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LINEAR AND NON-LINEAR STUDY OF MAGNETO-DIFFUSIVE CONVECTION IN DUFOUR-SORET INDUCED NANOFLUID LAYER

The present paper aims at analytical study of linear and unsteady non-linear Rayleigh Benard triply diffusive magneto convection in a Maxwell nanofluid layer with Dufour -Soret effects. The effects of Dufour parameter, Soret parameter, magnetic field, Lewis number, Modified diffusivity ratio, Concentration Rayleigh-Darcy number and Solutal Rayleigh number on the stability of the system have been investigated. Dufour parameter which was observed to have stabilizing behaviour in absence of magnetic field shows dual nature here in presence of magnetic field. The thermal Nusselt number here decreases with increase in magnetic field resulting in decrease of heat transport. This behaviour is same as was observed for Newtonian fluids.

2010 Mathematics Subject Classification: 76E06, 76E25, 76R50, 80A19, 80A21.

Keywords **-** Nanofluid, Magnetic field, Double diffusion, Dufour-Soret parameter.

1. Introduction

Heat transfer mechanism has been improved by replacing micro sized particles with nano sized particles in conventional fluids. The term nanofluid was first coined by Choi [5], he described the future and hope of the application of nanotechnology. Kumar et al. [7] established the utility of a particular nanofluid for its heat transfer application. Wong and Leon [16] focussed on the broad range of present and future applications of nanofluids. The problem of thermal convection for a Newtonian fluid layer was discussed by Chandrasekhar [4] taking varying assumptions of hydro-dynamics and hydro-magnetism.

Maxwell nanofluid taking into account thermophoresis and Brownian diffusion was studied by Jaimala et. al [6]. The study of magnetic field effects on the onset of convection has important applications in physics and engineering. In metal casting and in cooling systems of electronic devices, magnetic field effects are of great importance. The nanofluid can be taken as a working medium in order to get effective heat performance of such devices. Rayleigh Benard Magneto-convection arises due to combined effect of buoyancy force and magnetic field induced Lorentz force. A non-dimensional parameter called Chandrasekhar number gets introduced due to Lorentz force.

Sheikhloleslami et al. [11] investigated about the effect of radiation on nanofluid free convective heat transfer in presence of magnetic field. The combined effect of a vertical magnetic field and the boundaries on the onset of convection in an electrically nanofluid layer heated from below was investigated by Agrawal et al. [1]. Effect of magnetic field considering internal heating after filling the space between plates with nanofluid was also studied by Yadav et al. [17]. Non-linear Rayleigh-Benard magnetoconvection in temperature sensitive Newtonian liquids was studied by Aruna [2] demonstrating the diminishing heat transport and stabilization of system with the increasing strength of the magnetic field.

In fluid flow problems, the phenomenon of generation of the concentration flux by temperature gradient is termed as Soret effect and the energy flux caused by a composition gradient is called the Dufour or diffusion thermo effect. Study on the Soret induced convective instability of a regular Newtonian fluid saturated in a porous medium has been done by many researchers. Wang and Tan [15] analysed the convective instability in Benard cells in a non-Newtonian fluid incorporating Soret factor. The impact of Soret parameter induced by the temperature gradient was studied by Singh et al. [13]. Bahlowl et al. [3] and Mansour et al. [8] also have much research work on Soret effect in different forms of fluid layers. Postelnicu [9] and Rajput and Shareef [10] also studied the combined effect of magnetic field and Soret parameter. Recently triple diffusive convection with Soret-Dufour effects in a Maxwell nanofluid saturated in Darcy porous medium was studied by Singh et al. [14] demonstrating the effects of different parameters on heat transfer.

Double diffusive magneto convection in nanofluid layers incorporating Soret factor was studied by the author [12] and the effects of all parameters on stability of system were investigated analytically as well as graphically. The literature survey indicates that no study has investigated the effect of magnetic field on triple diffusive convection in a nanofluid layer with Dufour-Soret factor. The present study examines the effect of vertical magnetic field on Dufour-Soret induced triple diffusive convection in a nanofluid layer. To find the behaviour of different parameters in presence of magnetic field, a comparison has been made with Singh et al. [14] and Aruna [2].

