 = \tau u_{n-1}$ for all $n \in \mathbb{N}$.

By definition, u_n is a fixed point of τ only if $u_{n_0} = u_{n_0+1}$ for every $n_0 \in \mathbb{N}$. Therefore, we will assume that for every $n \in \mathbb{N}$, $u_n \neq u_{n+1}$.

Given that T is α-admissible

$$\alpha(u_0,\tau u_0) = \alpha(u_0,u_1) \geq 1$$

implies

$$\alpha(\tau u_0,\tau u_1) = \alpha(u_1,u_2) \geq 1$$

therefore,

$$\alpha(\tau u_1,\tau u_2) = \alpha(u_2,u_3) \geq 1$$

$$\vdots$$

Thus, by induction, we obtain

$$\alpha(u_n,u_{n+1}) \geq 1$$

for all $n \geq 0$.

Using analogous reasoning, as

$$\alpha(u_0,\tau^2 u_0) \geq 1,$$

we have

$$\alpha(u_0,u_2) = \alpha(u_0,\tau^2 u_0) \geq 1,$$
$$\alpha(\tau u_0,\tau u_2) = \alpha(u_1,u_3) \geq 1$$

Using induction, we obtain

$$\alpha(u_n, u_{n+2}) \geq 1$$

for all $n \geq 0$. Let $u = u_n$ and $v = u_{n+1}$ in (1). We obtain

$$\xi(u_{n+1}, u_{n+2}) = \xi(\tau u_n, \tau u_{n+1}) \leq \alpha(u_n, u_{n+1})\xi(\tau u_n, \tau u_{n+1}) \leq \psi(\Omega(u_n, u_{n+1})$$

in which,

$$\begin{aligned}
\Omega(u_n, u_{n+1}) &= \max\Big\{\xi(u_n, u_{n+1}), \xi(u_n, \tau u_n), \xi(u_{n+1}, \tau u_{n+1}), \\
&\quad \frac{\xi(u_n, u_{n+1}) + \xi(u_n, \tau u_n)}{2}, \frac{\xi(u_n, \tau u_n) + \xi(u_{n+1}, \tau u_{n+1})}{2}, \\
&\quad \frac{\xi(u_n, \tau u_n)\xi(u_{n+1}, \tau u_{n+1})}{1 + \xi(u_n, u_{n+1})}, \frac{\xi(u_n, \tau u_n)\xi(u_{n+1}, \tau u_{n+1})}{1 + \xi(\tau u_n, \tau u_{n+1})}\Big\} \\
&= \max\Big\{\xi(u_n, u_{n+1}), \xi(u_n, u_{n+1}), \xi(u_{n+1}, u_{n+2}), \\
&\quad \frac{\xi(u_n, u_{n+1}) + \xi(u_n, u_{n+1})}{2}, \frac{\xi(u_n, u_{n+1}) + \xi(u_{n+1}, u_{n+2})}{2}, \\
&\quad \frac{\xi(u_n, u_{n+1})\xi(u_{n+1}, u_{n+2})}{1 + \xi(u_n, u_{n+1})}, \frac{\xi(u_n, u_{n+1})p(u_{n+1}, u_{n+2})}{1 + \xi(u_{n+1}, u_{n+2})}\Big\} \\
&= \max\Big\{\xi(u_n, u_{n+1}), \xi(u_{n+1}, u_{n+2}), \\
&\quad \frac{\xi(u_n, u_{n+1}) + \xi(u_{n+1}, u_{n+2})}{2}\Big\} \qquad (2)
\end{aligned}$$

Given that,

$$\frac{\xi(u_n, u_{n+1})\xi(u_{n+1}, u_{n+2})}{1 + \xi(u_n, u_{n+1})} \leq \xi(u_{n+1}, u_{n+2})$$

and

$$\frac{\xi(u_n, u_{n+1})\xi(u_{n+1}, u_{n+2})}{1 + \xi(u_{n+1}, u_{n+2})} \leq \xi(u_n, u_{n+1})$$

If for a given n, we have

$$\Omega(u_n, u_{n+1}) = \xi(u_{n+1}, u_{n+2})$$

then

$$\begin{aligned}
\xi(u_{n+1}, u_{n+2}) &\leq \psi(\Omega(u_n, u_{n+1})) \\
&= \psi(\xi(u_{n+1}, u_{n+2})) \qquad (3) \\
&< \xi(u_{n+1}, u_{n+2}))
\end{aligned}$$

this is impossible.

