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ON PREY-PREDATOR DYNAMICS WITH CERTAIN INTRINSIC BEHAVIOURS

Abstract. The present article investigates prey-predator interactions by incorporating not only the classical predation dynamics but also infighting among preys and retaliation against predators framed within the context of herd behaviour in prey populations. The study presents a thorough analysis that includes both stability and bifurcation assessments supported by detailed numerical simulations. The existence of transcritical and Hopf bifurcations is established and analyzed.

Keywords: Prey-predator interactions, stability, Hopf bifurcation, herd behavior.

Mathematics Subject Classification 2020: Primary: 92B05, 93E15, Secondery: 60H10, 34E10.

1. Introduction

The study of popualtion dynamics, nowadays, has become a prime topic in the arena of applied mathematics. The seed of the topic was planted by Lotka [9] and Volterra [14] by improving the theory of Malthus [12]. In the recent decades the theory has been further developed by several authors in several perspectives. For instance, we refer the works in [3, 4, 8, 13, 15] among many others. Very recently, in [10], the present authors with Mondal have studied prey-predator model in the light of Smith growth dynamics and established several significant results of population biology.

The number of preys attacked by a predator in certain time is known as functional response. C. S. Holling [5] introduced three kind of functional responses which have become a lucrative field of non-linear analysis in view of their ecological significance.

A fundamental feature of the behaviour of the forest dwelling animals such as buffaloes, sheep, deer is that they roam in groups in forests. Such behaviours are also exhibited by birds and fishes. When the animals move in herd the attack by predators are possible only from a boundary of the herd. In Savana region movement of preys and predators in groups are well known and established fact. Considering upon this phenomenon Ajraldi et al [1] proposed the herd behaviour model in population system. In order to develop the analytical aspects of such situations, they introduced square root functional response. But a lacuna of the square root model is that it needs critical analysis near the origin since a square root function is not differentiable at the origin and due to this fact it prevents the linearization of the model. Some minute aspects of this model was critically analyzed by Braza

[2] considering square root and Holling type II functional responses in a common framework. In spite of this limitation square root models are being considered by several authors [4] due to its realistic nature. Square root models with combination of other functional responses are prime topics of study in the field of mathematical ecology. Retaliation towards predators by preys is a common fact in Savana region wild lives. In [11], the present authors studied retaliation behaviours but they did not consider the infighting of preys.

The intra or infighting among populations is a common phenomenon. In [6], Kooi studied the dynamics of a predator-prey system where predators fight for captured prey besides searching and handling of preys. In real situation, it is seen that not only the predators exhibit infighting behaviours. Sometimes the preys also involve with infighting due to their intrinsic behaviour. For example, wild buffaloes show infighting behaviour. In addition wild buffaloes also defend in groups if they are attacked by wild animals like tiger. They take revenge or retaliation. In this paper we would like to analyze such situations i.e., we consider prey-predator model where preys move in groups, show infighting and also take revenge or retaliation towards to the predators.

The present article is organized as follows: After the introduction, the model is formulated in Section 2. Section 3 and Section 4 contain the analysis of stability and bifurcation respectively. The final section provides numerical simulations with graphical representations.

2. Derivation of predator-prey model with herd and infighting behaviour in preys and retaliation towards predators

It is known that the predator-prey model with logistic growth in the prey and Holling type II functional response [5] is given by

$$\frac{dX}{dt} = rX(1 - \frac{X}{N}) - \frac{\alpha XY}{1 + t_h \alpha X}, \tag{2.1}$$

$$\frac{dY}{dt} = -sY + \frac{c\alpha XY}{1 + t_h \alpha X}. \tag{2.2}$$

Here $X(t)$ and $Y(t)$ denote the prey population and predator population respectively. r is the growth rate of the prey population and N is its carrying capacity. In absence of prey population the death rate of the predator is s. The search coefficient of Y for X is α. c being biomass conversion or consumption rate. The average handling time of Y for X is t_h. Suppose T indicate the total time that each Y takes to collect food from X, T_s is the time taken by each Y looking for X and the time t_h that each Y takes handling X. Modifying the above model, Braza [2], developed prey-predator models as follows that have the interaction term as the square root of the prey population.

$$\frac{dX}{dt} = rX(1 - \frac{X}{N}) - \frac{\alpha\sqrt{X}Y}{1 + t_h \alpha\sqrt{X}}, \tag{2.3}$$

$$\frac{dY}{dt} = -sY + \frac{c\alpha\sqrt{X}Y}{1 + t_h\alpha\sqrt{X}}. \tag{2.4}$$

