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NEUTROSOPHIC BLOCK TOPOLOGICAL SPACES INDUCED BY NEUTROSOPHIC GRAPHS

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Abstract. This work introduces neutrosophic block topological spaces generated from block decompositions of neutrosophic graphs, establishing formal topologies on vertex sets where blocks represent maximal connected subgraphs without cut-vertices. Continuity and homeomorphism theories prove structural preservation including cut-vertex correspondence and block count invariance under topological equivalence.

Keywords: Neutrosophic Graph, Neutrosophic Block Topology, Cut-Vertices, Continuity, Homeomorphism.

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1. Introduction and Preliminaries

Modern networks often involve uncertainty that classical models cannot adequately represent. *Neutrosophic set theory*, proposed by Smarandache, independently models truth, indeterminacy, and falsity, allowing effective representation of ambiguous information [9, 10]. Extending this idea, *neutrosophic graphs* introduced by Broumi and Smarandache [2, 3] and further developed by Akram and Shahzadi [1] assign neutrosophic values to vertices and edges, capturing relational uncertainty. In classical graph theory, a *block* is a maximal connected subgraph without a cut-vertex and plays a key role in structural analysis [4, 5]. Topological methods have further enriched graph analysis through concepts such as continuity and connectedness [6, 8]. Recently, Macaso and Balingit [7] introduced *block topological spaces*, linking graph decomposition with topology. Building on these studies, **Mohanapriya Rajagopal** and **Durgadevi Shanmugasundaram** propose *neutrosophic block topological spaces*, integrating neutrosophic graphs with block-based topology to formalize continuity and homeomorphism under uncertainty.

2. Neutrosophic Block Topology Induced by Neutrosophic Graph

Let N_G be a neutrosophic graph with connected components $C_1, C_2, \ldots, C_j,\ j \in \mathbb{Z}^+$. Each component is first examined for cut-vertices; components without cut-vertices are considered neutrosophic blocks. If a component contains a cut-vertex $(v, \eta(v), \kappa(v), \lambda(v))$, it is decomposed into branches at that vertex, where each branch consists of vertices mutually reachable without passing through the cut-vertex. Each branch is then analyzed recursively: branches without cut-vertices form blocks, while those containing cut-vertices are further decomposed. This process continues until all resulting subgraphs contain no cut-vertices, yielding the complete set of neutrosophic blocks of N_G.

Block Collection: The complete set of neutrosophic blocks consists of all components, branches, and sub-branches that contain no cut-vertices. Consider the neutrosophic graph N_G shown in Figure 1.

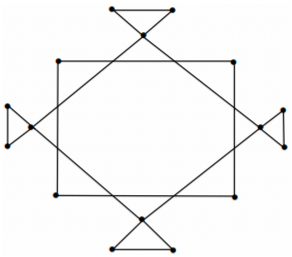


FIGURE 1. Neutrosophic Graph N_G

The components of N_G shown in Figure 2 :

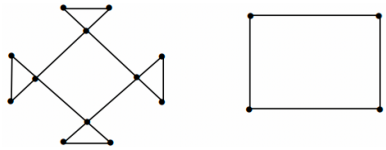


FIGURE 2. The Components of $N_G : C_1, C_2$

The distinct blocks of N_G shown in Figure 3:

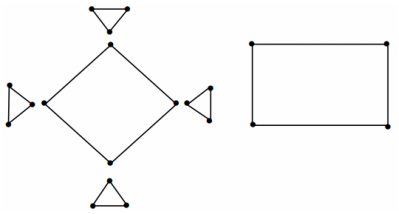


FIGURE 3. The Blocks of N_G

Definition 2.1. *Let N_G be a neutrosophic graph possessing blocks $B_1, B_2, \ldots, B_k$ with $k \in \mathbb{Z}^+$. The family of vertex sets $\Sigma_B(N_G) = \{V(B_1), V(B_2), \ldots, V(B_k)\}$, generates a topology on $V(N_G)$ through the following construction. Define $\Gamma_B(N_G)$ as the collection of all finite intersections of sets from $\Sigma_B(N_G)$, which serves as a topological base. The topology $\tau_B(N_G)$ consists of all arbitrary unions of sets in $\Gamma_B(N_G)$, together with the empty set and $V(N_G)$. We call $\tau_B(N_G)$ the neutrosophic block topology on N_G. The pair $(V(N_G), \tau_B(N_G))$ forms the neutrosophic block topological space associated with N_G. Sets belonging to $\tau_B(N_G)$ are said to be $\tau_B(N_G)$-open, while sets whose complements belong to $\tau_B(N_G)$ are $\tau_B(N_G)$-closed.*