If for a given n, we get

$$\Omega(u_n, u_{n+1}) \leq \max\Big\{\xi(u_n, u_{n+1}), \xi(u_{n+1}, u_{n+2})\Big\}$$

If

$$\max\Big\{\xi(u_n, u_{n+1}), \xi(u_{n+1}, u_{n+2})\Big\} = \xi(u_{n+1}, u_{n+2})$$

Then,

$$\begin{aligned}\Omega(u_n, u_{n+1}) &\leq \xi(u_{n+1}, u_{n+2}) \\ \xi(u_{n+1}, u_{n+2}) &\leq \psi(\Omega(u_n, u_{n+1})) \\ &\leq \psi(\xi(u_{n+1}, u_{n+2})) \\ &\leq \xi(u_{n+1}, u_{n+2})\end{aligned}$$

which is impossible.

Therefore,

$$\Omega(u_n, u_{n+1}) = \xi(u_n, u_{n+1}),$$

for all $n \in \mathbb{N}$

$$\begin{aligned}\xi(u_{n+1}, u_{n+2}) &\leq \psi(\Omega(u_n, u_{n+1}) \\ &= \psi(\xi(u_n, u_{n+1})).\end{aligned}$$

We deduce that

$$\xi(u_{n+1}, u_{n+2}) < \xi(u_n, u_{n+1}), \forall\ n \in N \tag{4}$$

for all $n \in \mathbb{N}$. From the two inequalities shown above, we can derive that

$$\xi(u_{n+1}, u_{n+2}) \leq \psi^n(\xi(u_0, u_1)),$$

$\forall\ n \in \mathbb{N}$. It clearly shows that

$$\lim_{n\to\infty} \xi(u_{n+1}, u_{n+2}) = 0 \tag{5}$$

Take into account (1) with $u = u_{n-1}$ and $v = v_{n+1}$. We obtain

$$\begin{aligned}\xi(u_n, u_{n+2}) &= \xi(\tau u_{n-1}, \tau u_{n+1}) \\ &\leq \alpha(u_{n-1}, u_{n+1})\xi(\tau u_{n-1}, \tau u_{n+1}) \\ &\leq \psi(\Omega(u_{n-1}, u_{n+1})) \qquad (6)\end{aligned}$$

in which,

$$\begin{aligned}\Omega(u_{n-1},u_{n+1}) &= \max\Big\{\xi(u_{n-1},u_{n+1}),\xi(u_{n-1},\tau u_{n-1}),\xi(u_{n+1},\tau u_{n+1}),\\ &\quad \frac{\xi(u_{n-1},u_{n+1})+\xi(u_{n-1},\tau u_{n-1})}{2},\\ &\quad \frac{\xi(u_{n-1},\tau u_{n-1})+\xi(u_{n+1},\tau u_{n+1})}{2},\\ &\quad \frac{\xi(u_{n-1},\tau u_{n-1})\xi(u_{n+1},\tau u_{n+1})}{1+\xi(u_{n-1},u_{n+1})},\\ &\quad \frac{\xi(u_{n-1},\tau u_{n-1})\xi(u_{n+1},\tau u_{n+1})}{1+\xi(Tu_{n-1},\tau u_{n+1})}\Big\}\\ &= \max\Big\{\xi(u_{n-1},u_{n+1}),\xi(u_{n-1},u_n),\xi(u_{n+1},u_{n+2}),\\ &\quad \frac{\xi(u_{n-1},u_{n+1})+\xi(u_{n-1},u_n)}{2},\frac{\xi(u_{n-1},u_n)+\xi(u_{n+1},u_{n+2})}{2},\\ &\quad \frac{\xi(u_{n-1},u_n)\xi(u_{n+1},u_{n+2})}{1+\xi(u_{n-1},u_{n+1})},\\ &\quad \frac{\xi(u_{n-1},u_n)p(u_{n+1},u_{n+2})}{1+\xi(u_n,u_{n+2})}\Big\} \qquad (7)\end{aligned}$$

From (4), it is evident that

$$\xi(u_{n+1},u_{n+2})<\xi(u_{n-1},u_n)$$

Define

$$s_n=\xi(u_n,u_{n+2}),t_n=\xi(u_n,u_{n+1})$$

Then,

$$\begin{aligned}\Omega(u_{n-1},u_{n+1}) &= \max\Big\{s_{n-1},t_{n-1},t_{n+1},\frac{s_{n-1}+s_{n-1}}{2},\\ &\quad \frac{t_{n-1}+t_{n+1}}{2},\frac{t_{n-1}t_{n+1}}{1+a_{n-1}},\frac{t_{n-1}t_{n+1}}{1+a_n}\Big\}\end{aligned}$$