Now, here we develop a model where the preys are involved with intra species fighting due to their intrinsic behaviour. Assume β is the rate of death of preys due to the infighting and the death rate for the predator is γ due to retaliation of the preys towards the predators. In that case the above model reduces to

$$\frac{dX}{dt} = rX(1 - \frac{X}{N}) - \frac{\alpha\sqrt{X}Y}{1 + t_h\alpha\sqrt{X}} - \beta X^2, \tag{2.5}$$

$$\frac{dY}{dt} = -sY + \frac{c\alpha\sqrt{X}Y}{1 + t_h\alpha\sqrt{X}} - \gamma Y. \tag{2.6}$$

Here $X(0) > 0$ and $Y(0) \geq 0$.

3. Stability analysis of predator-prey model with herd and infighting behaviour in preys and retaliation towards predators

3.1. **Positivity and boundedness of the model:** Let us investigate the positivity and boundedness of the system of equations (2.5) and (2.6). The right hand sides of (2.5) and (2.3) are continuous functions of the dependent variables X and Y. Integrating both sides of the equations of the system, we have

$$X(t) = X(0)exp\left[\int_0^t \left(r\left(1 - \frac{X}{N}\right) - \frac{\alpha Y}{1 + t_h\alpha X} - \beta X\right) dx\right]$$

$$Y(t) = Y(0)exp\left[\int_0^t \left(-s + \frac{c\alpha\sqrt{X}}{1 + t_h\alpha\sqrt{X}} - \gamma\right) dx\right]$$

In view of the above expressions of $X(t)$ and $Y(t)$ it can be inferred that $X(t)$ and $Y(t)$ remain non-negative for the infinite time, if they starts from an interior point of

$$R_+^2 = \{(X(t), Y(t)) \in R^2 : X(t) > 0, Y(t) \geq 0\}.$$

Thus R_+^2 is positively invariant for the system considered.

Regarding the uniform boundedness, let us prove the folllowing

Theorem 3.1. *The solutions of the system of equations* (2.5) *and* (2.6) *consisting the model with non-negative initial conditions* $(X(0), Y(0))$ *starting from the interior of* R_+^2 *are uniformly bounded.*

Proof. Let

$$W(t) = X(t) + \frac{1}{c}Y(t). \tag{3.1}$$

Hence, along the solution trajectories of the model

$$\frac{dW}{dt} = rX(1 - \frac{X}{N}) - (\beta + \gamma + \frac{s}{c})Y. \tag{3.2}$$

By virtue of (3.1) and (3.2)

$$\frac{dW}{dt} + \theta W = (r+\theta)X - \frac{r}{N}X^2 - (\beta + \gamma + \frac{s-\theta}{c})Y.$$

Choosing θ such that $0 < \theta < s$, we have from above

$$\begin{aligned} \tfrac{dW}{dt} + \theta W &\leq (r+\theta)X - \tfrac{r}{N}X^2 \\ &= \tfrac{r}{N}\left(\tfrac{N^2(r+\theta)^2}{4r^2} - \left(\tfrac{N(r+\theta)}{2r} - X\right)^2\right) \\ &\leq \tfrac{N(r+\theta)^2}{4r}. \end{aligned}$$

Thus from above

$$\frac{dW}{dt} + \theta W \leq P, \tag{3.3}$$

where $P = \frac{N(r+\theta)^2}{4r}$. In view of differential inequality from (3.3), one obtains

$$0 \leq W(t) \leq \frac{P}{\theta}(1 - e^{-\theta t}) + W(0)e^{-\theta t}.$$

As $t \to \infty$, we infer $0 \leq W(t) \leq \frac{P}{\theta} + \epsilon$ for $0 < \epsilon < W(0)$. Hence, in view of positivity of $X(t)$ and $Y(t)$, we conclude that all solutions initiating in R^2_+ are restricted in the region $D = \left\{(X(t), Y(t)) \in R^2 : X(t) + \frac{1}{c}Y(t) \leq \frac{P}{\theta} + \epsilon, \epsilon > 0\right\}$. Thus the solutions of the system are uniformly bounded. Hence the model is biologically well posed. □

3.2. **Equlibrium points and their stability:** Using, MAPLE we find the equilibrium points of the above model are

(i) $X = 0, Y = 0$

(ii) $X = \frac{Nr}{N\beta + r}$, $Y = 0$.

