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CONTROLLING INSURGENCY IN NORTHEAST INDIA BY GENERATION OF EMPLOYMENT AND AWARENESS: A SOCIO – MATHEMATICAL STUDY

Abstract

Insurgency is a persistent issue in Northeast India. This region of India is a habitat of different tribes with distinct ethnicity and cultural diversity. However, due to the mountainous and hilly territory Northeast India not well connected with the mainland. Therefore, it has been deprived of various socioeconomic development opportunities like the rest part of India. This deprivation fused with social, political and religious prosecutions has many times resulted in outburst of extremism and insurgency in different parts of Northeast India. It has caused unnecessary waste of lives and a major concern for law & order. Foreign powers also instigate such radical ideologies for their own vested interest. Therefore, eradication of this problem of insurgency from Northeast India is essentially required not only for this region but also for whole India. In this article we propose a deterministic mathematical model using the nonlinear differential equations to establish the hypothesis that the issue of insurgency in Northeast India can be resolved by adequate socioeconomic developments. We consider generation of employments and awareness against insurgency through proper education as the vital indicators of socioeconomic development. The boundedness and positivity of solutions have been studied. The existence and stability of the equilibrium solutions have been analyzed analytically as well as numerically. It has been shown that even in a society where unemployed youth are more prone to get involved in extremism due to their socioeconomic frustrations, insurgency can be eradicated by creating new jobs in a sufficiently higher rate and generating awareness against insurgency through appropriate education. Therefore, our results can be applied while framing various governmental policies for Northeast India to resolve the persistent issue of insurgency in this region.

Keywords: Insurgency, Employment, Northeast India, Mathematical model, Stability analysis, Numerical analysis, Critical values, Control mechanisms.

Mathematics Subject Classification 2020: 91D10, 97M10.

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1 Introduction

Insurgency has been one of the major problems that India had to encounter in the Northeast India since the time of India's independence. The Nagas were the first to raise their voice for a free Naga homeland and against forceful imposition of India's rule in the Naga hills. The Naga National Council (NNC) was formed under the leadership of A. Z. Phizo in 1946. The NNC under the leadership of Phizo declared Naga independence on 14 August 1947 and led an armed insurrection movement from early 1950s when their demand for a sovereign Naga state was turned down. The problem did not come down even after the formation of Nagaland in December 1963 due to refusal of NNC to recognize the new state. A peace accord was signed in 1975 in Shillong but the stalemate continued because Issac Swu and Thuingleng Muivah denounced the accord and they along with S S Kaphlang formed the Nationalist Socialist Council of Nagaland (NSCN) on January 31, 1980. The situation has improved considerably since the time of singing the "Indo-Naga" ceasefire between NSCN (I-M) and the Government of India on July 1, 1997 (Kotwal, 2000). However, the problem still remains unresolved (Karmakar, 2022). Following the footsteps of the Naga insurgency, the Mizo National Front (MNF) under the leadership of Phu Laldenga declared independence in Mizo hills and began a secessionist movement. With the formation of Mizoram state in 1987, the problem of Mizo insurgency had been resolved (Saikia, 2022). Tripura merged with the Union of India on September 9, 1949. But, disgruntlement among the tribal communities continued. The problem of insurgency turned into a serious law and order problem in Tripura in the 1980s with formation of Tripura National Volunteer (TNV) in 1981, National Liberation Front of Tripura (NLFT) in 1989 and All Tripura Tiger Force (ATTF) in 1990 demanding for political sovereignty of Tripura (Ghosh, 2003). Similarly, Manipur also merged with the Union of India on October 15, 1949. However, a section of the Meiteis was unhappy with the post-merger political development in Manipur fuelling secessionist tendencies. A number of separatist groups such as United National Liberation Front (UNLF), People's Liberation Army (PLA), People's Revolutionary Party of Kangleipak (PREPAK) were formed in 1960s and 1970s to achieve independence through armed struggle (Singh, 2015; Sundari & Sasikala, 2020) Assam has also been inflicted by insurgency problem. The insurgent groups like United Liberation Front of Assam (ULFA), National Democratic Front of Bodoland (NDFB), Bodo Territorial Tiger Force (BTLF) carried on their subversive activities against India's security establishment (Bhuyan, 2019).

1.1 Causes for the growth of insurgency in the Northeast India

There are different reasons that caused to the growth of insurgency and reasons which factored to sustain insurgency in the Northeast India. All these factors did not have similar effect in all the states and but incited the growth of ethnic nationalism in the entire Northeast.

1.1.1 Ethnicity in the Northeast India

Northeast India is home of different ethnic groups. Majority of these ethnic groups belong to the Sino-Tibetan and Tibeto-Burman groups of people. Therefore, in term of ethnic affiliation, they have closer proximity with the people of South East Asia than the people of mainland India (Hazarika, 2013). The British colonial administration, *albeit,* exploited the resources of Northeast India, yet it provided protection to the many Northeast hill tribes from the exploitation of the plain businessmen by promulgating the Bengal Eastern Frontier Regulations of 1887. The regulations restricted the entry of outsiders without permit obtaining from designated officers and provided internal autonomy to the hill tribes (Bengal Eastern Frontier Regulations, 2020). Naturally, following British decision to withdraw from India and India's attainment of independence, when there was jubilation and celebration, many hill tribes looked dejected thinking about their future political future. There was confusion whether they would get similar political protection following their integration with India or they would face cultural assimilation in the process of *Indianization* (Nag, 1998). Ethnicity, which De Vos defined as subjective, symbolic or emblematic use to differentiate themselves from other groups (Brass, 1991) was employed by many ethnic groups in the Northeast to reiterate their distinctive cultural identities and need for a homeland to protect and preserve their distinctiveness. Their apprehension has been addressed by the Government of India. New states such as Arunachal Pradesh, Nagaland, Mizoram, Meghalaya were created. Autonomous District Council under Sixth Schedule of the Indian Constitution were created in Mizoram, Tripura, Assam and Meghalaya were formed to address the ethnic demands of the small minority ethnic groups. The Government of India retained the Inner Line Regulations of the Bengal Eastern Regulations of 1887 in states like Arunachal Pradesh, Nagaland, Mizoram and Manipur. The constitution of India accepted cultural pluralism as symbol of national identity giving importance to the cultural values of every community in the country (Mohammed, 2016).

1.1.2 Illegal migration and demographic changes in the Northeast India

Following partition of the Indian sub-continent and formation of India and Pakistan, there had been an influx of refugees from East Pakistan (now Bangladesh) in Assam, Tripura and Meghalaya. The situation of minorities did not improve even after emergence of Bangladesh as an

independent nation state in 1971. Northeast Indian states sharing international border with Bangladesh in the western side continued to experience influx of illegal migration from Bangladesh. The influx of refugee and illegal migration from Bangladesh brought a dramatic change in demographic landscape in the Northeast India particularly in Tripura and Assam with the indigenous peoples converting into minority in Tripura. Anti-foreigner agitation in Assam, Tripura and Meghalaya turned violent threatening law and order situation in the Northeast India. Naturally, the Government of India decided to insert Section 6A in the Citizenship Act defining March 25, 1971 as the prescribed cut-off date for detection and deportation of foreigners in Assam. The Government of India promulgated several acts such as the Passport (Entry into India) Act, 1920 restricting and regulating the entry of the foreigners into India, the Foreigners (Tribunals) Order of 1964 empowering the district magistrates in all States and Union Territories to set up tribunals to decide on whether a person staying illegally is a foreigner or not. The illegal Migrants (Determination by Tribunals) Act, 1983 was enacted for detection and deportation of all illegal migrants who entered into India after March 25, 1971 (Singh, 2019).