2. Mathematical Formulations

We consider a layer of nanofluid confined between two infinite horizontal surfaces separated by a distance a, with z-axis vertically upward. Lower surface is maintained at higher temperature T_l^* and upper surface is maintained at temperature T_r^*. A uniform vertical magnetic field $M^* = (0, 0, M_0^*)$ is applied (See Fig. 1)

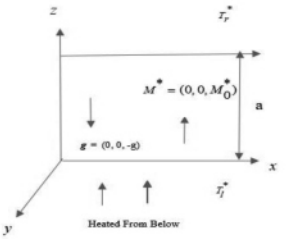


Fig. 1: Physical configuration of the problem

The governing equations for conservation of mass, momentum, energy and concentration of salt and nanoparticles in non-dimensional form on defining

$$(X,Y,Z)=\frac{(x^*,y^*,z^*)}{a},\quad t=\frac{t^*\alpha_m}{\sigma a^2},\quad (u_{1d},u_{2d},u_{3d})=\frac{(u_{1d}^*,u_{2d}^*,u_{3d}^*)a}{\alpha_m},\ p=\frac{p^*K}{\mu\alpha_m},\psi=\frac{\psi^*-\psi_0^*}{\psi_0^*},$$

$$(M_X,M_Y,M_Z)=\frac{(M_X^*,M_Y^*,M_Z^*)}{M_0^*},\ S=\frac{S^*-S_r^*}{S_l^*-S_r^*},T=\frac{T^*-T_r^*}{T_l^*-T_r^*},\quad \lambda=\frac{\lambda^*\alpha_m}{a^2}\quad\text{and}\quad S=\frac{S^*-S_r^*}{S_l^*-S_r^*}\ \text{are as}$$

follows:

$$\nabla.\boldsymbol{q}=0\ , \tag{1}$$

$$\boldsymbol{q}=(1+\frac{\lambda}{\sigma}\frac{\partial}{\partial t})[\{-\nabla p-R_m\hat{\boldsymbol{e}}_Z-R_n\psi\hat{\boldsymbol{e}}_Z+R_aT\hat{\boldsymbol{e}}_Z+\frac{R_s}{Ln}S\hat{\boldsymbol{e}}_Z\}+\frac{P_1}{P_{1m}}QD_a(\nabla\times M)\times M], \tag{2}$$

$$\frac{\partial T}{\partial t}+(\boldsymbol{q}.\nabla)T=\nabla^2T+\frac{N_b}{Le}\nabla\psi.\nabla T+\frac{N_aN_b}{Le}\nabla T.\nabla T+N_{tc}\nabla^2S\ , \tag{3}$$

$$\frac{1}{\sigma}\frac{\partial S}{\partial t}+\frac{1}{\in}\boldsymbol{q}.\nabla S=\frac{1}{Ln}\nabla^2S+N_{ct}\nabla^2T\quad , \tag{4}$$

$$\frac{1}{\sigma}\frac{\partial\psi}{\partial t}+\frac{1}{\in}(\boldsymbol{q}.\nabla)\psi=\frac{1}{Le}\nabla^2\psi+\frac{N_a}{Le}\nabla^2T\ , \tag{5}$$

$$\frac{1}{\sigma}\frac{\partial\boldsymbol{M}}{\partial t}+\frac{1}{\in}(\boldsymbol{q}.\nabla)\boldsymbol{M}=\frac{1}{\in}(\boldsymbol{M}.\nabla)\boldsymbol{q}+\frac{\mathrm{P}_1}{P_{1m}}\nabla^2\boldsymbol{M}\quad , \tag{6}$$

Here $R_a(=\frac{\rho g\beta Ka(T_l^*-T_r^*)}{\mu\alpha_m})$, $R_n(=\frac{(\rho_P-\rho)\psi_0^*gKa}{\mu\alpha_m})$, $R_m(=\frac{\rho_P\psi_0^*+\rho(1-\psi_0^*)gKa}{\mu\alpha_m})$,

$R_s(=\frac{\rho\beta_c gaK(S_l^*-S_r^*)}{\mu S_d})$ are thermal, concentration, basic density and solutal Rayleigh Darcy number respectively,

$P_1(=\frac{\mu}{\rho\alpha_m})$ and $P_{1m}(=\frac{\mu}{\rho\eta})$ are Prandtl numbers, $Q(=\frac{\mu_e M_0^{*2}a^2}{4\pi\mu\eta})$ is Magnetic Chandrasekhar number,