(iii)$X = \frac{(s+\gamma)^2}{\alpha^2(c - t_h(s+\gamma))^2}$, $Y = \frac{c(s+\gamma)}{\alpha^2(c-t_h(s+\gamma))^2}\left(\frac{r-\beta}{\alpha^2} - \frac{r}{N}\frac{(s+\gamma)^2}{(c-t_h(s+\gamma))^2}\right)$.

The Jacobian of the system is

$$\begin{bmatrix} r - \frac{2rX}{N} - \frac{\alpha Y}{2\sqrt{X}(1+t_h\alpha\sqrt{X})^2} - + \frac{\alpha^2 Y t_h}{2(1+t_h\alpha\sqrt{X})^2} - 2\beta X & -\frac{\alpha\sqrt{X}}{1+t_h\alpha\sqrt{X}} \\ \frac{c\alpha Y}{2\sqrt{X}(1+t_h\alpha X)} - \frac{c\alpha^2\sqrt{X}Yt_h}{(1+t_h\alpha X)^2} & -s + \frac{c\alpha\sqrt{X}}{1+t_h\alpha X} - \gamma \end{bmatrix}.$$

3.3. **Local stability for case (i).** In this subsection, we discuss the nature of stability for $X = 0$ and $Y = 0$. Since $\frac{1}{\sqrt{X}}$ is indeterminate for $X = 0$ and the square root function is not differentiable at $X = 0$, the stability of this case cannot be determined by simply evaluating the Jacobian matrix at $X = 0$ and $Y = 0$. So let us consider a deleted neighbourhood of $(0, 0)$, say $(\epsilon, 0)$, where ϵ is arbitrarily small and tends to zero. At that point the Jacobian matrix is of the form

$$\begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix},$$

where $a_{11} = r - \frac{2r\epsilon}{N} - 2\beta\epsilon$. $a_{12} = -\frac{\alpha\sqrt{\epsilon}}{1+t_h\alpha\sqrt{\epsilon}}$. $a_{22} = -s + \frac{c\alpha\sqrt{\epsilon}}{1+t_h\alpha\epsilon} - \gamma$. Thus the eigenvalues are $r - \frac{2r\epsilon}{N} - 2\beta\epsilon$ and $-s + \frac{c\alpha\sqrt{\epsilon}}{1+t_h\alpha\epsilon} - \gamma$. It is seen that in such a case the values of r, s and γ are very crucial to determine the stability. The above discussion leads us to state the following:

Theorem 3.2. *A deleted neighbourhood $(0, \epsilon)$ of the trivial equilibrium point is locally stable if $r < \frac{2r\epsilon}{N} + 2\beta\epsilon$ and $s + \gamma > \frac{c\alpha\sqrt{\epsilon}}{1+t_h\alpha\epsilon}$.*

3.4. **Local stability for case (ii).** In this subsection, we investigate the nature of stability for $X = \frac{Nr}{N\beta+r}$, $Y = 0$. and prove the following:

Theorem 3.3. *The equilibrium point $X = \frac{Nr}{N\beta+r}$, $Y = 0$ is locally stable $1 > \frac{r}{N\beta+r}$ and $s + \gamma > \frac{c\alpha\sqrt{\frac{Nr}{N\beta+r}}}{1+\frac{t_hNr\alpha}{N\beta+r}}$.*

Proof. The Jacobian matrix for case (i) is of the form

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},$$

where

$$a_{11} = -r(1 - \frac{r}{N\beta+r}) - \frac{r^2}{N\beta+r} - \frac{2\beta Nr}{N\beta+r}$$

$$a_{12} = -\frac{\alpha\sqrt{\frac{Nr}{N\beta+r}}}{1+t_h\alpha\sqrt{\frac{Nr}{N\beta+r}}}.$$

$$a_{21} = 0.$$

$$a_{22} = -s - \gamma + \frac{c\alpha\sqrt{\frac{Nr}{N\beta+r}}}{1+\frac{t_hNr\alpha}{N\beta+r}}.$$

The eigenvalues are $-r(1 - \frac{r}{N\beta+r}) - \frac{r^2}{N\beta+r} - \frac{2\beta Nr}{N\beta+r}$ and $-s - \gamma + \frac{c\alpha\sqrt{\frac{Nr}{N\beta+r}}}{1+\frac{t_hNr\alpha}{N\beta+r}}$. Both the eigenvalues will be negative if $1 > \frac{r}{N\beta+r}$ and $s + \gamma > \frac{c\alpha\sqrt{\frac{Nr}{N\beta+r}}}{1+\frac{t_hNr\alpha}{N\beta+r}}$. □