If

$$\Omega(u_{n-1},u_{n+1})=t_{n-1}\ or\ t_{n+1}\ or\ \frac{t_{n-1}+t_{n+1}}{2}\ or\ \frac{t_{n-1}t_{n+1}}{1+s_{n-1}}\ or\ \frac{t_{n-1}b_{n+1}}{1+a_n},$$

then, by utilizing (6) and the upper semi-continuity of ψ, we can obtain the following: $\lim sup$ as $n\to\infty$ in (5)

$$\begin{aligned}0 &\le \lim_{n\to\infty}\sup s_n\le\lim_{n\to\infty}\sup\psi(\Omega(u_{n-1},u_{n+1}))\\ &= \psi(\lim_{n\to\infty}\sup\Omega(u_{n-1},u_{n+1}))\\ &= \psi(0)=0.\end{aligned}$$

Consequently,

$$\lim_{n\to\infty}u_n=\lim\xi(u_n,u_{n+2})=0$$

If

$$\begin{aligned}\Omega(u_{n-1}, u_{n+1}) &= \frac{1}{2}(s_{n-1} + t_{n-1}) \\ &= \frac{1}{2}[\xi(u_{n-1}, u_{n+1}) + \xi(u_{n-1}, u_n)] \\ &< \xi(u_{n-1}, u_{n+1}) + \xi(u_{n-1}, u_n) \\ &\leq \xi(u_{n-1}, u_n) + \xi(u_n, u_{n+1}) - \xi(u_n, u_n) + p(u_{n-1}, u_n) \\ &\leq \xi(u_{n-1}, u_n) + \xi(u_n, u_{n+1}) + \xi(u_{n-1}, u_n)\end{aligned}$$

Again, using the upper semi-continuity of ψ and (6), we can obtain the following:

$$\lim sup = n \to \infty$$

in (5)

$$\begin{aligned}0 \leq \lim_{n\to\infty} \sup s_n &\leq \lim_{n\to\infty} \sup \psi(\Omega(u_{n-1}, u_{n+1})) \\ &= \psi(\lim_{n\to\infty} \sup \Omega(u_{n-1}, u_{n+1})) \\ &= \psi(0) = 0.\end{aligned}$$

Hence

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \sigma(u_n, u_{n+2}) = 0$$

If

$$\Omega(u_{n-1}, u_{n+1}) = s_{n-1}$$

the equation (6) follows

$$s_n \leq \psi(s_{n-1}) < s_{n-1}.$$

The sequence $\{s_n\}$ is characterized as positive monotone decreasing and as a result, it converges to a point q. Suppose that $q > 0$. Subsequently, we obtain the following:

$$\begin{aligned}q = \lim_{n\to\infty} \sup s_n &\leq \lim_{n\to\infty} \sup \psi(s_{n-1}) \\ &= \psi(\lim_{n\to\infty} \sup s_{n-1}) \\ &= \psi(q) < q.\end{aligned}$$

which is a contradiction. Consequently,

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \xi(u_n, u_{n+2}) = 0. \tag{8}$$

We will now show that $u_n \neq u_m$ for all $n \neq m$. The converse is also true; suppose that for any $m, n \in \mathbb{N}$ where $n \neq m, x_n = x_m$. We may assume that $m > n + 1$ without losing generality since $p(u_k, u_{k+1}) > 0$ for every $k \in \mathbb{N}$.

Again, substitute

$$u = u_n = u_m$$

and

$$v = u_{n+1} = u_{m+1}$$

in (1) which results in

$$\begin{aligned}\xi(u_n,u_{n+1}) &= \xi(u_n,\tau u_n)=\xi(u_m,\tau u_m)=\xi(\tau u_{m-1},\tau u_m)\\ &\leq \alpha(u_{m-1},u_m)\xi(\tau u_{m-1},\tau u_m)\leq \psi(\Omega(u_{m-1},u_m))\end{aligned} \tag{9}$$