$D_a(=\frac{K}{a^2})$ is Darcy number, $N_{ct}(=\frac{S_{ct}(T_l^*-T_r^*)}{\alpha_m(S_l^*-S_r^*)})$ is Soret parameter, $N_{tc}(=\frac{S_{tc}(S_l^*-S_r^*)}{\alpha_m(T_l^*-T_r^*)})$ is Dufour parameter,

$N_a(=\frac{B_t(T_l^*-T_r^*)}{B_dT_r^*\psi_0^*})$ and $N_b(=\frac{(\rho c)_P\psi_0^*\in}{(\rho c)_F})$ are modified diffusivity ratio and modified particle density increment

respectively, $Le(=\frac{\alpha_m}{B_d})$ and $Ln=\frac{\alpha_m}{S_d}$ are Lewis numbers for nanofluid and salt respectively. The boundary conditions

are given as

$\boldsymbol{q} = 0$, T=1, S =1, $\dfrac{\partial \psi}{\partial Z} + N_a \dfrac{\partial T}{\partial Z} = 0$ at Z = 0 (7)

$\boldsymbol{q} = 0$, T=0, $S = 0$, $\dfrac{\partial \psi}{\partial Z} + N_a \dfrac{\partial T}{\partial Z} = 0$ at Z = 1 (8)

On the basic state, we superimpose perturbations in the form

Let $\boldsymbol{q} = \boldsymbol{q}'$, $p = p_{bs} + p'$, $T = T_{bs} + T'$, $S = S_{bs} + S'$, $\psi = \psi_{bs} + \psi'$, and $\boldsymbol{M} = \hat{\boldsymbol{e}}_Z + \boldsymbol{M}'$,

where the primes denote infinitesimal small quantities. Ignoring the products of primed quantities and their derivatives, following linearised form of equations is obtained:

$$\left(\frac{1}{\sigma}\frac{\partial}{\partial t} - \frac{P_1}{P_{1m}}\nabla^2\right)\left[\nabla^2 u_3' - \left(1 + \frac{\lambda}{\sigma}\frac{\partial}{\partial t}\right)\left(R_a \nabla_H^2 T' - R_n \nabla_H^2 \psi' + \frac{R_s}{Ln}\nabla_H^2 S'\right)\right] = \left(1 + \frac{\lambda}{\sigma}\frac{\partial}{\partial t}\right) Q \frac{P_1}{P_{1m}} \frac{D_a}{\in} \nabla^2 \frac{\partial^2 u_3'}{\partial Z^2}, \tag{9}$$

$$\frac{\partial T'}{\partial t} - u_3' = \nabla^2 T' - \frac{N_a N_b}{Le}\frac{\partial T'}{\partial Z} - \frac{N_b}{Le}\frac{\partial \phi'}{\partial Z} + N_{tc}\nabla^2 S', \tag{10}$$

$$\frac{1}{\sigma}\frac{\partial S'}{\partial t} - \frac{u_3'}{\in} = \frac{1}{Ln}\nabla^2 S' + N_{ct}\nabla^2 T', \tag{11}$$

$$\frac{1}{\sigma}\frac{\partial \psi'}{\partial t} + \frac{1}{\in}(\boldsymbol{q}'.\nabla)\psi' + \frac{1}{\in}N_a u_3' = \frac{1}{Le}\nabla^2 \psi' + \frac{N_a}{Le}\nabla^2 T', \tag{12}$$

$$u_3' = 0, T' = 0, S' = 0, \frac{\partial \psi'}{\partial Z} + N_a \frac{\partial T'}{\partial Z} = 0 \text{ at Z = 0 and Z = 1} \tag{13}$$