3.5. **Local stability for case (iii).** In this subsection, we investigate the nature of stability for $X = \frac{(s+\gamma)^2}{\alpha^2(c-t_h(s+\gamma))^2}$, $Y = \frac{c(s+\gamma)}{\alpha^2(c-t_h(s+\gamma))^2}\left(\frac{r-\beta}{\alpha^2} - \frac{r}{N}\frac{(s+\gamma)^2}{(c-t_h(s+\gamma))^2}\right)$ and establish the following:

Theorem 3.4. *The equilibrium point*

$$X = \frac{(s+\gamma)^2}{\alpha^2(c - t_h(s+\gamma))^2} \text{ and } Y = \frac{c(s+\gamma)}{\alpha^2(c - t_h(s+\gamma))^2}\left(\frac{r-\beta}{\alpha^2} - \frac{r}{N}\frac{(s+\gamma)^2}{(c - t_h(s+\gamma))^2}\right)$$

is locally asymptotically stable if

$$\frac{r\alpha^2}{N}.\frac{(s+\gamma)^2}{(c - t_h(s+\gamma))^2} < r - \beta$$

$$< \frac{r\alpha(s+\gamma)^2\big(4rc - \alpha(c - t_h(s+\gamma))\big) + (r-\beta)N(c - t_h(s+\gamma))^3}{2N\alpha^3 rc(c - t_h(s+\gamma))^2}.$$

Proof. The Jacobian matrix for case (ii) is of the form

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},$$

where

$$a_{11} = (r-\beta) - \frac{r\alpha(s+\gamma)^2\big(4rc - \alpha(c - t_h(s+\gamma))\big) + (r-\beta)N(c - t_h(s+\gamma))^3}{2N\alpha^3 rc(c - t_h(s+\gamma))^2}.$$

$$a_{12} = -\frac{s+\gamma}{c}.$$

$$a_{21} = \frac{1}{2}\left(\frac{r-\beta}{\alpha^2} - \frac{r}{N}\frac{(s+\gamma)^2}{(c - t_h(s+\gamma))^2}\right).$$

$$a_{22} = 0.$$

□

The eigenvalues λ of the above Jacobian is given by

$$\lambda^2 - B_1\lambda + B_2 = 0,$$

where $B_1 = a_{11} + a_{12}$ and $B_2 = a_{11}a_{22} - a_{12}a_{21}$. Assume

$$r - \beta > \frac{r\alpha^2}{N}.\frac{(s+\gamma)^2}{(c - t_h(s+\gamma))^2}. \tag{3.4}$$

Then $a_{22} = 0$, $a_{12} < 0$ and $a_{21} > 0$. So the determinant of the Jacobian for this case is positive. Consequently, $B_2 > 0$. Hence, by Routh Hurwitz criterion theorem [15], the solution will be asymptotically stable if $a_{11} + a_{22} < 0$. Now, $a_{22} = 0$. Hence, the solution will be asymptotically stable if $a_{11} < 0.$, i.e.,

$$r - \beta < \frac{r\alpha(s+\gamma)^2\big(4rc - \alpha(c - t_h(s+\gamma))\big) + (r-\beta)N(c - t_h(s+\gamma))^3}{2N\alpha^3 rc(c - t_h(s+\gamma))^2}. \tag{3.5}$$

Combining (3.4) and (3.5), we get the result.

4. Bifurcation Analysis:

In the present section, we shall analyze some aspects of local bifurcation analysis of the model. The nature of solutions depend upon some changes of the associated parameters. The value of a parameter for which the nature of the solution structurally changes is known as critical value and the changing phenomenon is known as bifurcation. In the following we exhibit the presence of transcritical and Hopf bifurcation in the system.