Example 2.2. *Consider the graph H_N shown in Figure 4*

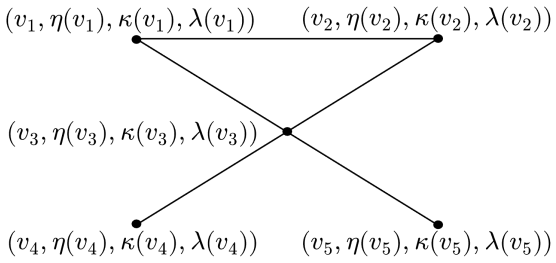


FIGURE 4. H_N

The blocks of H_N are shown in Figure 5 and Figure 6

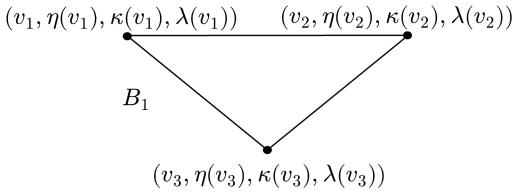


FIGURE 5. The Blocks of H_N

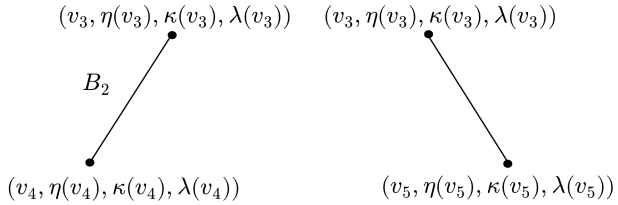


FIGURE 6. The Blocks of H_N

$$\Sigma_B(H_N) = \left\{ \begin{array}{l} \{(v_1, \eta(v_1), \kappa(v_1), \lambda(v_1)),\ (v_2, \eta(v_2), \kappa(v_2), \lambda(v_2)),\ (v_3, \eta(v_3), \kappa(v_3), \lambda(v_3))\}, \\ \{(v_3, \eta(v_3), \kappa(v_3), \lambda(v_3)),\ (v_4, \eta(v_4), \kappa(v_4), \lambda(v_4))\}, \\ \{(v_3, \eta(v_3), \kappa(v_3), \lambda(v_3)),\ (v_5, \eta(v_5), \kappa(v_5), \lambda(v_5))\} \end{array} \right\}.$$

Taking the finite intersections of sets in $\Sigma_B(N_G)$, we obtain

$$\Gamma_B(H_N) = \left\{ \begin{array}{l} \{(v_1, \eta(v_1), \kappa(v_1), \lambda(v_1)),\ (v_2, \eta(v_2), \kappa(v_2), \lambda(v_2)),\ (v_3, \eta(v_3), \kappa(v_3), \lambda(v_3))\}, \\ \{(v_3, \eta(v_3), \kappa(v_3), \lambda(v_3)),\ (v_4, \eta(v_4), \kappa(v_4), \lambda(v_4))\}, \\ \{(v_3, \eta(v_3), \kappa(v_3), \lambda(v_3)),\ (v_5, \eta(v_5), \kappa(v_5), \lambda(v_5))\}, \\ \{(v_3, \eta(v_3), \kappa(v_3), \lambda(v_3))\} \end{array} \right\}.$$

Hence, the neutrosophic block topology is

$$\tau_B(H_N) = \left\{ \begin{array}{l} \varnothing, \\ \{(v_3, \eta(v_3), \kappa(v_3), \lambda(v_3))\}, \\ \{(v_1, \eta(v_1), \kappa(v_1), \lambda(v_1)),\ (v_2, \eta(v_2), \kappa(v_2), \lambda(v_2)),\ (v_3, \eta(v_3), \kappa(v_3), \lambda(v_3))\}, \\ \{(v_3, \eta(v_3), \kappa(v_3), \lambda(v_3)),\ (v_4, \eta(v_4), \kappa(v_4), \lambda(v_4)),\ (v_5, \eta(v_5), \kappa(v_5), \lambda(v_5))\}, \\ \{(v_1, \eta(v_1), \kappa(v_1), \lambda(v_1)),\ (v_2, \eta(v_2), \kappa(v_2), \lambda(v_2)),\ (v_3, \eta(v_3), \kappa(v_3), \lambda(v_3)),\ (v_4, \eta(v_4), \kappa(v_4), \lambda(v_4))\}, \\ \{(v_1, \eta(v_1), \kappa(v_1), \lambda(v_1)),\ (v_2, \eta(v_2), \kappa(v_2), \lambda(v_2)),\ (v_3, \eta(v_3), \kappa(v_3), \lambda(v_3)),\ (v_5, \eta(v_5), \kappa(v_5), \lambda(v_5))\}, \\ V(H_G) \end{array} \right\}.$$