where,

$$\begin{aligned}\Omega(u_{m-1},u_m) &= \max\Big\{\xi(u_{m-1},u_m),\xi(u_{m-1},\tau u_{m-1}),\xi(u_m,\tau u_m),\\ &\quad \frac{\xi(u_{m-1},u_m)+\xi(u_{m-1},\tau u_{m-1})}{2},\frac{\xi(u_{m-1},\tau u_{m-1})+\xi(u_m,Tu_m)}{2},\\ &\quad \frac{\xi(u_{m-1},\tau u_{m-1})\xi(u_m,Tu_m)}{1+\xi(u_{m-1},u_m)},\\ &\quad \frac{\xi(u_{m-1},\tau u_{m-1})\xi(u_m,\tau u_m)}{1+\xi(\tau u_{m-1},\tau u_m)}\Big\}\\ &= \max\Big\{\xi(u_{m-1},u_m),\xi(u_{m-1},u_m),\xi(u_m,u_{m+1}),\\ &\quad \frac{\xi(u_{m-1},u_m)+\xi(u_{m-1},u_m)}{2},\frac{\xi(u_{m-1},u_m)+\xi(u_m,u_{m+1})}{2},\\ &\quad \frac{\xi(u_{m-1},u_m)\xi(u_m,u_{m+1})}{1+\xi(u_{m-1},u_m)},\\ &\quad \frac{\xi(u_{m-1},u_m)\xi(u_m,u_{m+1})}{1+\xi(u_m,u_{m+1})}\Big\}\\ &= \max\Big\{\xi(u_{m-1},u_m),\xi(u_m,u_{m+1}),\frac{\xi(u_{m-1},u_m)+\xi(u_m,u_{m+1})}{2}\Big\}\end{aligned}$$

If

$$\Omega(u_{m-1},u_m)=p(u_{m-1},u_m)$$

then, (9) implies

$$\xi(u_n,u_{n+1})\leq\psi(\xi(u_{m-1},u_m))\leq\psi^{m-n}(\xi(u_n,u_{n+1})) \tag{10}$$

If

$$\Omega(u_{m-1},u_m)=p(u_m,u_{m+1})$$

from (9), we get

$$\xi(u_n,u_{n+1})\leq\psi(\xi(u_m,u_{m+1})\leq\psi^{m-n+1}(\xi(u_n,u_{n+1})) \tag{11}$$

If

$$\Omega(u_{m-1},u_m) \;<\; \max\{\xi(u_{m-1},u_m),\xi(u_m,u_{m+1)}\}$$

If

$$\max\{\xi(u_{m-1},u_m),\xi(u_m,u_{m+1})\}=\xi(u_{m-1},u_m)$$

(9) implies

$$\xi(u_n,u_{n+1})\leq\psi(\xi(u_{m-1},u_m))\leq\psi^{m-n}(\xi(u_n,u_{n+1})). \tag{12}$$

If

$$\max\{\xi(u_{m-1},u_m),\xi(u_m,u_{m+1})\}=\xi(u_m,u_{m+1})$$

(9) implies

$$\xi(u_n, u_{n+1}) \leq \psi(\xi(u_m, u_{m+1})) \leq \psi^{m-n+1}(\xi(u_n, u_{n+1})) \tag{13}$$

The inequalities (10), (11), (12), (13) imply and using the property (iii) of ψ

$$\xi(x_n, x_{n+1}) < \xi(x_n, x_{n+1}),$$

which is impossible.

We are going to show that $\{x_n\}$ is a Cauchy sequence,

$$\lim_{n\to\infty} \xi(u_n, u_{n+k}) = 0,$$

for all $k \in \mathbb{N}$. In (5) and (8), we have already established the cases for $k = 1$ and $k = 2$, respectively.

Let k be an arbitrary integer such that $k \geq 3$. We discuss two cases:

Case 1: Let $k = 2m + 1$, where $m \geq 1$. Utilizing characteristic (iv) of partial metric space, we obtain

$$\begin{aligned}
\xi(u_n, u_{n+k}) &= \xi(u_n, u_{n+2m+1}) \leq \xi(u_n, u_{n+1}) \\
&\quad +\xi(u_{n+1}, u_{n+2m+1}) - \xi(u_{n+1}, u_{n+1}) \\
&\leq \xi(u_n, u_{n+1}) + \xi(u_{n+1}, u_{n+2m+1}) \\
&\leq \xi(u_n, u_{n+1}) + \xi(u_{n+1}, u_{n+2}) \\
&\quad +\xi(u_{n+2}, u_{n+2m+1}) - \xi(u_{n+2}, u_{n+2}) \\
&\leq \xi(u_n, u_{n+1}) + \xi(u_{n+1}, u_{n+2}) + \xi(u_{n+2}, u_{n+2m+1}) \\
&\quad \vdots \\
&\leq \xi(u_n, u_{n+1}) + \xi(u_{n+1}, u_{n+2}) + \cdots + \xi(u_{n+2m}, u_{n+2m+1}) \\
&\leq \sum \lim_{q=n}^{n+2m} \xi(u_q, u_{q+1})
\end{aligned}$$

$$\begin{aligned}
&\leq \sum \lim_{q=n}^{n+2m} \psi^q(\xi(u_0, u_1) \\
&\leq \sum \lim_{q=n}^{\infty} \psi^q(\xi(u_0, q_1) \to 0
\end{aligned}$$

as $n \to \infty$.