In the previous section, we have seen that the point $(0, \epsilon)$ is asymptotically stable if $r < \frac{2r\epsilon}{N} + 2\beta\epsilon$ and $s + \gamma > \frac{c\alpha\sqrt{\epsilon}}{1+t_h\alpha\epsilon}$. Now if we change r such that $r > \frac{2r\epsilon}{N} + 2\beta\epsilon$ without changing the other parameters, then the point becomes unstable and axial equilibrium arises. If $1 > \frac{r}{N\beta+r}$, then the axial equilibrium point is stable. Hence the point $(0, \epsilon)$ exchanges its stability with the axial equilibrium. If we change r again such that $1 < \frac{r}{N\beta+r}$, the axial equilibrium point becomes unstable. Now, we are in a position to state the following:

Theorem 4.1. *The equilibrium point $(0, \epsilon)$ undergoes through transcritical bifurcation for $r = \frac{2r\epsilon}{N} + 2\beta\epsilon$, and the axial equilibrium point undergoes transcritical bifurcation for r satisfying $\frac{r}{N\beta+r} = 1$ with certain specific conditions.*

Now we prove the following result related to Hopf bifurcation.

Theorem 4.2. *At the equilibrium point*

$$X = \frac{(s+\gamma)^2}{\alpha^2(c - t_h(s+\gamma))^2},\ Y = \frac{c(s+\gamma)}{\alpha^2(c - t_h(s+\gamma))^2}\left(\frac{r-\beta}{\alpha^2} - \frac{r}{N}\frac{(s+\gamma)^2}{(c - t_h(s+\gamma))^2}\right)$$

for $r - \beta > \frac{r\alpha^2}{N}\cdot\frac{(s+\gamma)^2}{(c-t_h(s+\gamma))^2}$, the system undergoes Hopf bifurcation with respect to the parameter β for the critical value $\beta_0 = r\left(1 - \frac{\alpha(s+\gamma)^2\left(4rc - \alpha(c-t_h(s+\gamma))\right)}{1-N(c-t_h(s+\gamma))^3}\right)$.

Proof. Consider the Jacobian matrix

$$\begin{bmatrix} a_{11} & 12 \\ a_{21} & a_{22} \end{bmatrix},$$

of the system for $X = \frac{(s+\gamma)^2}{\alpha^2(c-t_h(s+\gamma))^2}$, $Y = \frac{c(s+\gamma)}{\alpha^2(c-t_h(s+\gamma))^2}\left(\frac{r-\beta}{\alpha^2} - \frac{r}{N}\frac{(s+\gamma)^2}{(c-t_h(s+\gamma))^2}\right)$. For $r - \beta > \frac{r\alpha^2}{N}\cdot\frac{(s+\gamma)^2}{(c-t_h(s+\gamma))^2}$ and $\beta = r\left(1 - \frac{\alpha(s+\gamma)^2\left(4rc-\alpha(c-t_h(s+\gamma))\right)}{1-N(c-t_h(s+\gamma))^3}\right)$, $a_{12} < 0$ and $a_{21} > 0$. Again $a_{22} = 0$. In this case the characteristic equation of the Jacobian matrix is $\lambda^2 = a_{12}a_{21} < 0$. Thus the eigenvalues are purely imaginary. Verifying the transversality condition, one can conclude that the model undergoes Hopf bifurcation at the considered point for $\beta = r\left(1 - \frac{\alpha(s+\gamma)^2\left(4rc-\alpha(c-t_h(s+\gamma))\right)}{1-N(c-t_h(s+\gamma))^3}\right)$. □

5. Numerical simulations

Theoretical results are better understood whenever they are supplemented by graphical representations as graphs are nice mathematical tools that provide quick concept of any theory. This is why we represent the stability, phase portrait and the aspects of bifurcation graphically in the subsequent part of the article.

5.1. **Nullclines and solution curves.** Let us now exhibit the graphs for tne nullclines and solution curves for different numerical values of the parameters. Figures 1 and 2 show the existence of locally stable interior equilibrium that verifies Theorem 3.4. Figure 3 ensures the stability of the point $(0, \epsilon)$ in the tune of Theorem 3.2. Figure 4 verifies the existence of axial equilibrium point and its stability associated with Theorem 3.3.

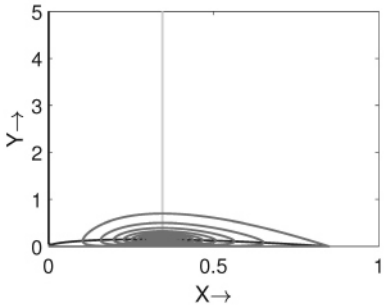


Fig. 1: Nullclines and solution curve for $c = 5, s = 5, r = 1, N = 200, \alpha = 2, \gamma = 0.8, \beta = 0.15, t_h = 1$. The Figure shows the existence of stable interior equilibrium.

