Journal of Indian Acad. Math. ISSN: 0970-5120
Vol. 48, No. 1 (2026) pp. 112–120.

GENERATING FUNCTIONS FOR TWO VARIABLE LEGENDRE POLYNOMIALS

B. S. Desale[1] and G. S. Kadu[2]

Abstract. In this paper we have used Weisner's group theoretic method to derive generating functions involving two variable Legendre polynomials $P_n(x, y)$ by suitable interpretation to the index variable n.

Keywords: Two variable Legendre polynomials, group theoretic method, generating functions.

2010 AMS Subject Classification: Primary 33C50; Secondary 33C70.

1. Introduction

It was Weisner [1, 2, 3] who introduced the group theoretic method to obtain generating functions for hypergeometric, Hermite, and Bessel functions. The Weisner's method was the subject of Miller's [4] research, and he presented it in a systematic manner to establish a solid foundation for us. During the same period, McBride ([5]) used this method to obtain generating functions for Laguerre, modified Laguerre, Bessel, and Gegenbauer polynomials. Recently, M. G. Bin-saad and M. J. S. Shahwan [6], H. M. Srivastava et al. [7], G. Yasmin and A. Muhyi [8], and S. Khan and M. Ali [9] have used the method adopted by Miller to obtain relations for multivariable special functions, namely 2D-Laguerre, 2I4V1P Hermite, 3V2P Hermite, and 3V Laguerre-Hermite polynomials, respectively. In this paper, we have used a method adopted by McBride to obtain generating functions for two-variable Legendre polynomials.

The two-variable extension of Legendre polynomials (TVLP) introduced by G. Dattoli et al. [10] is defined as

$$P_n(x,y) = \frac{(2n)!}{2^{2n}(n!)^2}(-x)^n {}_2F_1\left(-\left[\frac{n}{2}\right], \frac{1-n^*}{2}; \frac{1-2n}{2}; \frac{4y}{x^2}\right) \tag{1}$$

where

$$n^* = \begin{cases} n, & \text{if } n \text{ is even} \\ n+1, & \text{if } n \text{ is odd} \end{cases}$$

and its generating function is given by,

$$\sum_{n=0}^{\infty} P_n(x,y)t^n = \frac{1}{\sqrt{1+xt+yt^2}}. \tag{2}$$

In the following section, we will use group theoretic method to find the generating functions of TVLP.

2. Group-Theoretic Method

The first and important step in the group theoretic method is to obtain two independent differential recurrence relations; using (2), we have determined the following differential recurrence relations for TVLP.

$$(x^2-4y)\frac{\partial P_n(x,y)}{\partial x} = n\left[xP_n(x,y)+2yP_{n-1}(x,y)\right], \tag{3}$$

$$(4y-x^2)\frac{\partial P_n(x,y)}{\partial x} = (n+1)\left[2P_{n+1}(x,y)+xP_n(x,y)\right], \tag{4}$$

$$(4y-x^2)\frac{\partial P_n(x,y)}{\partial y} = n[2P_n(x,y)+xP_{n-1}(x,y)]. \tag{5}$$

By using the above recurrence relations (3) and (4), we verify that the following partial differential equation is satisfied by TVLP:

$$(4y-x^2)D^2P_n(x,y)-2xDP_n(x,y)+n(n+1)P_n(x,y)=0, \text{where } D=\frac{\partial}{\partial x}. \tag{6}$$

We can recast the differential equation (6) in the form of an operator as follows:

$$L\left(x,y,\frac{\partial}{\partial x},n\right)P_n(x,y)=\left[(4y-x^2)D^2-2xD+n(n+1)\right]P_n(x,y)=0. \tag{7}$$

Now we replace n by $z\frac{\partial}{\partial z}$ and $P_n(x,y)$ by $f(x,y,z)$ in (7), to get the following partial differential equation

$$\left[(4y-x^2)\frac{\partial^2}{\partial x^2}-2x\frac{\partial}{\partial x}+2z\frac{\partial}{\partial z}+z^2\frac{\partial^2}{\partial z^2}\right]f(x,y,z)=0. \tag{8}$$