1.1.3 Lack of economic development and sense of deprivation & exploitation

Geographically Northeast India is connected with the mainland India only through the Jalpaiguri corridor. The region did not witness much economic development due to its geographical isolation, road and connectivity problem and lack of attention of Government of India in economic development policies caused resentment and insecurity amongst the people in the Northeast India. The economic underdevelopment exacerbated the employment problem luring many youths to join various insurgent groups. The Mizo nationalism which culminated into a secessionist movement under the umbrella of MNF was a ramification of economic deprivation from the Government of Assam during the outbreak of rat famine in the Mizo hills in 1959 (Benjamin, 2022). The Government of India converted the Look East Policy into Act East Asia Policy in 2014. Northeast India has been projected as India's gateway to South East Asia. Since then, Government of India has been developing the road and connectivity in the Northeast to accelerate the process of economic development in the region. The development of tourism sector is given top priority. The Government of India is encouraging in entrepreneurship proving training to the youths. The Tribal Welfare Department, the nodal department for tribal development, in every state in the Northeast India provides short terms loans to become entrepreneurs.

The intensity of the insurgency movement has substantially declined in the Northeast India not only due to some proactive political measures but also largely it is because of economic growth and development that Northeast India has been experiencing in the last two decades. G Kishan

Reddy, the Union Minister of Development of Northeast Region said in 2022 in the Parliament that the notable decline in insurgency in the region was due to Government of India's agenda of transforming Northeast India with a new phase of peace and prosperity (Singh, 2022). Therefore, if the pace of economic development can be accelerated, the problem of ethnic insurgency can be contained.

2 Mathematical models to contain the insurgency problem

A thorough understanding of the insurgency problem, namely, the causation, propagation, prevention etc. is essentially required to develop the effective and viable intervention strategies to mitigate the issues and challenges of insurgency and associated violence. In this regard mathematical models can provide an important insight in this direction. Applications of differential equations and related mathematical techniques to study various socio-economic problems was first pioneered by Lewis F. Richardson (Richardson, 1960). He studied the causes of war, its propagation, impacts of arms race between countries and the eruption of war using mathematical models (Richardson, 1935). In recent times various mathematical models have been studied by various researchers to analyze the dynamics of political party growth, the spread of crime etc. (see for instance Crisosto et al., 2010; Hayward, 1999; Jeffs et al., 2016; McMillon, Simon & Morenoff, 2014; Mohammad & Roslan, 1870; Perc, Donnay & Helbing, 2013; Romero et al., 2011; Sooknanan, Bhatt & Comissiong, 2013, 2016 and the references therein). A comprehensive review of the mathematical models to study the dynamics of crimes can be found in (D'Orsogna & Perc, 2015; Sooknanan & Comissiong, 2018). Use of compartmental models to study various socio-economic issues such as insurgency, terrorism, fanatic behavior, violence, radicalization etc. can be found in Camacho (2013), Castillo-Chavez (2003), Galam (2016), Helbing et al. (2015), McCluskey (2018), Nathan (2018), Santoprete (2018) and the references therein. Chuang (2018) studied an age-structured model of radicalization and a bistable model of terrorism (Chuang, 2017). A game theoretic terrorism model was proposed and analyzed by Short (2017). A mathematical study involving terrorist and fanatic was proposed by Camacho (2013). In this study the population has been divided into core and non-core groups. In the first model two core populations have been considered with no interactions between the subpopulations and both of the core-groups are taken from the same general designated population. They have shown that, with increasing implementation of efforts to recruit and retain, terrorist groups will be able to influence vulnerable individuals. They have however, did not consider the impacts of governmental interventions to combat the terrorism. A deterministic model of incorporating the allocation of optimal human resources to implement counter-terrorism operations was proposed by Udoh (2019). They studied possible strategies to allocate counter-terrorism resources towards providing a long-term mitigation

of the terrorism problem. In this article coupled differential equations have been used with system variables representing the internal and external dynamics of the insurgent organization. In their model, they have not considered the effect of detention/rehabilitation facility. The model proposed by Castillo-Chavez (2003) was extended by Cherif, et al. (2010) where the authors developed a mathematical model of insurgency process. They have considered the spread of radical ideologies, recruitment of new terrorists, impacts of fanatisms etc. including control of terrorism and radicalization process. A deterministic model to study the radicalization process in Kenya is proposed by Ngari (2016). Authors proposed control of the insurgency problem through rehabilitation centres but did not consider implementation of detention facility. A compartmental model to analyse the impacts of de-radicalization programs is studied by Sandler (2014). The authors have divided the general population in the following four compartments, namely, Susceptible (S), Extremists (E), Recruiters (R), and Under Treatment or deradicalization programs (T). They however, did not incorporate the possibility of force recruitment of the susceptible or already treated individuals. A five compartmental model including the vaccinated compartment that incorporates the individuals who have already gone through Countering Violent Extremism (CVE) programs before being radicals is studied by Santoprete (2019). In this study the authors analysed the impacts of CVE programs to counter terrorism. Chuang (2019) proposed a mathematical model with two cross sections to analyse the propagation of insurgency among different sub populations. The first model exhibits the possibility of an individual to progress from susceptible to moderate group before becoming a terrorist. Whereas, in the second model authors have considered two radical groups, which emerge from the susceptible before becoming fully radical without any possibility of control strategies. An optimal control model to implement counter-terrorism was studied by Bayon et al. (2019). The authors proposed “fire” and “water” control strategies but did not consider surveillance, detention/rehabilitation, facilities, force recruitment etc. Applications of optimal control strategies using Differential Transformation Method (DTM) is proposed by Akande (2017). Interestingly their result shows that implementation of the best control strategy does not always imply optimality but accuracy in result compared to other possible control mechanisms. Udwadia (2006) developed a dynamical model of terrorism incorporating the impacts of direct governmental interventions through military and police to control the terrorist population. Their study concluded that that using military and police actions may reduce the threat of insurgency. A dynamic model considering suicide bombers and political influence is studied by Geller (2015). Their study provided the emphasized and de-emphasized impacts of the forces in controlling the insurgency problem.

3 Mathematical models of unemployment

Adequate generation of employment is a vital indicator of economic development. Unemployment is an increasing global concern and according to Global Employment Trends 2012 out of a global labor force of 3.3 billion people all over the world almost 200 million people are unemployed. In a pioneering work to understand the dynamics of the problem of unemployment Nikolopoulos (2003) proposed a mathematical model and provided few measures to control unemployment. Some notable studies in this field have been performed by Misra (2011, 2013). Misra (2013) assumed a constant rate of increase of the unemployed individuals. Whereas, their number decreases due to death and emigration, at fixed constant rate proportional to the number of unemployed individuals They have also considered that the unemployed people acquire jobs at a rate proportional to the number of present unemployed persons and the number of available vacancies. A delay mathematical model incorporating the job creation delay has been studied by Misra (2013). According to their analysis if the rate of getting employment increases or the number of newly created vacancies increases then number of unemployed people decreases. Their study also suggests that creation of new job opportunities can be a solution of unemployment problem. Hence, they suggested that government must create new jobs parity with the present number of unemployed individuals to overcome the unemployment crisis.

In this article we propose a deterministic mathematical model with an aim to control the issues and challenges of the insurgency problem in Northeast India by generation of adequate employment opportunity which is considered as one of the important indicators of socioeconomic development. We also incorporate the awareness against the insurgency and violence among the people of Northeast India. In fact, proper awareness can be generated only with appropriate education which can only be sustained with adequate socioeconomic development.