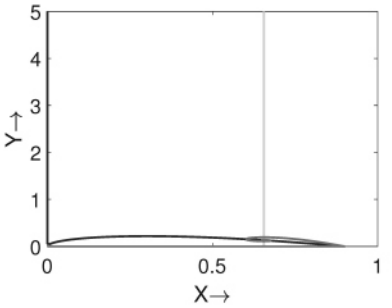


Fig. 2: Nullclines and solution curve for $c = 5, s = 5, r = 0.1, N = 200, \alpha = 1.5, \gamma = 1, \beta = 0.1, t_h = 1$. The Figure shows the existence of stable interior equilibrium.

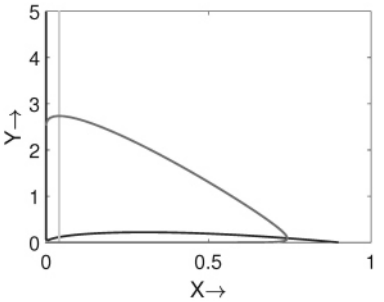


Fig. 3: Nullclines and solution curve for $c = 5, s = 0.5, r = 1, N = 200, \alpha = 1.5, \gamma = 1, \beta = 0.1, t_h = 1$. The figure shows existence of stable trivial equilibrium.

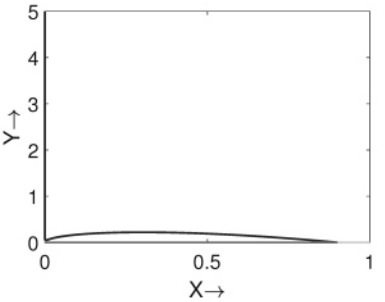


Fig. 4: Nullclines and solution curve for $c = 1, s = 0.5, r = 1, N = 200, \alpha = 1.5, \gamma = 1, \beta = 0.1, t_h = 1$. The figure shows existence of stable axial equilibrium

5.2. **Phase portrait and time series.** In the following, we plot phase portrait and time series of the model by using MATLAB for different numerical values of the associated parameters and verify existence of bifurcation. From Figures 7 and 8, it is seen that for change of s the asymptotically stable interior equilibrium point becomes unstable or vice versa which verifies the existence of Hopf bifurcation at interior equilibrium point. Since in the assumed condition of Theorem 4.2, β and s are related, so for change of β we must have Hopf bifurcation at the interior equilibrium. Thus the numerical representation agrees with Theorem 4.2.

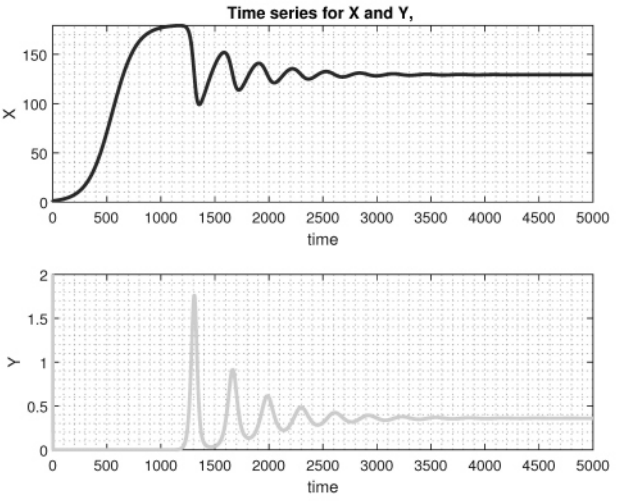


Fig. 5: $s = 3.2, c = 4, r = .01, N = 200, \alpha = 0.9t_h = 1, \gamma = 0.444, \beta = 0.001.$ The figure establishes Hopf bifurcation.

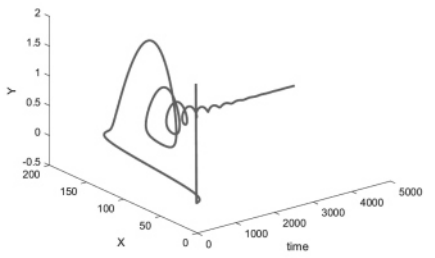


Fig. 6:
$s = 3.2, c = 4, r = .01, N = 200, \alpha = 0.9t_h = 1, \gamma = 0.444, \beta = 0.001.$ The figure establishes Hopf bifurcation.