We can recast the partial differential equation (8) in the form of operator as

$$L\left(x,y,\frac{\partial}{\partial x},z\frac{\partial}{\partial z}\right)f(x,y,z)=0.$$

Now we proceed to find power raising and lowering operators, which is one of the key steps in the group theoretic method. By simple calculations, we have determined that the following linear partial differential operators

$$A=z\frac{\partial}{\partial z}, \tag{9}$$

$$B=\frac{(4y-x^2)}{yz}\frac{\partial}{\partial x}+\frac{x}{y}\frac{\partial}{\partial z}, \tag{10}$$

and

$$C=z(4y-x^2)\frac{\partial}{\partial x}-xz^2\frac{\partial}{\partial z}-xz \tag{11}$$

are such that their action on $P_n(x,y)z^n$ is as follows:

$$A[P_n(x,y)z^n]=nP_n(x,y)z^n, \tag{12}$$

$$B[P_n(x,y)z^n]=-2nP_{n-1}(x,y)z^{n-1}, \tag{13}$$

$$C[P_n(x,y)z^n]=2(n+1)P_{n+1}z^{n+1}. \tag{14}$$

Here, operator (10) is a power-lowering operator, whereas operator (11) is a power-raising operator. Both these operators play an important role in determining the generating functions of TVLP. Now we proceed to determine commutator relations satisfied by operators A, B, and C, which are presented in the form of the following proposition.

Proposition 2.1. *The operators A, B and C satisfy the following commutator relations*

(i) $[A, B] = -B$
(ii) $[B, C] = -8A - 4$
(iii) $[A, C] = C$

Proof. Let $f = f(x, y, z) \in \mathcal{C}^2$ be an arbitrary function.
Firstly, we will prove the commutator relation satisfied by operators A and B. Now

$$ABf = z\frac{\partial}{\partial z}\left[\frac{(4y - x^2)}{yz}\frac{\partial f}{\partial x} + \frac{x}{y}\frac{\partial f}{\partial z}\right].$$

Further simplification leads to

$$ABf = \frac{-(4y - x^2)}{yz}\frac{\partial f}{\partial x} + \frac{(4y - x^2)}{y}\frac{\partial^2 f}{\partial z \partial x} + \frac{zx}{y}\frac{\partial^2 f}{\partial z^2}. \tag{15}$$

Similarly, we get

$$BAf = \frac{x}{y}\frac{df}{dz} + \frac{(4y - x^2)}{y}\frac{\partial^2 f}{\partial x \partial z} + \frac{xz}{y}\frac{\partial^2 f}{\partial z^2}. \tag{16}$$

Using equations (15) and (16), we get

$$[A, B]f = ABf - BAf = -Bf.$$

Therefore,

$$[A, B] = -B.$$

Now we will prove the commutator relation for operators B and C; by doing elementary calculations, we get

$$\begin{aligned}[B, C]f &= \frac{-(4y - x^2)z}{y}\frac{\partial f}{\partial z} - \frac{(4y - x^2)}{y}f - \frac{2x^2 z}{y}\frac{\partial f}{\partial z} - \frac{x^2 f}{y} - \frac{z(4y - x^2)}{y}\frac{\partial f}{\partial z} \\ &= -8z\frac{\partial f}{\partial z} - 4f \\ &= (-8A - 4)f.\end{aligned}$$

Therefore,

$$[B, C]f = BCf - CBf = -8z\frac{\partial f}{\partial z} - 4f = (-8A - 4)f. \tag{17}$$

Hence $[B, C] = -8A - 4$.
Finally, we will prove the commutator relation satisfied by operators A and C. Doing some simplification, we get

$$[A, C]f = z(4y - x^2)\frac{\partial f}{\partial x} - xz^2\frac{\partial f}{\partial z} - xzf = Cf.$$

Therefore

$$[A, C] = C. \tag{18}$$

□

Now by using the commutator relations proved in proposition 2.1, we prove the following proposition.