Case 2: Let $k = 2m$, where $m \geq 2$. By applying the property (iv) of partial metric space, we get

$$\begin{aligned}
\xi(u_n, u_{n+k}) = \xi(u_n, u_{n+2m}) &\leq \xi(u_n, u_{n+2}) \\
&+ \xi(u_{n+2}, u_{n+2m}) - \xi(u_{n+2}, u_{n+2}) \\
&\leq \xi(u_n, u_{n+2}) + \xi(u_{n+2}, u_{n+2m}) \\
&\leq \xi(u_n, u_{n+2}) + \xi(u_{n+2}, u_{n+3}) \\
&+ \xi(u_{n+3}, u_{n+2m}) - \xi(u_{n+3}, u_{n+3}) \\
&\leq \xi(u_n, u_{n+2}) + \xi(u_{n+2}, u_{n+3}) + \xi(u_{n+3}, u_{n+2m}) \\
&\vdots \\
&\leq \xi(u_n, u_{n+2}) + \xi(u_{n+2}, u_{n+3}) \\
&+ \cdots + \xi(u_{n+2m-1}, u_{n+2m}) \\
&\leq \xi(u_n, u_{n+2}) + \sum \lim_{q=n+2}^{n+2m-1} \psi^q(\xi(u_0, u_1)) \\
&\leq \xi(u_n, u_{n+2}) + \sum \lim_{q=n+2}^{\infty} \psi^q(\xi(u_0, u_1)) \to 0
\end{aligned}$$

as $n \to \infty$. Since,

$$lim_{n\to\infty}\xi(u_n, u_{n+2}) = 0$$

because of (8). In the two examples given above, we can see that

$$\lim_{n\to\infty} \xi(u_n, u_{n+k}) = 0$$

for all $k \geq 3$. So, we may say that $\{u_n\}$ is a Cauchy sequence in (X, ξ). Since (X, ξ) is complete, there is a point $u^* \in X$ that

$$\lim_{n\to\infty} p(u_n, u^*) = 0. \tag{14}$$

Next, we shall show that the fixed point τ is the limit u^* of the sequence $\{u_n\}$. Allow us to begin by assuming that τ is a curve. Then,

$$\lim_{n\to\infty} \xi(\tau u_n, \tau u^*) = \lim_{n\to\infty} \xi(u_{n+1}, \tau u^*) = 0.$$

We conclude that

$$u^* = Tu^*.$$

In other words, u^* is a fixed point in τ.

Let us assume that u is α-regular. After that, there is a subsequence $\{u_{n_k}\}$ of $\{u_n\}$ such that

$$\alpha(u_{n_k-1}, u^*) \geq 1$$

$\forall\ k \in \mathbb{N}$. From inequality (1), where $u = u_{n_k}$ and $v = u^*$, we get

$$\begin{aligned}
\xi(u_{n_k+1}, \tau u^*) &= \xi(\tau u_{n_k}, \tau u^*) \\
&\leq \alpha(u_{n_k}, u^*)\xi(\tau u_{n_k}, \tau u^*) \\
&\leq \psi(\Omega(u_{n_k}, u^*))
\end{aligned} \tag{15}$$

where,

$$\begin{aligned}\Omega(u_{n_k},u^*) &= \max\Big\{\xi(u_{n_k},u^*),\xi(u_{n_k},\tau u_{n_k}),p(u^*,\tau u^*),\\ &\quad \frac{\xi(u_{n_k},u^*)+\xi(u_{n_k},\tau u_{n_k})}{2},\\ &\quad \frac{\xi(u_{n_k},\tau u_{n_k}),\xi(u^*,\tau u^*)}{2},\\ &\quad \frac{\xi(u_{n_k},\tau u_{n_k})\xi(u^*,\tau u^*)}{1+\xi(u_{n_k},u^*)},\\ &\quad \frac{\xi(u_{n_k},\tau u_{n_k})\xi(u^*,Tx^*)}{1+\xi(\tau u_{n_k},\tau u^*)}\Big\}\\ &= \max\Big\{\xi(u_{n_k},u^*),\xi(u_{n_k},u_{n_k+1}),\xi(u^*,\tau u^*),\\ &\quad \frac{\xi(u_{n_k},u^*),+\xi(u_{n_k},u_{n_k+1})}{2},\\ &\quad \frac{\xi(u_{n_k},u_{n_k+1}),\xi(u^*,Tu^*)}{2},\\ &\quad \frac{\xi(u_{n_k},u_{n_k+1})\xi(u^*,\tau u^*)}{1+\xi(u_{n_k},u^*)},\\ &\quad \frac{\xi(u_{n_k},u_{n_k+1})\xi(u^*,Tu^*)}{1+\xi(u_{n_k+1},\tau u^*)}\Big\} \end{aligned} \tag{16}$$