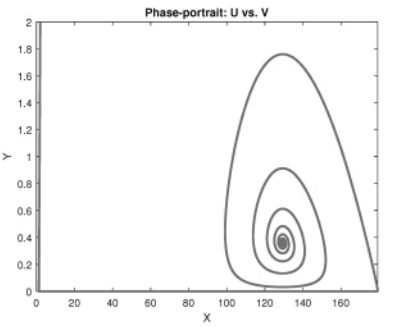


Fig. 7:
$s = 3.2, c = 4, r = .01, N = 200, \alpha = 0.9t_h = 1, \gamma = 0.444, \beta = 0.001$ The figure establishes Hopf bifurcation.

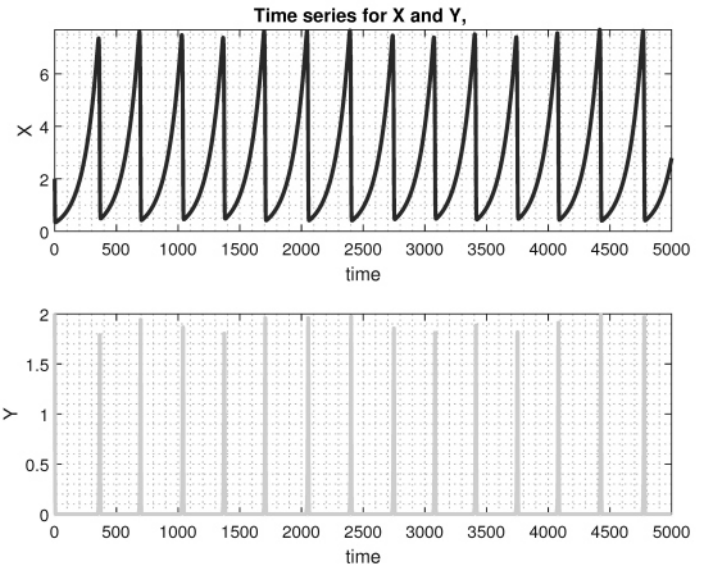


Fig. 8: $s = 2, c = 4, r = .01, N = 200, \alpha = 0.9t_h = 1, \gamma = 0.444, \beta = 0.001$ The figure establishes existence of limit cycle.

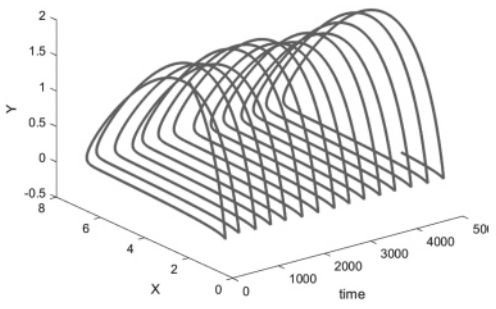


Fig. 9:
$s = 2, c = 4, r = .01, N = 200, \alpha = 0.9t_h = 1, \gamma = 0.444, \beta = 0.001$ The figure establishes existence of limit cycle.

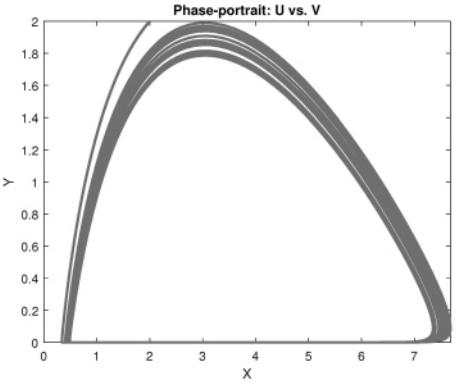


Fig. 10:
$s = 2, c = 4, r = .01, N = 200, \alpha = 0.9t_h = 1, \gamma = 0.444, \beta = 0.001.$ The figure establishes existence of limit cycle.

5.3. **Bifurcation diagrams.** In this sub-section, using MATLAB, we plot two bifurcation diagrams for different numerical values of the parameters exhibited against each figure.