3. Linear Study

Following the linear stability theory by Chandrasekhar [4], the perturbations are taken of the form

$$\left(\psi', T', u_3', S'\right) = \left[\Phi(Z), \Theta(Z), \Omega(Z), \Psi(Z)\right] e^{st + iLX + iMY}, \tag{14}$$

where L and M are dimensionless wave numbers in X and Y directions respectively. On substituting the above values and employing Galerkin method to solve equations (9)-(12) together with the boundary condition (13) and taking first estimation as N=1, we have $\Omega = A_1 \sin \pi Z$, $\Theta = B_1 \sin \pi Z$, $\Phi = -N_a C_1 \sin \pi Z$, $\Psi = D_1 \sin \pi Z$ .Taking the determinant of above matrix equation as zero, the following Rayleigh number is obtained

$$R_a = \frac{\sigma}{\alpha^2}\left[\begin{array}{l} R_1\alpha^2(\lambda s + \sigma)(\sigma A\chi^2 + s)(\sigma\chi^2 + sLe)(\chi^2 + s)\{\chi^2(\in N_{ct} - 1) - s\} \\ -R_n N_a \alpha^2(\lambda s + \sigma)(A\chi^2\sigma + s)[(\chi^2\sigma + sLn)\{\chi^2(\in + Le) + sLe\} - \chi^4 N_{tc} Ln\sigma(1 + LeN_{ct})] \\ + \in(\chi^2\sigma + sLn)(\chi^2\sigma + sLe)(\chi^2 + s)\{A\sigma\chi^4 + B\pi^2\chi^2(\sigma + \lambda s) + s\chi^2\}\{(\chi^2\sigma + sLn)(\chi^2 + s) - \chi^4 N_{tc} N_{ct} Ln\sigma \\ \hline (\sigma\chi^2 \in + sLn \in -\chi^2 N_{tc} Ln\sigma)(\sigma\chi^2 + sLe)(\lambda s + \sigma)(A\chi^2\sigma + s) \end{array}\right], \tag{15}$$

where, $A = \dfrac{P_1}{P_{1m}}, B = Q\dfrac{P_1}{P_{1m}}\dfrac{D_a}{\in}$, $\chi^2 = \pi^2 + \alpha^2$.

Taking s=0 in equation (15),

$$R_a^{st} = \frac{\chi^2(\in \chi^2 + QD_a\pi^2)(1 - LnN_{ct}N_{tc}) - R_n N_a \alpha^2\{\in + Le - LnN_{tc}(1 + LeN_{ct})\} - R_s\alpha^2(1 - \in N_{ct})}{\alpha^2(\in - LnN_{tc})} . \tag{16}$$

From equation (16), the minimum Rayleigh number is given at critical wave number

$$\alpha_c = \pi\left(1 + \frac{QD_a}{\in}\right)^{1/4} .$$

In absence of magnetic field, result is same as obtained by Singh et al. [14].

Taking $s = i\omega$ in equation (15), we get oscillatory Rayleigh Number.

4. Nonlinear Stability Analysis

To predict the amplitude of convective motion and rate of heat transfer, we discuss the nonlinear stability theory. For simplicity, we consider the case of two-dimensional rolls, assuming all physical entities to be dependent of y. Introducing stream function γ and magnetic potential function ϕ such that

$$u = \frac{\partial \gamma}{\partial Z}, w = -\frac{\partial \gamma}{\partial X}, H_X = \frac{\partial \phi}{\partial Z}, H_Z = -\frac{\partial \phi}{\partial X}$$

Dimensionless equations are

$$\nabla_1^2 \gamma + (1 + \frac{\lambda}{\sigma}\frac{\partial}{\partial t})[(R_a \frac{\partial T}{\partial X} + \frac{R_s}{Ln}\frac{\partial S}{\partial X} - R_n \frac{\partial \psi}{\partial X}) - \frac{QP_1 D_a}{P_{1m}}\frac{\partial}{\partial Z}\nabla_1^2 \phi] = (1 + \frac{\lambda}{\sigma}\frac{\partial}{\partial t})\frac{QP_1 D_a}{P_{1m}} J(\nabla_1^2 \phi, \phi) \quad (17)$$

$$\frac{\partial \gamma}{\partial X} - \nabla_1^2 T + \frac{N_b}{Le}\frac{\partial \psi}{\partial Z} + \frac{N_a N_b}{Le}\frac{\partial T}{\partial Z} = -\frac{\partial T}{\partial t} + J(\gamma, T) + N_{tc}\nabla_1^2 S \quad (18)$$

$$\frac{1}{\in}\frac{\partial \gamma}{\partial X} - N_{ct}\nabla_1^2 T - \frac{1}{Ln}\nabla_1^2 S = -\frac{1}{\sigma}\frac{\partial S}{\partial t} + \frac{1}{\in} J(\gamma, S) \quad (19)$$