Let $k\to\infty$ in (16), we get

$$\Omega(u_{n_k},u^*)=\xi(u^*,Tu^*).$$

Consequently, by taking the limit as $k\to\infty$ in inequality (16), we get

$$\xi(u^*,\tau u^*)\le\psi(\xi(u^*,\tau u^*))<\xi(u^*,\tau u^*),$$

which implies that

$$u^*=\tau u^*,$$

that is, u^* is a fixed point of τ.

Let us consider two distinct fixed points of τ, denoted as u^* and v^*, where $u^*\neq v^*$. According to the hypothesis, $\alpha(u^*,v^*)\ge 1$. Therefore, substituting $u=u^*$ and $v=v^*$ into (1) yields

$$\begin{aligned}\xi(u^*,v^*)=\xi(\tau u^*,\tau v^*) &< \alpha(u^*,v^*)\xi(\sigma u^*,\sigma v^*)\\ &< \psi(\Omega(u^*,v^*))\end{aligned} \tag{17}$$

where

$$\begin{aligned}\Omega(u^*,v^*) &= \max\Big\{\xi(u^*,v^*),\xi(v^*,\tau v^*),\xi(u^*,\tau u^*),\frac{\xi(u^*,v^*)+\xi(u^*,\tau v^*)}{2},\\ &\quad \frac{\xi(u^*,\tau u^*)+\xi(v^*,\tau v^*)}{2},\frac{\xi(u^*,\tau u^*)\xi(v^*,\tau v^*)}{1+\xi(u^*,v^*)},\\ &\quad \frac{\xi(u^*,\tau u^*)\xi(v^*,\tau v^*)}{1+\xi(\tau u^*,\tau v^*)}\Big\}\\ &= \xi(u^*,v^*).\end{aligned} \tag{18}$$

Therefore, we get

$$\xi(u^*, v^*) \leq \psi(\xi(u^*, v^*)) < \xi(u^*, v^*),$$

which is possible only if $\sigma(u^*, v^*) = 0$, that is $u^* = v^*$. Thus, τ possesses a unique fixed point. □

Example 3.3. *Assume that $X = \{1, 2, 3\}$ is a finite set. Define*

$$\xi : X \times X \to [0, \infty)$$

as:

$$\begin{aligned}
&\xi(1,1) = \xi(2,2) = \xi(3,3) = 0\\
&\xi(1,2) = \xi(2,1) = 3\\
&\xi(2,3) = \xi(3,2) = \xi(1,3) = \xi(3,1) = 8
\end{aligned}$$

In this case, ξ is a partial metric on X, and (X, ξ) is a complete partial metric space.

Define $\tau : X \to X$ as

$$\tau 3 = 2, \tau 1 = \tau 2 = 1,$$

Define $\alpha(u, v)$ as $\alpha(u, v) = 1$, $\psi(t) = \dfrac{t}{2}$.

Finally, for $u = 1, 2$ and $v = 1, 2$, we get

$$\alpha(u, v)\xi(\tau u, \tau v) = 0 \leq \psi(\Omega(u, v)) = 0$$

Conversely, for $u = 1, 2$ and $v = 3$

$$\alpha(u, 3)\xi(\tau u, \tau 3) = \xi(1, 2) = 3$$

$$\begin{aligned}
\Omega(u,3) = \max\Big\{&\xi(u,3), \xi(u,\tau u), \xi(3,\tau 3), \frac{\xi(u,3)+\xi(u,\tau u)}{2},\\
&\frac{\xi(u,\tau u)+\xi(3,\tau 3)}{2}, \frac{\xi(u,\tau u)\xi(3,\tau 3)}{1+\xi(u,3)}, \frac{\xi(u,\tau u)\xi(3,\tau 3)}{1+\xi(\tau u,\tau 3)}\Big\}\\
= \max\Big\{&8, 0 \text{ or } 3, 8, \frac{8}{2} \text{ or } \frac{11}{2}, \frac{8}{2} \text{ or } \frac{11}{2}, \frac{(0 \text{ or } 3)\times 8}{1+8}, \frac{(0 \text{ or } 3)\times 8}{1+8}\Big\}\\
= \max\Big\{&8, 0 \text{ or } 3, 4 \text{ or } \frac{11}{2}, 0 \text{ or } \frac{24}{9}\Big\}\\
= 8.&
\end{aligned}$$

and hence

$$\alpha(u, 3)\xi(\tau u, \tau 3) = \xi(1, 2) = 3 \leq \psi(8) = 4$$

The contraction condition is evident when $u = 3$ and $v = 3$. It is evident that τ satisfies the conditions of theorem (3.2) and has a unique fixed point at $u = 1$.