Proposition 2.2. *The operators given by (9), (10), and (11) commute with the operator*

$$\frac{(4y-x^2)}{y}L$$

where L is the operator given by (7).

Proof. Let $f = f(x, y, z) \in \mathcal{C}^2$ be a arbitraray function. Here,

$$Lf = (4y-x^2)\frac{\partial^2 f}{\partial x^2} - 2x\frac{\partial f}{\partial x} + 2z\frac{\partial f}{\partial z} + z^2\frac{\partial^2 f}{\partial z^2} \tag{19}$$

and

$$CBf = \frac{(4y-x^2)^2}{y}\frac{\partial^2 f}{\partial x^2} - \frac{2x(4y-x^2)}{y}\frac{\partial f}{\partial x} + \frac{z(4y-x^2)}{y}\frac{\partial f}{\partial z} - \frac{x^2z^2}{y}\frac{\partial^2 f}{\partial z^2} - \frac{x^2z}{y}\frac{\partial f}{\partial z}. \tag{20}$$

Using (19) and (20), we have

$$\left[\frac{(4y-x^2)}{y}L - CB\right] f = 4\left[z^2\frac{\partial^2}{\partial z^2} + z\frac{\partial}{\partial z}\right] f = 4A^2 f.$$

Since f is arbitrary, we conclude that

$$\frac{(4y-x^2)}{y}L = CB + 4A^2. \tag{21}$$

Using (17) and (18), we have following assertions

$$BC - CB = -8A - 4 \tag{22}$$

$$A^2C - CA^2 = 2CA + C. \tag{23}$$

Now using (21), (22) and (23), we have

$$\begin{aligned}\left[\frac{(4y-x^2)}{y}L, C\right] f &= \left[\frac{(4y-x^2)}{y}LC - C\frac{(4y-x^2)}{y}L\right] f \\ &= [C(BC-CB) + 4(A^2C - CA^2)]f \\ &= 0.\end{aligned}$$

Hence $\frac{(4y-x^2)}{y}L$ commutes with C. Similarly, we can verify that the operator $\frac{(4y-x^2)}{y}L$ commutes with the operators A and B respectively. □

Now we proceed to find the extended form of a group of operators; the operators $\exp(aA)$,
$\exp(bB)$ and $\exp(cC)$ are called the extended forms of operators A, B and C, where a, b, c are arbitrary constants. In order to find the extended form of the group of operator corresponding to C, we firstly will change the form of the differential operator $C = C_1\frac{\partial}{\partial x} + C_2\frac{\partial}{\partial y} + C_3\frac{\partial}{\partial z} + C_4$ to $E = C_1\frac{\partial}{\partial x} + C_2\frac{\partial}{\partial y} + C_3\frac{\partial}{\partial z}$ and finally to $D = \frac{\partial}{\partial X}$ by using change of variables. If a function $\phi(x, y, z)$ is such that $C[\phi] = 0$, then $\phi^{-1}C\phi = E = C_1\frac{\partial}{\partial x} + C_2\frac{\partial}{\partial y} + C_3\frac{\partial}{\partial z}$, refer McBride [5] for the proof. Hence, firstly, we will find a function ϕ such that $C[\phi] = 0$.