4 Model Formulation

Let, us denote $U(t), R(t), T(t), J(t)$ and $E(t)$ respectively be the densities of unemployed individuals, recruiters of insurgency groups, insurgents, available jobs and employed individuals at any time t in any designated area. Let Δ be the constant input of unemployed people, especially youth, who can be exploited by the insurgent groups. β be the rate at which recruiters interact with those unemployed people. However, α fraction of the unemployed people who get convinced by the recruiters to participate in insurgency groups become recruiters and $(1-\alpha)$ portion becomes insurgents themselves. Recruiters can become insurgents at a rate σ_1 and the insurgents can become recruiters at a rate σ_2. New jobs are created at a rate ϕ depending on the density of present unemployed people. Unemployed people get jobs at a rate γ. The parameter γ also provides a measurement of the available skills and training of the unemployed individuals to obtain the new

job opportunities. Various job opportunities are shut down or lapsed at a rate m. The employed people lose their jobs and again become unemployed at a rate θ. The removal rate of unemployed and employed people due to migration, retirement or natural death is considered to be μ. However, the recruiters and insurgents experience some additional death rate due to be killed by police/army, incarcerated in jail etc. Their extra death rate is represented by δ. It is important to mention that, even employed individuals may get influenced by insurgency ideologies. However, in our work we do not consider that case, as one of the objectives of our study is to understand the impacts of generation of new employments on controlling the insurgency. We propose the following schema diagram to provide a pictorial demonstration of the interactions between different system classes:

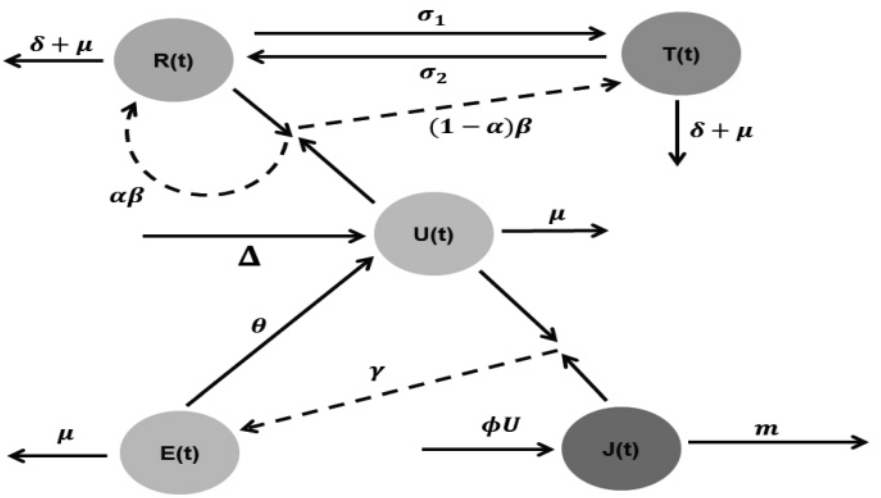


Figure 1. *Schema diagram of the interactions between system populations.*

We propose the following deterministic mathematical model using nonlinear ordinary differential equations based on the interactions described in the schema diagram given in Fig. 1:

$$\begin{aligned}
\frac{dU}{dt} &= \Delta - \beta UR - \gamma UJ + \theta E - \mu U,\\
\frac{dR}{dt} &= \alpha\beta UR - \sigma_1 R + \sigma_2 T - (\delta + \mu)R,\\
\frac{dT}{dt} &= (1-\alpha)\beta UR + \sigma_1 R - \sigma_2 T - (\delta + \mu)T,\\
\frac{dJ}{dt} &= \phi U - mJ,\\
\frac{dE}{dt} &= \gamma UJ - \theta E - \mu E.
\end{aligned} \tag{1}$$

If the rate of awareness against insurgency increases then insurgency should decrease. Moreover, if adequate job opportunities are generated for people of Northeast India, then their quality of life will increase through proper socio-economic development. Let, c be the rate of awareness against the insurgency among the people. $c \in [0,1]$, where $c = 1$ implies 100% awareness and $c = 0$ signifies absence of awareness. We therefore, modify Model System (1) to the following model incorporating the effect of awareness against insurgency:

$$\begin{aligned}
\frac{dU}{dt} &= \Delta - \frac{(1-c)\beta UR}{a+J} - \gamma UJ + \theta E - \mu U, \\
\frac{dR}{dt} &= \frac{(1-c)\alpha\beta UR}{a+J} - \sigma_1 R + \sigma_2 T - (\delta+\mu)R, \\
\frac{dT}{dt} &= \frac{(1-c)(1-\alpha)\beta UR}{a+J} + \sigma_1 R - \sigma_2 T - (\delta+\mu)T, \\
\frac{dJ}{dt} &= \phi U - mJ, \\
\frac{dE}{dt} &= \gamma UJ - \theta E - \mu E.
\end{aligned} \tag{2}$$

Here a is a constant. We will study this Model System (2) under the initial conditions given by:

$$U(0) > 0, R(0) \geq 0, T(0) \geq 0, J(0) > 0, T(0) > 0. \tag{3}$$

5 Analytical results

In this section we derive different analytical results such as positivity and boundedness of the solutions of the Model System (2), existence and stability of the equilibrium points etc.

5.1 Positivity and boundedness

Proposition 5.1 *All solutions of System (2) are positively invariant and uniformly bounded in* Γ *for all large* t *where,*

$$\Gamma = \left\{(U,R,T,J,E) \in \mathbb{R}_+^5 \;\middle|\; 0 < U(t) + R(t) + T(t) + E(t) \leq \frac{\Delta}{\mu}, 0 \leq J(t) \leq \frac{\Delta\phi}{\mu m}\right\}.$$

Proof. First, we show that all the solutions of the System (2) starting with initial conditions (2) are positive, using a lemma proposed by Nagumo (1952)**.**

Lemma 5.1 *Consider a system* $\dot{X} = F(X)$ *where* $F(X) = [F_1(X), F_2(X), \dots, F_n(X)]$, $X \in \mathbb{R}^n$ *with initial condition* $X(0) = X_0 \in \mathbb{R}^n$. *If for* $X_i = 0, i = 1,2,\dots,n$ *we get* $F_i(X)|_{X_i=0} \geq 0$, *then any solution of* $\dot{X} = F(X)$ *with given initial condition, say,* $X(t) = X(t; X_0)$ *will be positive i.e.,* $X(t) \in \mathbb{R}_+^n$.

It can be easily seen from (2) that $\frac{dU}{dt}|_{U=R=T=J=E=0} = \Delta > 0$ and when $U = R = T = J = E = 0$ then $\frac{dR}{dt} = \frac{dT}{dt} = \frac{dJ}{dt} = \frac{dE}{dt} = 0$. Hence following Lemma 3.1 all solutions of (2) starting with initial conditions (3) are positive.

Next, we establish the boundedness of the solutions of (1) for all large t. Let, $W(t) = U(t) + R(t) + T(t) + J(t) + E(t)$. Differentiating $W(t)$ along the solutions of (2), we find

$\dot{W}(t) = \dot{U}(t) + \dot{R}(t) + \dot{T}(t) + \dot{E}(t) \Rightarrow \dot{W}(t) \leq \Delta - \mu W \Rightarrow \lim_{t\to\infty} W(t) \leq \frac{\Delta}{\mu}$.

Therefore, the solutions $U(t), R(t), T(t), E(t)$ of (2) are all bounded above by $\frac{\Delta}{\mu}$. Again, from the 4th equation of (2) one can calculate that

$$\frac{dJ}{dt} = \phi U - mJ \Rightarrow \frac{dJ}{dt} + mJ \leq \frac{\phi\delta}{\mu} \Rightarrow \lim_{t\to\infty} J(t) \leq \frac{\Delta\phi}{\mu m}.$$

Therefore, for all large t, solutions of System (2) are positively invariant and uniformly bounded in the region Γ. Hence the proposition is proved.