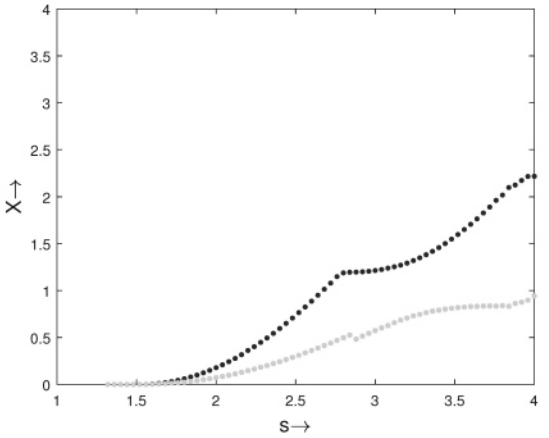


Fig. 11: Hopf Bifurcation diagram for
$c = 4, r = 0.01, N = 200, \alpha = 0.9, t_h = 1, \gamma = 0.444, \beta = 0.001.$

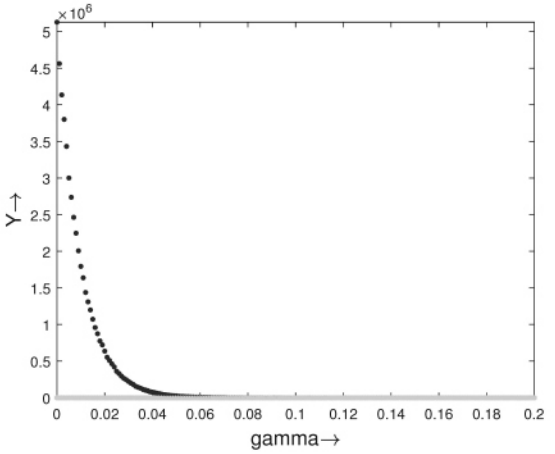


Fig. 12: Hopf Bifurcation diagram for
$c = 5.5, t_h = 1, r = 0.01, \alpha = 4.7, s = 1, \beta = 0.001.$

Conclusion. The stability and local bifurcations are analyzed for prey predator system with infighting and herd behaviour in preys and the preys showing retaliation behaviour towards the predators. Theoretically the existence of Hopf bifurcations are shown and graphical representations are done for different numeric values of

the associated parameters from where the aspects of stability and bifurcations are easily understood.

Future scope of study. It will be an interesting study if one considers Allee effect in preys and Hunting cooperation among predators in the perspective of present model.

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References

[1] Ajraldi, V., Pittarino, M. Venturino, E., Modelling Herd behavior in population system, Nonlinear Analysis, RWA, 12(2011), 2319-2338.

[2] Braza, P. A., Predator-prey dynamics with square root functional response, Nonlinear Analysis: real world Applications, 13(2012), 1837-1843.

[3] Cosner, C., De Angelis, D. L., Effects of spatial grouping on functional response of predator, Theoretical population Biology, 56(1999), 65-75.

[4] Debnath, S., Majumdar, P., Sarkar, S. and Ghosh, U., Complex dynamical behaviour of a delayed prey-predator model with square root functional response in presence of fear in the prey, Int J Modelling and simulation, 43(2023), 612-637.

[5] Holling, C. S., The components of predation as revealed by a study of small mamal predation of the European pine sawfly. Can. Entomol, 91(1959), 293-320.

[6] Kooi, B. W., Modelling the dynamic traits involved in fighting predator-prey system, J. Math. Biol. 71(2015), 1575-1605.

[7] Kaushik, R. and Banerjee, S., Predator-prey system: Prey's counter-attack on juvenile predators shows opposite side of the same ecological coin, Applied Mathematics and Computation, 388(2021), 125330.

[8] Kuang, Y., Beretta, E., Global qualitative analysis of a ratio dependent predator-prey system, J. Math. Biol., 36(1998), 389-406.

[9] Lotka, A. J., Elements of physical biology, Williams and Wilkins Co., Inc., Baltimore.

[10] Maiti, T., Mondal, B. and Sarkar, A., Dynamic Interplay: Cooperative hunting strategies in a predator-prey model with Smith growth dynamics, Brazilian J. Physics, 54(2024):82, 11pp.

[11] Maiti, and Sarkar, A., On prey-predator model with herd and retaliation behaviours in preys, Appl. Math. E Notes, (To appear in 2025).

[12] Malthus, T., An essay on the principle of population, J. Joston, St. Paul's Church-Yard, London, 1978.

[13] Mondal, B., Roy S., Ghosh, U. and Tiwari, P. K., A systematic study of autonomous and non-autonomous predator prey models for the combined effects of fear, refuge, cooperation and harvesting, Eur. Phys J. Plus, 2022, 137: 724.

[14] Volterra, V., Variazionj e fluttauazionidel numero d' individui in specie animals conviventi, Mem. Acad. Lincei, 2(1926), 31-33.

[15] Wiggins, S., Stephen, W. and Martin, G., Introduction to applied nonlinear dynamical systems and chaos, Vol 2, No 3, New York, Springer, 2003.

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