4. Some consequences

Definition 4.1. *Consider a partially ordered set $(X, \leq)$. For any pair of elements $u, v \in X$, where $u \leq v$ implies $\tau u \leq Tv$, we say that the mapping $\tau : X \to X$ is non-decreasing with respect to $\leq$.*

Definition 4.2. *Consider a partially ordered metric space $(X, \xi, \leq)$. The regular partial metric space X is defined as the set of all sequences in X such that for every $k \in \boldsymbol{N}$, there exists a subsequence $\{u_{n_k}\}$ of $\{u_n\}$ such that $u_{n_k} \leq u, \forall\ u, v \in X$.*

Theorem 4.3. *Let $(X, \xi, \leq)$ represent a partially ordered complete partial metric space, and let $\tau : X \to X$ denote a non-decreasing self-mapping. Assuming the following conditions are satisfied:*

(i) $\exists\ \psi \in \Psi$ for which $\xi(\tau u, \tau v) \leq \psi(\Omega(u, v))$
where

$$\Omega(u,v) = \max\Big\{\xi(u,v), \xi(u,\tau u), \xi(v,\tau v), \frac{\xi(u,v)+\xi(u,\tau u)}{2}, \frac{\xi(u,\tau u)+\xi(v,\tau v)}{2}, \frac{\xi(u,\tau u)\xi(v,\tau v)}{1+\xi(u,v)}, \frac{\xi(u,\tau u)\xi(v,\tau v)}{1+\xi(\tau u,\tau v)}\Big\}$$

(ii) $\exists\ u_0 \in X$ such that $u_0 \leq \tau u_0$ and $u_0 \leq \tau^2 u_0$.
(iii) Either X is regular or τ is continuous. Consequently, τ possesses a fixed point $u^* \in X$ and $\{\tau^n u_0\}$ converges to u^*.

Proof. Let $\alpha : X \times X \to [0, \infty)$ be a mapping defined by:

$$\alpha(u,v) = \begin{cases} 1 & \text{if } u \leq v \text{ or } v \leq u; \\ 0 & \text{otherwise .} \end{cases}$$

Now that we have shown that Theorem 3.2 holds, we can say that τ has a fixed point, which is the limit of the sequence $\{\tau^n u_0\}$. □

Theorem 4.4. *Assume $(X, \xi, \leq)$ is a partially ordered complete Partial metric space, and $\tau : X \to X$ is a non-decreasing self mapping. Assume the following criteria are met:*

(i) $\exists\ \psi \in \Psi$ for which $\xi(\tau u, \tau v) \leq \psi(\Omega(u, v))$ where

$$\Omega(u,v) = \max\{\xi(u,v), \xi(u,\tau u), \xi(v,\tau v), \frac{\xi(u,v)+\xi(u,\tau u)}{2}, \frac{\xi(u,\tau u)+\xi(v,\tau v)}{2}, \frac{\xi(u,\tau u)\xi(v,\tau v)}{1+\xi(u,v)+\xi(u,\tau v)+\xi(v,\tau u)}, \frac{\xi(u,\tau v)\xi(u,v)}{1+\xi(u,\tau u)+\xi(v,\tau u)+\xi(v,\tau v)}\}$$

(ii) $\exists\ u_0 \in X$ such that $u_0 \leq \tau u_0$ and $u_0 \leq \tau^2 u_0$.
(iii) Either X is regular or τ is continuous. Consequently, τ possesses a fixed point $u^* \in X$ and $\{\tau^n u_0\}$ converges to u^*.

Proof. Consider a mapping $\alpha : X \times X \to [0, \infty)$ defined by:

$$\alpha(u, v) = \begin{cases} 1 & \text{if } u \leq v \text{ or } v \leq u; \\ 0 & \text{otherwise .} \end{cases}$$

The existence conditions of Theorem 3.2 show that τ has a fixed point, which is the limit of the sequence $\{\tau^n u_0\}$.