5.2 Equilibrium points and their existence conditions

Model System (2) has two equilibrium points. Namely insurgency free equilibrium point $P_0(U_0, 0, 0, J_0, E_0)$ and insurgency equilibrium $P^*(U^*, R^*, T^*, J^*, E^*)$. The population densities at the equilibrium P_0 are given by $J_0 = \frac{\phi U_0}{m}, E_0 = \frac{\Delta}{\mu} - U_0$ and U_0 is given by the positive roots of the following quadratic equation:

$$A_1 U^2 + A_2 U + A_3 = 0,$$

where $A_1 = \frac{\gamma\phi}{m} > 0$, $A_2 = \theta + \mu > 0$, $A_3 = -\frac{\Delta(\theta+\mu)}{\mu} < 0$. Clearly, this equation has exactly one positive root given by: $U_0 = \frac{-A_2 + \sqrt{A_2^2 - 4A_1A_3}}{2A_1}$.

The equilibrium P_0 will be feasible if $0 < U_0 < \frac{\Delta}{\mu}$. (4)

Again, at $P^*(U^*, R^*, T^*, J^*, E^*)$ we have

$$E^* = \lambda(U^*)^2,$$

$$J^* = \frac{\phi U^*}{m},$$

$$R^* = \frac{B_1(U^*)^3 + B_2(U^*)^2 + B_3(U^*) + B_4}{B_5 U^*},$$

$$T^* = \frac{B_6(U^*)^3 + B_7(U^*)^2 + B_8(U^*) + B_9}{B_{10} U^*},$$

where $\lambda = \frac{\gamma\phi}{m(\theta+\mu)} > 0$, $B_1 = \phi\left(\theta\lambda - \frac{\gamma\phi}{m}\right)$, $B_2 = \left(\theta\lambda - \frac{\gamma\phi}{m}\right)am - \mu\phi$, $B_3 = \phi\Delta - \mu am$, $B_4 = \Delta am$, $B_5 = \beta m(1-c)$, $B_6 = -\mu B_5\lambda - (\delta+\mu)B_1$, $B_7 = -\mu B_5 - B_2(\delta+\mu)$, $B_8 = B_5\Delta - B_3(\delta+\mu)$, $B_9 = -B_4(\delta+\mu)$, $B_{10} = B_5(\delta+\mu)$. The equilibrium density U^* is found to be the positive roots of the following equation:

$$D_1(U^*)^4 + D_2(U^*)^3 + D_3(U^*)^2 + D_4(U^*) + D_5 = 0,$$

where,

$$D_1 = B_{10}\bar{C}_1B_1 + B_5\bar{C}_3B_6,$$
$$D_2 = B_{10}(\bar{C}_1B_2 + B_1\bar{C}_2) + B_5(\bar{C}_3B_6\bar{C}_4 + \bar{C}_3B_8),$$
$$D_3 = B_{10}(B_3\bar{C}_1 + B_2\bar{C}_2) + B_5(\bar{C}_3B_7\bar{C}_4 + \bar{C}_3B_8),$$
$$D_4 = B_{10}(B_4\bar{C}_1 + B_3\bar{C}_2) + B_5(\bar{C}_3B_8\bar{C}_4 + B_4\bar{C}_3),$$
$$D_5 = B_{10}B_4\bar{C}_2 + B_5\bar{C}_3B_9\bar{C}_4,$$
$$\bar{C}_1 = (1-c)\alpha\beta - \frac{\phi}{m}(\delta+\mu+\sigma_1),$$
$$\bar{C}_2 = -a(\delta+\mu+\sigma_1), \bar{C}_3 = \frac{\sigma_2\phi}{m}, \bar{C}_4 = \frac{am}{\phi}.$$

However, due to the high parametric complexity of the equilibrium population densities at P^* we will verify its existence numerically.

5.3 Local stability analysis of the equilibrium points

First, we discuss the local stability of the equilibrium point P_0 using linearization method. The Jacobian matrix evaluated at P_0 is given by

$$W_1(U_0, 0, 0, J_0, E_0) = \begin{pmatrix} m_{11} & m_{12} & 0 & m_{14} & m_{15} \\ 0 & m_{22} & m_{23} & 0 & 0 \\ 0 & m_{32} & m_{33} & 0 & 0 \\ m_{41} & 0 & 0 & m_{44} & 0 \\ m_{51} & 0 & 0 & m_{54} & m_{55} \end{pmatrix},$$

where,

$$m_{11} = \gamma J_0 - \mu, m_{12} = -\frac{(1-c)\beta U_0}{a+J_0}, m_{14} = -\gamma U_0, m_{15} = \theta,$$
$$m_{22} = \frac{(1-c)\alpha\beta U_0}{a+J_0} - \sigma_1 - (\delta+\mu), m_{23} = \sigma_2,$$
$$m_{32} = \frac{(1-c)(1-\alpha)\beta U_0}{a+J_0} + \sigma_1, \quad m_{33} = -\sigma_1 - (\delta+U_0),$$
$$m_{41} = \phi, m_{44} = -m, m_{51} = \gamma J_0, m_{54} = \gamma U_0, m_{55} = -\theta - \mu.$$

The characteristic equation of the Jacobian matrix W_1 is given by

$$\lambda^5 + V_1\lambda^4 + V_2\lambda^3 + V_3\lambda^2 + V_4\lambda + V_5 = 0$$

where,

$$\begin{aligned}
&V_1 = \frac{X_2}{X_1}, \quad V_2 = \frac{X_3}{X_1}, \quad V_3 = \frac{X_4}{X_1}, \quad V_4 = \frac{X_5}{X_1}, \quad V_5 = \frac{X_6}{X_1}, \quad X_1 = R_6,\\
&X_2 = R_5, \quad X_3 = O_4 + R_4 + T_4, \quad X_4 = O_3 + R_3 + T_3,\\
&X_5 = O_2 + R_2 + T_2, \quad X_6 = O_1 + R_1 + T_1, \quad O_1 = N_1N_2 + N_5N_2,\\
&O_2 = N_1N_3 + N_5N_3 + N_6N_2, \quad O_3 = N_1N_4 + N_5N_4 + N_6N_3,\\
&O_4 = N_6N_4, \quad N_1 = m_{15}m_{41}m_{54}, \quad N_2 = m_{33}m_{22} - m_{32}m_{23},\\
&N_3 = -(m_{33} + m_{22}), \quad N_4 = 1, \quad N_5 = -m_{15}m_{51}m_{44},\\
&N_6 = m_{15}m_{51}, \quad R_1 = Q_1Q_4, \quad R_2 = Q_1Q_5 + Q_2Q_4,\\
&R_3 = Q_1 + Q_2Q_5 + Q_3Q_4, \quad R_4 = Q_2 + Q_3Q_5 + Q_4,\\
&R_5 = Q_3 + Q_5, \quad R_6 = Q_6, \quad Q_1 = P_1P_3, \quad Q_2 = P_1P_4 + P_2P_3,\\
&Q_3 = P_2P_4 + P_3, \quad Q_4 = P_5, \quad Q_5 = P_6, \quad Q_6 = 1,\\
&P_1 = m_{55}m_{11}, \quad P_2 = -(m_{55} + m_{11}), \quad P_3 = m_{44}, \quad P_4 = -1,\\
&P_5 = m_{22}m_{33} - m_{32}m_{23}, \quad P_6 = -(m_{22} + m_{33}), \quad T_1 = S_1S_3,\\
&T_2 = S_2S_3 + S_1S_4, \quad T_3 = S_1 + S_2S_4, \quad T_4 = S_2,\\
&S_1 = -m_{14}m_{41}m_{55}, \quad S_2 = m_{41}m_{14}, \quad S_3 = m_{22}m_{33} - m_{23}m_{32},\\
&S_4 = -(m_{22} + m_{33}).
\end{aligned} \tag{5}$$

Hence, we obtain the following proposition:

Proposition 5.2 *If the insurgency-free equilibrium P_0 exists following (4) then it will be is locally asymptotically stable if and only if:*

$$V_1 > 0, V_5 > 0, V_1V_2 > V_3, V_1(V_2V_3 + V_5) > V_3^2 + V_1^2V_4,$$
$$V_1V_2V_3V_4 + 2V_1V_4V_5 + V_2V_3V_5 > V_1V_2^2V_5 + V_1^2V_4^2 + V_4V_3^2 + V_5^2$$

where the expressions of $V_i, i = 1, 2, 3, 4, 5$ are given in (4).