2.1. **Continuity in Neutrosophic Block Topological Space.**

Definition 2.3. *A function $f : (V(N_G), \tau_B(N_G)) \to (V(N_H), \tau_B(N_H))$ is said to be continuous if and only if for every $U \in \tau_B(NH)$, we have $f^{-1}(U) \in \tau_B(NG)$, where $f^{-1}(U) = \{\, (v, \eta(v), \kappa(v), \lambda(v)) \in V(NG) \ :\ f(v, \eta(v), \kappa(v), \lambda(v)) \in U \,\}$.*

Theorem 2.4. *Let N_G and N_H be neutrosophic graphs. A function $f : (V(N_G), \tau_B(N_G)) \to (V(N_H), \tau_B(N_H))$ is continuous if and only if for every neutrosophic block B of N_H, the set $f^{-1}(V(B))$ belongs to $\tau_B(N_G)$.*

Proof. *Assume f is continuous. Let B be any neutrosophic block of NH. Since $V(B) \in \Sigma_B(NH)$, it is $\tau_B(NH)$-open. By continuity, $f^{-1}(V(B)) \in \tau_B(N_G)$.*

Conversely, suppose that for every neutrosophic block B of N_H, $f^{-1}(V(B)) \in \tau_B(N_G)$. Let U be an arbitrary $\tau_B(N_H)$-open set. By definition of neutrosophic block topology,

$$U = \bigcup_{\alpha} \left(\bigcap_{j} V(B_j^{\alpha}) \right),$$

where each B_j^{α} is a neutrosophic block of N_H. Taking preimages,

$$f^{-1}(U) = \bigcup_{\alpha} f^{-1} \left(\bigcap_{j} V(B_j^{\alpha}) \right) = \bigcup_{\alpha} \left(\bigcap_{j} f^{-1}(V(B_j^{\alpha})) \right).$$

By hypothesis, each $f^{-1}(V(B_j^{\alpha}))$ is $\tau_B(NG)$-open. Since $\tau_B(NG)$ is closed under finite intersections and arbitrary unions, we have $f^{-1}(U) \in \tau_B(N_G)$. Hence, f is continuous.

Theorem 2.5. *Let N_G and N_H be neutrosophic graphs. If $(v, \eta(v), \kappa(v), \lambda(v))$ is a neutrosophic cut-vertex of N_G and $f : (V(N_G), \tau_B(N_G)) \to (V(N_H), \tau_B(N_H))$ is continuous, then $f((v, \eta(v), \kappa(v), \lambda(v)))$ is either a neutrosophic cut-vertex of N_H or an isolated vertex of N_H.*

Proof. *Since $(v, \eta(v), \kappa(v), \lambda(v))$ is a cut-vertex of the neutrosophic graph N_G, and since every cut-vertex induces a singleton which is $\tau_B(N_G)$-open (see [7], Theorem 3), it follows that $\{(v, \eta(v), \kappa(v), \lambda(v))\} \in \tau_B(N_G)$. Assume f is continuous. If $f((v, \eta(v), \kappa(v), \lambda(v)))$ is neither a cut-vertex nor an isolated vertex in N_H, then $\{\, f((v, \eta(v), \kappa(v), \lambda(v))) \,\} \notin \tau_B(NH)$. However, by continuity, the preimage of $\{\, f((v, \eta(v), \kappa(v), \lambda(v))) \,\}$ under f must be $\tau_B(N_G)$-open, which contains $(v, \eta(v), \kappa(v), \lambda(v))$.*

This leads to a contradiction. Therefore, $f((v, \eta(v), \kappa(v), \lambda(v)))$ must be either a cut-vertex or an isolated vertex of N_H.