Now $C[\phi] = 0$, implies that

$$z(4y - x^2)\frac{\partial \phi}{\partial x} - xz^2\frac{\partial \phi}{\partial z} - xz\phi = 0.$$

For $z \neq 0$, we get

$$(4y - x^2)\frac{\partial \phi}{\partial x} - xz\frac{\partial \phi}{\partial z} - x\phi = 0.$$

To get ϕ, we solve the following system

$$\frac{dx}{(4y - x^2)} = \frac{dz}{-xz} = \frac{d\phi}{x\phi}.$$

The general solution of the above system is $\sigma(z\phi, y) = 0$, where σ is an arbitrary function. We choose a special case,

$$\phi^{-1} = \frac{z}{y}. \tag{24}$$

Now for any function $u(x, y, z)$, we get

$$\phi^{-1}C\phi u = z(4y - x^2)\frac{\partial u}{\partial x} - xz^2\frac{\partial u}{\partial z}.$$

Thus

$$E = \phi^{-1}C\phi = z(4y - x^2)\frac{\partial}{\partial x} - xz^2\frac{\partial}{\partial z}. \tag{25}$$

Now we have to choose variables X, Y & Z, so that E will be transformed into $k\frac{\partial}{\partial X}$ where $k^2 = 1$. We put

$$\begin{aligned}
\frac{\partial u}{\partial x} &= \frac{\partial u}{\partial X}\frac{\partial X}{\partial x} + \frac{\partial u}{\partial Y}\frac{\partial Y}{\partial x} + \frac{\partial u}{\partial Z}\frac{\partial Z}{\partial x} \\
\frac{\partial u}{\partial y} &= \frac{\partial u}{\partial X}\frac{\partial X}{\partial y} + \frac{\partial u}{\partial Y}\frac{\partial Y}{\partial y} + \frac{\partial u}{\partial Z}\frac{\partial Z}{\partial y} \\
\frac{\partial u}{\partial z} &= \frac{\partial u}{\partial X}\frac{\partial X}{\partial z} + \frac{\partial u}{\partial Y}\frac{\partial Y}{\partial z} + \frac{\partial u}{\partial Z}\frac{\partial Z}{\partial z}
\end{aligned}$$

in

$$Eu = z(4y - x^2)\frac{\partial u}{\partial x} - xz^2\frac{\partial u}{\partial z}$$

to get

$$Eu = \left[z(4y - x^2)\frac{\partial X}{\partial x} - xz^2\frac{\partial X}{\partial z}\right]\frac{\partial u}{\partial X} + \left[z(4y - x^2)\frac{\partial Y}{\partial x} - xz^2\frac{\partial Y}{\partial z}\right]\frac{\partial u}{\partial Y} + \left[z(4y - x^2)\frac{\partial Z}{\partial x} - xz^2\frac{\partial Z}{\partial z}\right]\frac{\partial u}{\partial Z}.$$

We set

$$z(4y - x^2)\frac{\partial Y}{\partial x} - xz^2\frac{\partial Y}{\partial z} = 0, z(4y - x^2)\frac{\partial Z}{\partial x} - xz^2\frac{\partial Z}{\partial z} = 0, z(4y - x^2)\frac{\partial X}{\partial x} - xz^2\frac{\partial X}{\partial z} = k.$$

For $z \neq 0$, we have

$$(4y - x^2)\frac{\partial Y}{\partial x} - xz\frac{\partial Y}{\partial z} = 0, \tag{26}$$

$$(4y - x^2)\frac{\partial Z}{\partial x} - xz\frac{\partial Z}{\partial z} = 0, \tag{27}$$

$$(4y - x^2)\frac{\partial X}{\partial x} - xz\frac{\partial X}{\partial z} = kz^{-1}. \tag{28}$$

General solutions of (26), (27) and (28) are

$$\sigma_1\left(\frac{4y-x^2}{z^2},Y\right)=0,\tau\left(y,Z\right)=0 \text{ and } \eta\left(\frac{x}{4z}-\frac{Xy}{k},y\right)=0 \tag{29}$$

respectively, where σ_1,τ & and η are arbitrary functions. We choose the following particular solutions

$$Y=\frac{4y-x^2}{z^2}, \tag{30}$$

$$Z=y, \tag{31}$$

and

$$X=\frac{x}{4yz}. \tag{32}$$

Solving (30), (31) and (32), gives us

$$x=\frac{8Z^{\frac{3}{2}}X}{\sqrt{Y+16Z^2X^2}},y=Z,z=\frac{2Z^{\frac{1}{2}}}{\sqrt{Y+16Z^2X^2}}, \tag{33}$$

which transforms E to $\frac{\partial}{\partial X}$. Now we are in a position to derive an extended form of operators. Firstly, we will find an extended form of the operator C.