Several specific cases can be inferred from the above results. □

Corollary 4.5. *Consider a complete partial metric space (X, σ) and let $\tau : X \to X$, $\alpha : X \times X \to \mathbb{R}$. Assume the following criteria are satisfied:*

(i) τ is an α-admissible mapping.

(ii) τ holds $\xi(\tau u, \tau v) \leq k_0 \Omega(u, v)$

where

$$\Omega(u, v) = \max\{\xi(u, v), \xi(u, \tau v), \xi(v, \tau v), \frac{\xi(u, v) + \xi(u, \tau u)}{2}, \frac{\xi(u, \tau u) + \xi(v, \tau v)}{2}, \frac{\xi(u, \tau u)\xi(v, \tau v)}{1 + \xi(u, v) + \xi(u, \tau v) + \xi(v, \tau u)}, \frac{\xi(u, \tau v)\xi(u, v)}{1 + \xi(u, \tau u) + \xi(v, \tau u) + \xi(v, \tau v)}\}$$

for some $k_0 \in [0, 1)$.

(iii) $\exists\ u_0 \in X$ such that $\alpha(u_0, \tau u_0) \geq 1$ and$\alpha(u_0, \tau^2 u_0) \geq 1$.

(iii) Either X is α-regular or τ is continuous.

As a result, $\{\tau^n u_0\}$ converges to u^* and τ has a fixed point $u^* \in X$. Additionally, if

$$\alpha(u, v) \geq 1$$

for every $u, v \in F(\tau)$, consequently, τ possesses a unique fixed point in X.

Proof. Define a mapping $\psi(t) = kt$. Evidently $\psi \in \boldsymbol{\psi}$. There is a unique fixed point for T according to theorem 3.2. □

Corollary 4.6. *Consider a complete partially ordered Partial metric space $(X, \xi, \leq)$ and let $\tau : X \to X$ be a non-decreasing self mapping. Assume that the following criteria are met:*

(i)

$$\xi(\tau u, \tau v) \leq k_0(\Omega(u, v))$$

where

$$\Omega(u, v) = \max\{\xi(u, v), \xi(u, \tau u), \xi(v, \tau v), \frac{\xi(u, v) + \xi(u, \tau u)}{2}, \frac{\xi(u, \tau u) + \xi(v, \tau v)}{2}, \frac{\xi(u, \tau u)\xi(v, \tau v)}{1 + \xi(u, v) + \xi(u, \tau v) + \xi(v, \tau u)}, \frac{\xi(u, \tau v)\xi(u, v)}{1 + \xi(u, \tau u) + \xi(v, \tau u) + \xi(v, \tau v)}\}$$

for all $u, v \in X$ with $u \leq v$ and for some $k_0 \in [0, 1)$.

(ii) $\exists\ u_0 \in X$ *such that* $u_0 \leq \tau u_0$ *and* $u_0 \leq \tau^2 u_0$.

(iii) Either X *is regular or* τ *is continuous.*

Consequently, τ *possesses a fixed point* $u^* \in X$ *and* $\{\tau^n u_0\}$ *converges to* u^*.

Proof. Let $\alpha : X \times X \to [0, \infty)$ be a mapping defined by:

$$\alpha(u, v) = \begin{cases} 1 & \text{if } u \leq v \text{ or } v \leq u; \\ 0 & \text{otherwise .} \end{cases}$$

It follows from Corollary 4.5 that τ possesses a fixed point. □

5. Conclusion

Our exploration of fixed point results for (α, ψ)-rational type contractive conditions in partial metric spaces has yielded significant insights and contributions to the field of fixed point theory. Through our investigation, we have established novel theoretical frameworks and provided practical tools for analyzing the existence and uniqueness of fixed points in diverse settings. Our findings have extended the classical notion of contractive mappings to a more flexible and adaptable form, encompassing a broader class of mappings and partial metric spaces. This study opens up new avenues for further research in fixed point theory, with opportunities to explore more intricate contractive conditions and their applications in both theoretical and practical contexts. We anticipate that our contributions will inspire future advancements in the understanding and utilization of fixed point concepts in partial metric spaces and beyond.

Conflict of Interest

The authors have no conflicts of interest to declare. The study was conducted independently without any external funding, and the authors did not have any involvement from commercial entities that could have influenced the study design, data collection, analysis, interpretation, or publication.

Acknowledgements

The authors like to express sincere gratitude to the referees for their helpful comments and suggestion to improve the quality of the manuscript.

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(Received, November 01, 2024)
(Revised, May 31, 2025)

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