The Jacobian matrix evaluated at the insurgency equilibrium P^* is given by,

$$W_2(U^*, R^*, T^*, J^*, E^*) = \begin{pmatrix} a_{11} & a_{12} & 0 & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 \\ a_{41} & 0 & 0 & a_{44} & 0 \\ a_{51} & 0 & 0 & a_{54} & a_{55} \end{pmatrix}$$

where,

$$a_{11} = -\frac{(1-c)\beta R^*}{a+J^*} - \gamma J^* - \mu, a_{12} = -\frac{(1-c)\beta U^*}{a+J^*}, \; a_{14} = \frac{(1-c)\beta U^* R^*}{(a+J^*)^2} - \gamma U^*, \; a_{15} = \theta,$$

$$a_{21} = \frac{(1-c)\alpha\beta R^*}{a+J^*}, a_{22} = \frac{(1-c)\alpha\beta U^*}{a+J^*} - \sigma_1 - (\delta+\mu), a_{23} = \sigma_2, \quad a_{24} = \frac{(1-c)\alpha\beta U^* R^*}{(a+J^*)^2},$$

$$a_{31} = \frac{(1-c)(1-\alpha)\beta R^*}{a+J^*}, a_{32} = \frac{(1-c)(1-\alpha)\beta U^*}{a+J^*} + \sigma_1, \quad a_{33} = -\sigma_1 - (\delta + U^*),$$

$$a_{34} = -\frac{(1-c)(1-\alpha)\beta U^* R^*}{(a+J^*)^2}, \quad a_{41} = \phi, \quad a_{44}(U^*, R^*, T^*, J^*, E^*) = -m, \quad a_{51} = \gamma J^*,$$

$a_{54} = \gamma U^*, \qquad a_{55} = -\theta - \mu.$

The characteristic equation of W_2 is given by:

$$\lambda^5 + U_1\lambda^4 + U_2\lambda^3 + U_3\lambda^2 + U_4\lambda + U_5 = 0$$

where,

$$\begin{aligned}
&U_1 = \frac{M_2}{M_1}, \quad U_2 = \frac{M_3}{M_1}, \quad U_3 = \frac{M_4}{M_1}, \quad U_4 = \frac{M_5}{M_1}, \quad U_5 = \frac{M_6}{M_1}, \quad M_1 = L_6, \quad M_2 = L_5,\\
&M_3 = G_4 + I_4 + L_4, \quad M_4 = G_3 + I_3 + L_3, \quad M_5 = G_2 + I_2 + L_2,\\
&M_6 = G_1 + I_1 + L_1, \quad G_1 = F_1F_3 + F_5F_3, \quad G_2 = -F_1F_4 + F_2F_3 - F_5F_4,\\
&G_3 = F_1 - F_2F_4 - F_5, \quad G_4 = F_2, \quad F_1 = -a_{15}a_{51}a_{44}, \quad F_2 = a_{15}a_{51},\\
&F_3 = a_{22}a_{33} - a_{23}a_{32}, \quad F_4 = a_{22} + a_{33}, \quad F_5 = a_{54}a_{15}a_{41},\\
&I_1 = H_5H_7 + H_1H_3, \quad I_2 = -H_1H_4 + H_2H_3 - H_5H_8 + H_6H_7,\\
&I_3 = -H_2H_4 + H_5 - H_6H_8, \quad I_4 = H_6, \quad H_1 = -a_{41}\,a_{12}\,a_{55}, \quad H_2 = a_{41}a_{12},\\
&H_3 = a_{23}a_{34} - a_{24}a_{33}, \quad H_4 = -a_{24}, \quad H_5 = -a_{41}a_{14}a_{55}, \quad H_6 = a_{41}a_{14},\\
&L_1 = K_7 + K_1K_5, \quad L_2 = K_1K_6 + K_2K_5 + K_8, \quad L_3 = K_1 + K_2K_6 + K_9,\\
&L_4 = K_2 + K_3K_6 + K_4K_5 + K_{10}, \quad L_5 = K_3 + K_4K_6, \quad L_6 = K_4, \quad K_1 = J_1J_3.\\
&K_2 = -J_1 - J_2J_3, \quad K_3 = J_2J_3 + J_3, \quad K_4 = -1, \quad K_5 = J_4, K_6 = J_5, \qquad (6)\\
&K_7 = J_6J_9, \quad K_8 = J_6J_{10} + J_7J_9, \quad K_9 = J_7J_{10} + J_8J_9, K_{10} = j_8J_{10},\\
&J_1 = a_{55}a_{44}, J_2 = a_{55} + a_{44}, \quad J_3 = a_{11}, \quad J_4 = a_{22}a_{33} - a_{23}a_{32},\\
&J_4 = -(a_{22} + a_{33}), \quad J_6 = -a_{55}a_{44}a_{12}, \quad J_7 = a_{12}(a_{55} + a_{44}),\\
&J_8 = -a_{12}, \quad J_9 = a_{21}a33 - a_{23}a_{31}, \quad J_{10} = -a_{21}.
\end{aligned}$$

Hence, we obtain the following proposition:

Proposition 5.3 *If the insurgency equilibrium P^* exists then it will be is locally asymptotically stable if and only if:*

$$U_1 > 0, U_5 > 0, U_1U_2 > U_3, U_1(U_2U_3 + U_5) > U_3^2 + U_1^2U_4,$$
$$U_1U_2U_3U_4 + 2U_1U_4U_5 + U_2U_3U_5 > U_1U_2^2U_5 + U_1^2U_4^2 + U_4U_3^2 + U_5^2$$

where the expressions of U_i, $i = 1, 2, 3, 4, 5$ are given in (6).

6 Numerical simulations

In this section we perform numerical simulations of model System (2) using MATLAB 2015a. First, we consider a society that is highly prone to insurgency activities with the following characteristics and denote it by **Society – X**.

CH1. Low rate of awareness against insurgency practices. This basically implies low rate of the the parameter c that measures the mass awareness against insurgency, terrorism and violence.

CH2. High rate of interactions between recruiters of insurgent groups and unemployed persons. Namely value of the parameter β, that represents the rate of interaction between unemployed individuals and recruiters is considerably high.

CH3. Low employment rate. In this case the rate γ at which unemployed individuals get employed is low.

CH4. Low creation rate of new employment opportunities, which implies the rate of creation of new jobs (ϕ) is low.