Theorem 2.6. *Let N_G be a neutrosophic graph and N_H a neutrosophic empty graph (only isolated vertices) with $|V(N_G)| = |V(N_H)|$. Then every bijection $f : (V(N_G), \tau_B(N_G)) \to (V(N_H), \tau_B(N_H))$ is continuous.*

Proof. Since N_H is an empty graph, every vertex of N_H is isolated. Hence, each singleton $\{(w, \eta(w), \kappa(w), \lambda(w))\}$ forms a block of N_H, and therefore $\Sigma_B(N_H)$ consists of all singletons. Consequently, $\tau_B(N_H)$ is the discrete topology on $V(N_H)$ (see [7], Theorem 2). Thus every subset of $V(N_H)$ is $\tau_B(N_H)$-open. Let $U \subseteq V(N_H)$, then $U = \bigcup\{\{(w, \eta(w), \kappa(w), \lambda(w))\} : (w, \eta(w), \kappa(w), \lambda(w)) \in U\}$. Taking preimages,$f^{-1}(U) = \bigcup\{f^{-1}(\{(w, \eta(w), \kappa(w), \lambda(w))\}) : (w, \eta(w), \kappa(w), \lambda(w)) \in U\}$. Since f is bijective, each preimage is a singleton in $V(N_G)$; hence $f^{-1}(U)$ is a union of singletons. As $\tau_B(N_G)$ is closed under arbitrary unions, $f^{-1}(U) \in \tau_B(N_G)$, and therefore f is continuous. □

2.2. **Homeomorphism in Neutrosophic Block Topological Space.**

Definition 2.7. *Let N_G and N_H be neutrosophic graphs with block topological spaces $(V(N_G), \tau_B(N_G))$ and $(V(N_H), \tau_B(N_H))$ respectively. A function $f : (V(N_G), \tau_B(N_G)) \to (V(N_H), \tau_B(N_H))$ is called a* ***neutrosophic block topological homeomorphism*** *if:*

(i) *f is bijective (one-to-one and onto),*
(ii) *f is continuous from $(V(N_G), \tau_B(N_G))$ to $(V(N_H), \tau_B(N_H))$, and*
(iii) *$f^{-1} : V(N_H) \to V(N_G)$ is continuous from $(V(N_H), \tau_B(N_H))$ to $(V(N_G), \tau_B(N_G))$.*

In this case, we say $(V(N_G), \tau_B(N_G))$ and $(V(N_H), \tau_B(N_H))$ are homeomorphism, written as $(V(N_G), \tau_B(N_G)) \cong (V(N_H), \tau_B(N_H))$.

Theorem 2.8. *Let N_G and N_H be neutrosophic graphs. If $f : (V(N_G), \tau_B(N_G)) \to (V(N_H), \tau_B(N_H))$ is a neutrosophic block topological homeomorphism, then a vertex*

$$(v, \eta(v), \kappa(v), \lambda(v)) \in V(N_G)$$

is a neutrosophic cut-vertex if and only if $f(v, \eta(v), \kappa(v), \lambda(v)) \in V(N_H)$ is a neutrosophic cut-vertex. Moreover, the neutrosophic degree of $(v, \eta(v), \kappa(v), \lambda(v))$ is preserved under f.

Proof. Suppose $(v, \eta(v), \kappa(v), \lambda(v)) \in V(N_G)$ is a neutrosophic cut-vertex. Then

$$\{(v, \eta(v), \kappa(v), \lambda(v))\} \in \tau_B(N_G).$$

Since f is a homeomorphism, $f(\{(v, \eta(v), \kappa(v), \lambda(v))\}) = \{f(v, \eta(v), \kappa(v), \lambda(v))\} \in \tau_B(N_H)$. By the neutrosophic analogue of the block topology characterization, if

$$\{f(v, \eta(v), \kappa(v), \lambda(v))\}$$

is open in $\tau_B(N_H)$, then $f(v, \eta(v), \kappa(v), \lambda(v))$ must be either a cut-vertex or an isolated vertex of N_H.If $f(v, \eta(v), \kappa(v), \lambda(v))$ were isolated, then $f(v, \eta(v), \kappa(v), \lambda(v))$ would have no adjacent vertices in N_H. But since f is bijective and preserves adjacency through continuity, this contradicts $(v, \eta(v), \kappa(v), \lambda(v))$ being a cut-vertex in N_G. Thus, $f(v, \eta(v), \kappa(v), \lambda(v))$

must be a cut-vertex. Since f is a homeomorphism, it preserves neutrosophic values associated with open sets. Therefore, the degree of $f(v, \eta(v), \kappa(v), \lambda(v))$ equals the degree of $(v, \eta(v), \kappa(v), \lambda(v))$. The reverse follows symmetrically by applying the same reasoning to $f^{-1} : (V(N_H), \tau_B(N_H)) \to (V(N_G), \tau_B(N_G))$. □