$$\begin{aligned}e^{cC}f(x,y,z)&=e^{c\phi E\phi^{-1}}f(x,y,z)\\&=\phi(x,y,z)e^{cE}\left[\phi^{-1}(x,y,z)f(x,y,z)\right].\end{aligned}$$

Using (24), the above equation can be simplified as

$$e^{cC}f(x,y,z)=yz^{-1}e^{cE}\left[y^{-1}zf(x,y,z)\right]. \tag{34}$$

Now use of (33) in (34) will transform E to D, where $D=\frac{\partial}{\partial X}$. Making this substitution (34) can be recasted as

$$e^{cC}f(x,y,z)=yz^{-1}e^{cD}\left[\frac{2}{\sqrt{Z}\sqrt{Y+16Z^2X^2}}f\left(\frac{8Z^{\frac{3}{2}}X}{\sqrt{Y+16Z^2X^2}},Z,\frac{2Z^{\frac{1}{2}}}{\sqrt{Y+16Z^2X^2}}\right)\right]. \tag{35}$$

Now using Taylor's theorem in the right-hand side of the above equation gives

$$\begin{aligned}&e^{cC}f(x,y,z)\\&=yz^{-1}\left[\frac{2}{\sqrt{Z}\sqrt{Y+16Z^2(X+c)^2}}f\left(\frac{8Z^{\frac{3}{2}}(X+c)}{\sqrt{Y+16Z^2(X+c)^2}},Z,\frac{2Z^{\frac{1}{2}}}{\sqrt{Y+16Z^2(X+c)^2}}\right)\right].\end{aligned}$$

Finally, using reverse substitution, we get the following extended form of the operator C.

$$\begin{aligned}&e^{cC}f(x,y,z)\\&=\frac{1}{\sqrt{1+4c^2yz^2+2xzc}}\times f\left(\frac{x+4cyz}{\sqrt{1+4c^2yz^2+2xzc}},y,\frac{z}{\sqrt{1+4c^2yz^2+2xzc}}\right).\end{aligned} \tag{36}$$

Similarly, we can derive an extended form of the operators B and A, which are as follows:

$$e^{bB}f(x,y,z)=f\left(\frac{(xz+4b)\sqrt{y}}{\sqrt{yz^2+4b^2+2xzb}},y,\frac{\sqrt{yz^2+4b^2+2xzb}}{\sqrt{y}}\right) \tag{37}$$

$$e^{aA}f(x,y,z)=f\left(\frac{x}{z}e^{-a},y,z\right). \tag{38}$$

Using (36) and (37) we get,

$$e^{cC}e^{bB}f(x,y,z) = \frac{1}{\sqrt{1+4c^2yz^2+2xzc}} \times f(\zeta, y, \xi) \tag{39}$$

where,

$$\zeta = \frac{\sqrt{y}[zx(1+8bc)+4cyz^2(1+4bc)+4b]}{\sqrt{1+4c^2yz^2+2xzc}\sqrt{yz^2(1+4bc)^2+2xzb(1+4bc)+4b^2}},$$

$$\xi = \frac{\sqrt{yz^2(1+4bc)^2+2xzb(1+4bc)+4b^2}}{\sqrt{y}\sqrt{1+4c^2yz^2+2xzc}}.$$

In the next section, we will use the transformation operator (39) to derive some generating functions for TVLP.

3. Generating functions

Now, to obtain generating functions of TVLP we will transform $f(x,y,z)$ using (39). We discuss the following particular cases.