Based on the above discussion we consider the following parameter set to numerically simulate the characteristics CH1 to CH4 of Society – X.

$$\Delta = 5, \beta = 0.056, \alpha = 0.5, c = 0.3, \mu = 0.007, \theta = 0.15, m = 0.2, \gamma = 0.005, \\ \sigma_1 = 0.08, \sigma_2 = 0.06, \delta = 0.003, \phi = 0.1, a = 50. \tag{7}$$

For these parameter values one can calculate that $U_1 = 0.2232 > 0, U_5 = 11.43 > 0, U_1U_2 - U_3 = 0.6653 > 0, U_1(U_2U_3 + U_5) - (U_3^2 + U_1^2U_4) = 3.8772 > 0, U_1U_2U_3U_4 + 2U_1U_4U_5 + U_2U_3U_5 - (U_1U_2^2U_5 + U_1^2U_4^2 + U_4U_3^2 + U_5^2) = 0.00532 > 0$. Hence following Proposition 5.3 we obtain that the insurgency equilibrium P^* is locally asymptotically stable. It signifies that insurgency persists in a stable condition in the Society – X. The time evolutions of system trajectories have been depicted in Fig. 2. It shows that with lower level of awareness, rate of being employed, rate of new job creation and higher level of interactions between unemployed people and insurgent recruiters, insurgency prevails in the Society – X that has the aforementioned characteristics given in CH1 – CH4.

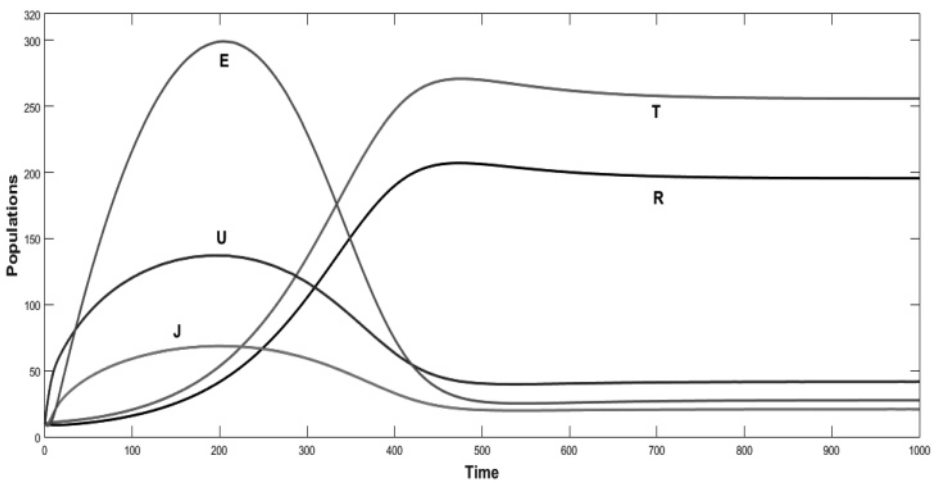


Figure 2. *Time evolution of model System (2) for the parameter values as given in (7).*

Next, we perform PRCC (Partial Rank Correlation Analysis) with $p-$value < 0.0001 to identify the most sensitive parameters of System (2). The PRCC analysis diagram is shown in Fig. 3. It clearly shows that the parameters c, β, γ and ϕ are the most sensitive parameters and act as the major regulators of the system dynamics and controlling the insurgency. Our objective is to check the impacts of the aforementioned four most sensitive parameters to control and if possible, eradicate insurgency from the Society – X.

6.1 Joint effect of c and β

First, we study the joint effect of the parameter c that measures the awareness against insurgency and β, the rate of interaction between unemployed individuals with recruiters of the insurgency organizations. It should be noted that c and β are interconnected. The basic hypothesis behind it is that if awareness against extremism increases then interactions between recruiters of insurgency groups and unemployed individuals will decrease and vice versa. This will provide an effective strategy to control insurgency through generation of awareness. The stability regions of the equilibrium points P_0 and P^* for joint variation of c and β are depicted in Fig. 4a.

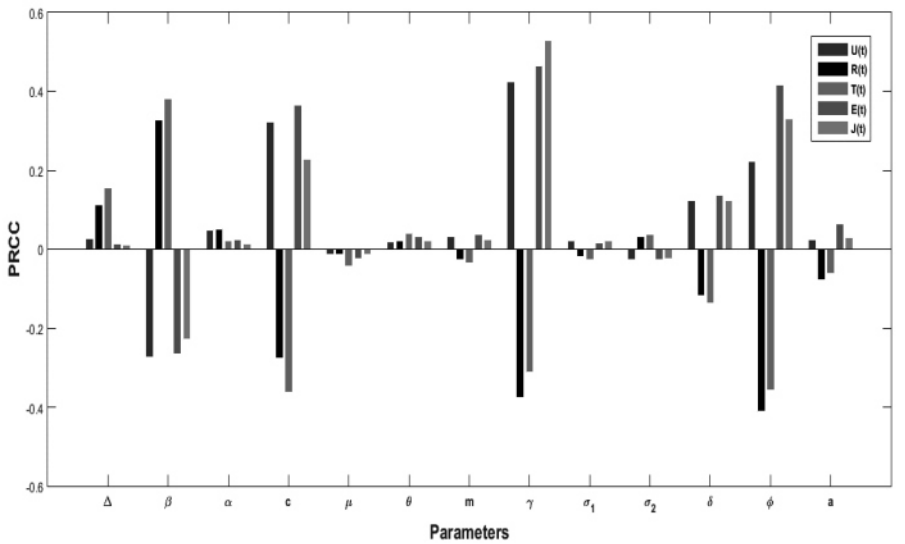


Figure 3. *PRCC analysis with $p-$value < 0.0001 of the parameters of System (2).*

It clearly conforms to our hypothesis that, by increasing awareness against insurgency practices, the rate of interactions between unemployed people and insurgency group recruiters can be decreased and thereby, insurgency can be contained. Now our objective is to free the Society – X from the

insurgency practices. We have considered $\beta = 0.056$ and $c = 0.3$ to simulate Society – X. Now from Fig. 4a one can see that for these values of β and c Society – X will not be free of insurgency (see Fig. 2). Now, we increase c from $c = 0.3$ to $c = 0.55$. From Fig. 4a we can assert that insurgency will not be eradicated, but the population density of insurgents and recruiters in Society – X will decrease. This case is depicted in Fig. 4b. The decrease in number of insurgents and recruiters can easily be seen by comparing Fig. 4b with Fig. 2.

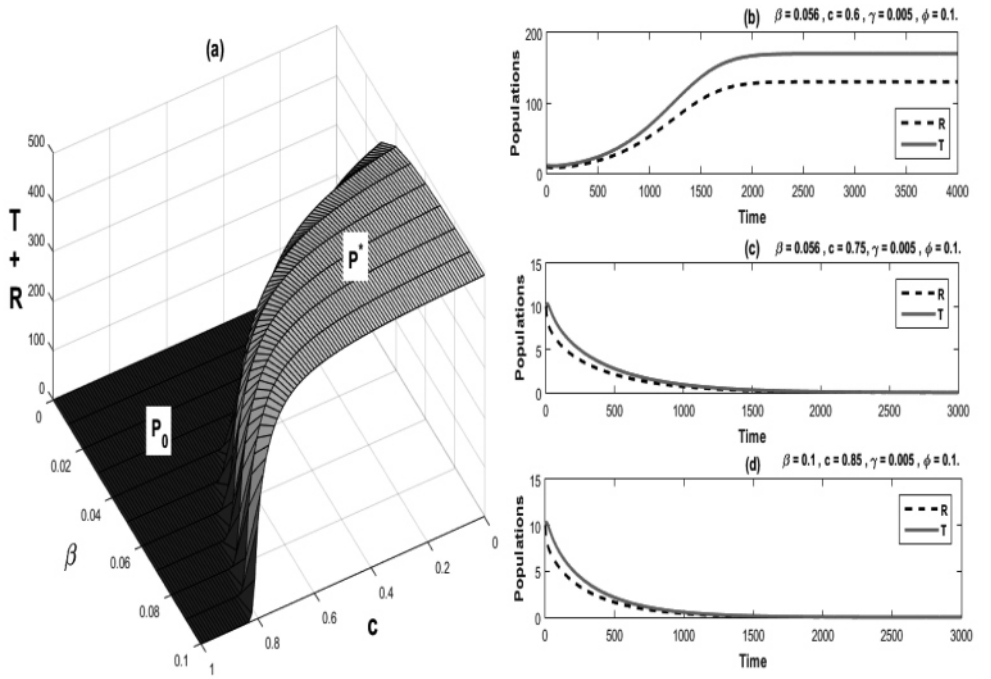


Figure 4. *(a) Stability regions of P_0 and P^* for joint variation of c and β. Time evolutions of the System (2) for different values of c and β (b) $\beta = 0.056, c = 0.55$, (c) $\beta = 0.056, c = 0.75$ and (d) $\beta = 0.1, c = 0.85$. Other parameters are as in (7).*