Theorem 2.9. *Let N_G and N_H be neutrosophic graphs. If $f : (V(N_G), \tau_B(N_G)) \to (V(N_H), \tau_B(N_H))$ is a neutrosophic block topological homeomorphism, then N_G and N_H have the same number of neutrosophic blocks. Moreover, for each block B of N_G, there exists a unique block B' of N_H such that $f(V(B)) = V(B')$.*

Proof. Let B be a neutrosophic block of N_G. Then $V(B) \in \Sigma_B(N_G)$, hence $V(B)$ is $\tau_B(N_G)$-open. Since f is a homeomorphism, $f(V(B)) \in \tau_B(N_H)$. By the definition of block topology, $\tau_B(N_H)$-open sets corresponding to maximal connected subgraphs without cut-vertices are precisely the vertex sets of blocks of N_H. Thus, $f(V(B)) = V(B')$ for some block B' of N_H.

To show uniqueness: If $f(V(B))$ were equal to both $V(B'_1)$ and $V(B'_2)$ for distinct blocks B'_1, B'_2 of N_H, then these blocks would share more than one vertex, contradicting the fact that blocks intersect in at most one cut-vertex. Since f is bijective, every block of N_H arises as the image of exactly one block of N_G. Hence, N_G and N_H have the same number of blocks. □

Theorem 2.10 (Composition of Homeomorphisms)**.** *Let N_{G_1}, N_{G_2}, and N_{G_3} be neutrosophic graphs. If*

$$f : (V(N_{G_1}), \tau_B(N_{G_1})) \to (V(N_{G_2}), \tau_B(N_{G_2})) \text{ and } g : (V(N_{G_2}), \tau_B(N_{G_2})) \to (V(N_{G_3}), \tau_B(N_{G_3}))$$

are neutrosophic block topological homeomorphisms, then the composition

$$g \circ f : (V(N_{G_1}), \tau_B(N_{G_1})) \to (V(N_{G_3}), \tau_B(N_{G_3}))$$

is also a neutrosophic block topological homeomorphism.

Proof. Since f and g are bijective, their composition $g \circ f$ is bijective. **Continuity:** Let $W \in \tau_B(N_{G_3})$ be open. Since g is continuous, $g^{-1}(W) \in \tau_B(N_{G_2})$. Since f is continuous, $f^{-1}(g^{-1}(W)) = (g \circ f)^{-1}(W) \in \tau_B(N_{G_1})$. Hence, $g \circ f$ is continuous.

Continuity of the inverse: We have $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. Since f^{-1} and g^{-1} are continuous, their composition is continuous. Therefore, $g \circ f$ is a homeomorphism. □

Corollary 2.11. *The relation "is homeomorphic to" ($\cong$) on neutrosophic block topological spaces is an equivalence relation.*

Proof. **Reflexivity:** For any N_G, the identity homeomorphism theorem shows that

$$(V(N_G), \tau_B(N_G)) \cong (V(N_G), \tau_B(N_G)).$$

Symmetry: If $(V(N_G), \tau_B(N_G)) \cong (V(N_H), \tau_B(N_H))$ via a homeomorphism f, then f^{-1} is a homeomorphism by inverse homeomorphism theorem. so $(V(N_H), \tau_B(N_H)) \cong (V(N_G), \tau_B(N_G))$.

Transitivity: If $(V(N_{G_1}), \tau_B(N_{G_1})) \cong (V(N_{G_2}), \tau_B(N_{G_2}))$ and $(V(N_{G_2}), \tau_B(N_{G_2})) \cong (V(N_{G_3}), \tau_B(N_{G_3}))$, then by Composition of homomorphism theorem, their composition gives $(V(N_{G_1}), \tau_B(N_{G_1})) \cong (V(N_{G_3}), \tau_B(N_{G_3}))$.

Therefore, $\cong$ is an equivalence relation. □

3. Conclusion

This work develops a rigorous framework for analyzing networks under uncertainty by integrating neutrosophic graph theory with topological concepts. It introduces neutrosophic block topological spaces to formally capture complex relational patterns, characterized through cut-vertex-based open sets and the preservation of block structures under continuous and homeomorphic mappings. The framework strengthens classical graph topology by accommodating uncertainty in relational structures and offers a solid mathematical foundation for complex network analysis. Future research may extend this study to dynamic and weighted neutrosophic networks, temporal block topologies, and evolving multi-criteria optimization models.

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