Case (i): $b=1, c=0$

For $b=1, c=0$ the equation (39) takes the following form:

$$e^{B}f(x,y,z) = f\left(\frac{\sqrt{y}(xz+4)}{\sqrt{yz^2+4+2xz}}, y, \frac{\sqrt{yz^2+4+2xz}}{\sqrt{y}}\right).$$

Now replace $f(x,y,z)$ by $P_n(x,y)z^n$ in the above equation, to get

$$e^{B}[P_n(x,y)z^n] = \left(\frac{yz^2+4+2xz}{y}\right)^{\frac{n}{2}} P_n\left(\frac{(xz+4)\sqrt{y}}{\sqrt{yz^2+4+2xz}}, y\right).$$

Using (13) in the above equation gives us

$$\sum_{q=0}^{\infty}\frac{2^q(-n)_q P_{n-q}(x,y)z^{n-q}}{q!} = \left(\frac{yz^2+4+2xz}{y}\right)^{\frac{n}{2}} P_n\left(\frac{(xz+4)\sqrt{y}}{\sqrt{yz^2+4+2xz}}, y\right).$$

Replace z^{-1} by t in the above equation to obtain the following generating function

$$\sum_{q=0}^{\infty}\frac{2^q(-n)_q P_{n-q}(x,y)t^{q}}{q!} = \left(\frac{y+4t^2+2xt}{y}\right)^{\frac{n}{2}} P_n\left(\frac{(x+4t)\sqrt{y}}{\sqrt{y+4t^2+2xt}}, y\right). \tag{40}$$

Case (ii) $b=0, c=1$

For $b=0, c=1$ the equation (39) takes the following form

$$e^{C}f(x,y,z) = \frac{1}{\sqrt{1+4yz^2+2xz}} \times f\left(\frac{x+4yz}{\sqrt{1+4yz^2+2xz}}, y, \frac{z}{\sqrt{1+4yz^2+2xz}}\right).$$

Now replace $f(x,y,z)$ by $P_n(x,y)z^n$ in the above equation, to get

$$e^{C}[P_n(x,y)z^n] = \frac{z^n}{(1+4yz^2+2xz)^{\frac{1+n}{2}}} P_n\left(\frac{x+4yz}{\sqrt{1+4yz^2+2xz}}, y\right).$$

Use of (14) in above equation leads to

$$\sum_{q=0}^{\infty}\frac{2^q(n+1)_q}{q!}P_{n+q}(x,y)z^{n+q}=\frac{z^n}{(1+4yz^2+2xz)^{\frac{1+n}{2}}}P_n\left(\frac{x+4yz}{\sqrt{1+4yz^2+2xz}},y\right).$$

Now we divide by z^n to get the following generating function

$$\sum_{q=0}^{\infty}\frac{2^q(n+1)_q}{q!}P_{n+q}(x,y)z^{q}=\frac{1}{(1+4yz^2+2xz)^{\frac{1+n}{2}}}P_n\left(\frac{x+4yz}{\sqrt{1+4yz^2+2xz}},y\right). \tag{41}$$

Case (iii) $b=\frac{1}{2},c=-\frac{1}{2}$

For $b=\frac{1}{2},c=-\frac{1}{2}$ the equation (39) takes the following form

$$e^{-\frac{C}{2}}e^{\frac{B}{2}}[f(x,y,z)]=\frac{1}{\sqrt{1+yz^2-xz}}\times f(\xi,y,\tau), \tag{42}$$

where

$$\xi=\frac{\sqrt{y}[2-xz]}{\sqrt{1+yz^2-xz}},\tau=\frac{1}{\sqrt{y}\sqrt{1+yz^2-xz}}. \tag{43}$$