However, following Fig. 4a if c is increased to $c = 0.75$ then insurgent and recruiter population dies out. This case is shown in Fig. 4c. It shows, if we wish to free Society – X from insurgency, the awareness (c) has to be at least 0.75. In this case it is calculated that $U_0 = 182.376 < \frac{\delta}{\mu} = 714.2857$. Hence, P_0 exists following (4) and as $V_1 = 21.781 > 0, V_5 = 2.398 > 0, V_1V_2 - V_3 = 0.0098 > 0, V_1(V_2V_3 + V_5) - (V_3^2 + V_1^2V_4) = 0.0734 > 0, V_1V_2V_3V_4 + 2V_1V_4V_5 + V_2V_3V_5 - (V_1V_2^2V_5 + V_1^2V_4^2 + V_4V_3^2 + V_5^2) = 11.2541 > 0$, following Proposition 5.2 we obtain that P_0 is

locally asymptotically stable. Therefore, all system trajectories converge to insurgency-free equilibrium P_0 and insurgency is removed from the Society – X. Interestingly, if we can increase the awareness against insurgency and extremism to a level higher than $c^* = 0.85$ (approximately, following Fig. 4a) which is basically the critical level of awareness to be generated, then from Fig. 4a one can easily see that even with high rate of interactions between unemployed individuals and recruiters of insurgency groups, the society can be made free of insurgency (Fig. 4d). Hence, utilizing Fig. 4a we can determine the level of awareness against insurgency practices is necessarily required to free Society – X of insurgency practices, if the rate of interactions between unemployed individuals and recruiters of insurgency groups is known.

6.2 Joint effect of γ and ϕ

The combined effect of γ and ϕ is an important aspect of our study. The parameter γ measures the rate of new employment of the unemployed individuals. Whereas, the parameter ϕ signifies the rate at which new employments are created. Evidently effects of these two parameters are interlinked. We consider the hypothesis that if ϕ increases, then more new jobs will be created and as a result unemployed individuals will be employed at a higher rate i.e., γ will increase. And as the unemployed individuals get employed, their economic condition will develop and they will stay away from insurgency. It is also important to under stand that mere creation of new jobs does not always imply less unemployment, as without proper training and skills required for the new jobs, unemployed will remain unemployed. Therefore, once new job opportunities are created, it is essential to provide the necessary skill development trainings to the unemployed individuals to increase the rate of employment among them. The stability regions of P_0 and P^* for varying ϕ and γ have been drawn in Fig. 5a. This figure provides an important estimate about the level of skill development and training to be given to the unemployed individuals, if the rate of new jobs creation is known. Now as mentioned before we wish to make the Society – X free of insurgency. So, keeping all other parameters fixed at (7), if we increase γ from $\gamma = 0.005$ to $\gamma = 0.05$, we can see by comparing Fig. 5b and Fig. 2, that insurgency is not eradicated, but surely decreased. Observing Fig. 5a, if we further increase γ to $\gamma = 0.135$, then Society – X can be made free of insurgency (Fig. 5c). Therefore, if the rate of new job creation is known, then we can determine the required rate of employment necessarily required to eradicate insurgency using Fig. 5a. Another interesting observation that can be derived from Fig. 5a, that if the rate of new job creation ϕ is higher than an upper critical level, in this case it is approximately $\phi^* = 0.35$, then even with low rate of employment Society – X can be made free of insurgency (Fig. 5d). However, if ϕ is below a lower critical lower level, which is in this case approximately $\phi_* = 0.55$, then even with quite high rate of

employment, say $\gamma = 0.2$, insurgency will prevail in the Society – X (Fig. 5e). Therefore, to free Society – X from insurgency, the rate of generation of new jobs must be higher than a critical value ϕ_*.

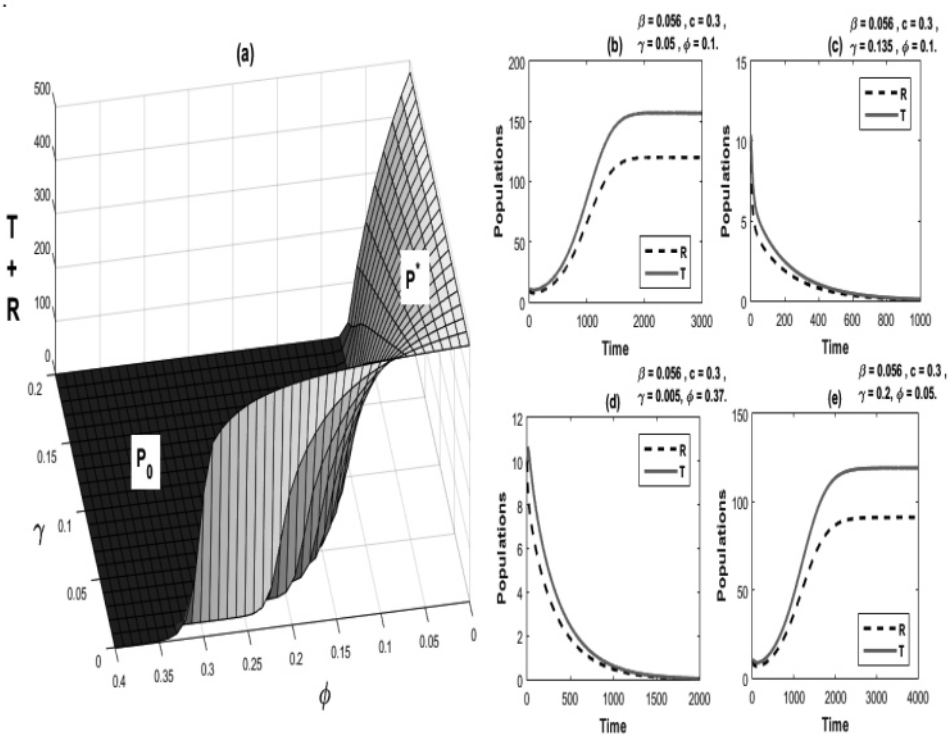


Figure 5. *(a) Stability regions of P_0 and P^* for joint variation of γ and ϕ. Time evolutions of the System (2) for different values of γ and ϕ (b) $\gamma = 0.05, \phi = 0.1$, (c) $\gamma = 0.135, \phi = 0.1$, (d) $\gamma = 0.005, \phi = 0.37$ and (e) $\gamma = 0.2, \phi = 0.05$. Other parameters are as in (7).*

6.3 Joint effect of c and ϕ

We now study the combined effect of awareness against insurgency (c) and the rate of generation of new jobs (ϕ) to mitigate the issue of insurgency in Society – X. It is evident that creation of new jobs will encourage the unemployed youth not to choose violence over socioeconomic development. Moreover, this mechanism to control insurgency is directly related to governmental policies & schemes. It is important to understand that the creation of new jobs is actually interlinked with the awareness against insurgency practices. If more new jobs are created unemployed youth will be more hopeful towards socioeconomic developments by being employed. For being employed they will require skill development training and proper educational qualifications. To acquire that they will go to educational institutes & participate in various

governmental skill development initiatives like Make in India, Digital India, Self-Reliant India, Skill India Programs etc. In this way they will get proper education and eventually become aware against the ways of extremism. Hence their possibility of being brainwashed by the recruiters of insurgency groups will be low. As a result, insurgency will decrease due to lack of new recruits.