Replace $f(x,y,z)$ by $P_n(x,y)z^n$ in the above equation to get,

$$e^{-\frac{C}{2}}e^{\frac{B}{2}}[P_n(x,y)z^n]=\frac{1}{\sqrt{1+yz^2-xz}}\times P_n\left(\frac{\sqrt{y}[2-xz]}{\sqrt{1+yz^2-xz}},y\right)\left(1+yz^2-xz\right)^{-\frac{n}{2}}y^{-\frac{n}{2}}.$$

Now using (13) and (14), we get

$$\sum_{p,q=0}^{\infty}\frac{(-n)_p(-1)^q(n-p+1)_q}{p!q!}P_{n-p+q}(x,y)z^{n-p+q}$$
$$=\frac{1}{\sqrt{1+yz^2-xz}}\times P_n\left(\frac{\sqrt{y}[2-xz]}{\sqrt{1+yz^2-xz}},y\right)\left(1+yz^2-xz\right)^{-\frac{n}{2}}y^{-\frac{n}{2}}. \tag{44}$$

Now we divide the above equation by z^n to get the following generating function

$$\sum_{p,q=0}^{\infty}\frac{(-n)_p(-1)^q(n-p+1)_q}{p!q!}P_{n-p+q}(x,y)z^{-p+q}$$
$$=\frac{1}{\sqrt{1+yz^2-xz}}\times\frac{\left(1+yz^2-xz\right)^{-\frac{n}{2}}y^{-\frac{n}{2}}}{z^n}\times P_n\left(\frac{\sqrt{y}[2-xz]}{\sqrt{1+yz^2-xz}},y\right). \tag{45}$$

4. Conclusion remarks

In this paper, we have used one of the powerful tools, i.e., Weisner's group theoretic method to derive generating functions of two-variable Legendre polynomials $P_n(x,y)$. In the beginning, we have determined the power maintaining, lowering, and raising group of operators A, B, and C. Subsequently, we have derived their extended forms and utilized them to find the generating functions, which are represented by the equations (40), (41), and (45).

Conflicts of Interest

The authors declare no conflict of interest.

REFERENCES

[1] Weisner, L., Group-theoretic origin of certain generating functions, Pacific J. Math 5, 1955, 1033-1039.
[2] Weisner, L., Generating functions for Hermite functions, Canadian Journal of Mathematics 11, 1959, 141-147.
[3] Weisner, L., Generating functions for Bessel functions, Canadian Journal of Mathematics 11, 1959, 148-155.
[4] Miller, W. J., Lie Theory and Special Functions, Academic, New York, 1968.
[5] McBride, E. B., Obtaining generating functions, Springer, New York, 1971.
[6] Bin-Saad, M. G. & Shahwan, M. J., Lie-type operational approach to modified 2D-Laguerre polynomials, Integral Transforms and Special Functions 33(1), 2022, 32-46.
[7] Srivastava, H. M., Yasmin, G. & Muhyi, A., Lie algebra representations and 2-index 4-variable 1-parameter Hermite polynomials, Note Mat 39(1), 2019, 65-87.
[8] Yasmin, G. & Muhyi, A., Lie algebra G (0, 1) and 3-variable 2-parameter Hermite polynomials, A. Afr. Mat 30, 2019 231-246.
[9] Khan, S. & Ali, M., Lie algebra K 5 and 3-variable Laguerre-Hermite polynomials, Revista de la Real Academia de Ciencias Exactas, Fsicas y Naturales. Serie A. Matemticas 113(2), 2019, 831-843.
[10] Dattoli, G., Germano, B., Martinelli, M. R. & Ricci, P. E., A novel theory of Legendre polynomials. Mathematical and Computer Modelling 54(1-2), 2011, 80-87.

(Received, July 26, 2025)
(Revised, September 07, 2025)

[1]Mathematics Department,
University of Mumbai, Mumbai 400 098, India.
E-mail: bhausaheb.desale@mathematics.mu.ac.in
[2]Department of Mathematics,
Arts, Commerce and Science College,
Lanja 416701, India.
E-mail: 4ganeshkadu@gmail.com