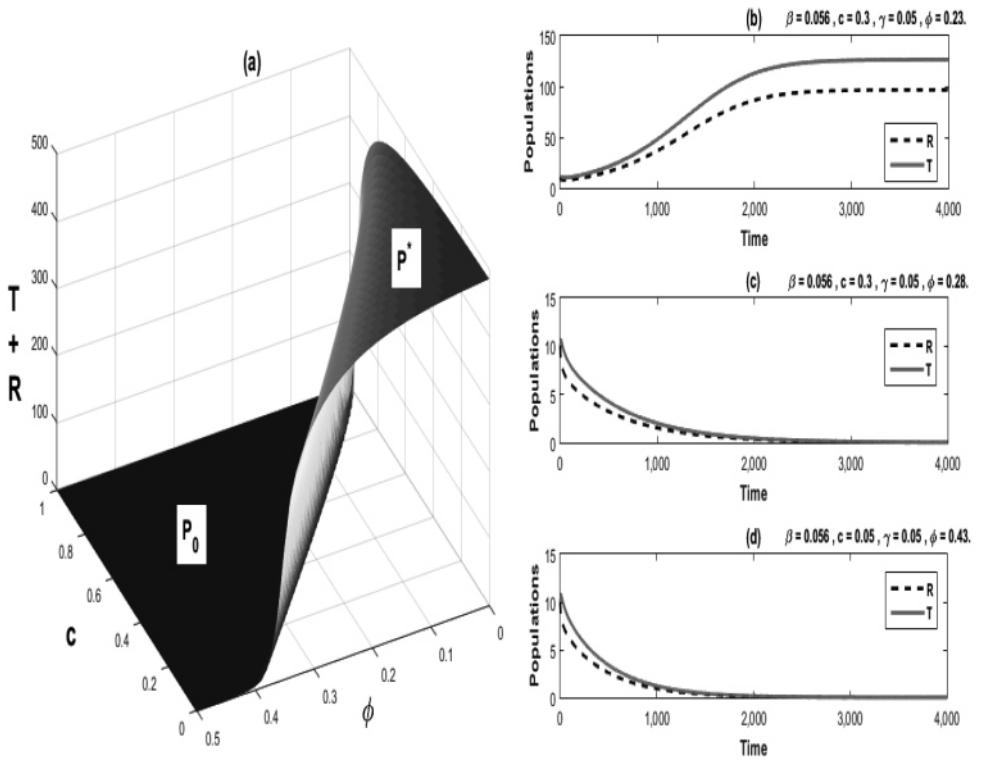


Figure 6. *(a) Stability regions of P_0 and P^* for joint variation of c and ϕ. Time evolutions of the System (2) for different values of c and ϕ (b) $c = 0.3, \phi = 0.23$, (c) $c = 0.3, \phi = 0.28$, (d) $c = 0.05, \phi = 0.43$. Other parameters are as in (7).*

The stability regions of the equilibrium points P_0 and P^* for varying c and ϕ have been drawn in Fig. 6a. It shows a reverse relation between c and ϕ, i.e., the level of awareness required to free Society – X from the menace of insurgency will be high if the rate of generation of new jobs is low and vice versa. The rate of new employments to be generated to make Society – X free of extremism for a known level of awareness against insurgency, can be decided using Fig. 6a. For, example we consider the case of Society – X where $c = 0.3$. The density of recruiters of insurgency groups and insurgents decrease in Society – X if ϕ is increased from 0.1 to 0.23. This claim can

easily be verified by comparing Fig. 2 and Fig. 6b. Moreover, if ϕ is increased further to 0.28, we can see from Fig. 6c that Society – X becomes free of insurgency. Another important observation can directly be obtained from Fig. 6a, that if the rate of generation of new jobs is higher that a critical level, say ϕ_c, then even with low awareness against insurgency, Society – X can be made free of insurgency. From Fig. 6a we assert that $\phi_c \approx 0.41$. The time evolution of System (2) with $c = 0.05$ and $\phi = 0.43 > \phi_c$ has been drawn in Fig. 6d. It shows when ϕ is more than the critical value ϕ_c, even with low level of $c = 0.05$, insurgency is eradicated.

7 Summary and discussion

In this article we have proposed and analyzed a deterministic mathematical model using nonlinear differential equations to establish the hypothesis that the issue of insurgency can be contained by adequate socioeconomic developments. The problem of insurgency is a persistent issue in Northeast India and socioeconomic developments can be a feasible way to mitigate this problem. For that purpose, we have considered generation of new employment opportunities and proper awareness against the radical ideologies through proper education, as the important indicators of socioeconomic developments. Through our analysis we have shown that a society, termed as Society – X with high rate of interaction between unemployed individuals & recruiters of insurgent groups (β), low rate of awareness against insurgency (c), low rate of new job creation (ϕ) & low rate of employment (γ) is prone to persistent insurgency activities. Sensitivity analysis using PRCC method shows that these four parameters are the major regulators to control and if possible, eradicate insurgency from the Society – X. These four parameters can individually regulate the insurgent population. But their combined effect is much more effective from real time application point of view. In light of this, we have explored three control mechanisms of insurgency depending on the combined effects of the parameters that are interconnected from realistic point of view. The first one is the joint effect of c and β. We have shown that when ϕ and γ are fixed, then c and β are negatively correlated in view of controlling insurgency. If fact if c is increased, then with low level of β, insurgency can be controlled in Society – X. On the other, to overcome the adverse effect of high rate of β, the level of c has to be increased. And if we can increase c more than a critical value c^* then even with high rate of β Society – X can be made free of insurgency. Another important mechanism to control insurgency in Society – X is to regulate the combined effect of γ and ϕ by considering fixed c and β. It has been shown that if ϕ increases, then γ will increase. As the unemployed individuals becomes employed, their economic condition will develop and eventually they will stay away from insurgency. It is also proven that if ϕ is higher than an upper critical level ϕ^*, then even with low γ Society – X can be made free of insurgency. However,

if ϕ is below a lower critical lower level ϕ_* , then even with quite high rate of γ, insurgency will prevail in the Society – X. Therefore, to free Society – X from insurgency, the rate of generation of new jobs must be higher than a critical value ϕ_*. Finally, we analyze the combined effect of the interlinked parameters c and ϕ. It is obtained that the required rate of c to free Society – X from insurgency has to be high if the rate of ϕ is low and vice versa. It has been further derived that if ϕ is higher that a critical level, say ϕ_c, then even with low rate of c, Society – X can be made free of insurgency. Therefore, our study shows that apart from generation of awareness, sufficiently higher creation rate of new employments can also be a useful tactic to control insurgency. If unemployed youths get jobs at a higher rate and their quality-of-life increases, they will not opt for insurgency practices and eventually the effect of insurgency will decrease due to lack of new recruits. Moreover, the critical values associated with the awareness against insurgency ideologies and the rate of generation of new employments determined by our study can be applied while framing various governmental policies for Northeast India to contain and even eradicate the persistent issue of insurgency in this region.

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