$$-\frac{N_a}{\in}\frac{\partial \gamma}{\partial X} - \frac{N_a}{Le}\nabla_1^2 T - \frac{1}{Le}\nabla_1^2 \psi = -\frac{1}{\sigma}\frac{\partial \psi}{\partial t} + \frac{1}{\in} J(\gamma, \psi) \quad (20)$$

$$\frac{1}{\in}\frac{\partial \gamma}{\partial Z} + \frac{P_1}{P_{1m}}\nabla_1^2 \phi = \frac{1}{\sigma}\frac{\partial \phi}{\partial t} - \frac{1}{\in} J(\gamma, \phi) \quad (21)$$

where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$.

We solve Eqs. (17) to (21) subjecting them to boundary conditions:

$$\gamma = 0, \frac{\partial^2 \gamma}{\partial Z^2} = 0, T = S = 0, \frac{\partial \Phi}{\partial Z} = 0, \frac{\partial \psi}{\partial Z} + N_a \frac{\partial T}{\partial Z} = 0 \quad (22)$$

at z=0 and z=1

We take the following Fourier series expression for γ, T, ψ, S, ϕ as

$$\gamma = A_{11}(t)\sin\alpha X \sin\pi Z$$

$$T = B_{11}(t)\cos\alpha X \sin\pi Z + B_{02}\sin 2\pi Z$$

$$\psi = -N_a C_{11}(t)\cos\alpha X \sin\pi Z - N_a C_{02}(t)\sin 2\pi Z$$

$$S = D_{11}(t)\cos\alpha X \sin\pi Z + D_{02}(t)\sin 2\pi Z$$

$$\phi = E_{11}(t)\sin\alpha X \cos\pi Z + E_{20}(t)\sin 2\alpha X$$

Putting in above equations we get

$$A_{11} = -\frac{\alpha}{\chi^2}[R_a B_{11} + \frac{R_s}{Ln} D_{11} + R_n N_a C_{11} + \frac{\lambda}{\sigma}(R_a \frac{dB_{11}}{dt} + R_n N_n \frac{dC_{11}}{dt} + \frac{R_s}{Ln}\frac{dD_{11}}{dt})] + Q\frac{P_1}{P_{1m}} D_a \frac{\pi\alpha}{\chi^2}(4\alpha^2 - \chi^2)E_{11}$$

$$-Q\frac{P_1}{P_{1m}} D_a \pi E_{11} - \frac{\lambda}{\sigma} Q D_a \pi \frac{P_1}{P_{1m}}\frac{dE_{11}}{dt} + \frac{\lambda}{\sigma} Q \frac{P_1}{P_{1m}} D_a \frac{\pi\alpha}{\chi^2}(4\alpha^2 - \chi^2)[E_{11}\frac{dE_{20}}{dt} + E_{20}\frac{dE_{11}}{dt}]$$

$$\frac{dB_{11}}{dt} = -(\alpha A_{11} + \chi^2 B_{11} + \pi\alpha A_{11} B_{02} + \chi^2 D_{11} N_{tc})$$

$$\frac{dB_{02}}{dt} = \frac{1}{2}(\pi\alpha A_{11} B_{11} - 8\pi^2 B_{02} - 8\pi^2 D_{02} N_{tc})$$

$$\frac{dC_{11}}{dt} = -\frac{\sigma}{\in}[\alpha A_{11} + \frac{\chi^2}{Le}(C_{11} - B_{11}) + \pi\alpha A_{11} C_{02}]$$

$$\frac{dC_{02}}{dt} = \frac{\sigma}{2 \in}[\pi\alpha A_{11}C_{11} - \frac{8\pi^2}{Le}(C_{02} - B_{02})]$$

$$\frac{dD_{11}}{dt} = -\sigma(\frac{\pi\alpha}{\in} A_{11}D_{02} + \frac{\alpha}{\in} A_{11} + \frac{\chi^2}{Ln} D_{11} + \chi^2 N_{ct} B_{11})$$

$$\frac{dD_{02}}{dt} = \frac{\sigma}{2}[\frac{\pi\alpha}{\in} A_{11}D_{11} - 8\pi^2(\frac{D_{02}}{Ln} + B_{02}N_{ct})]$$

$$\frac{dE_{11}}{dt} = \frac{\sigma}{\in}(\pi A_{11} - \chi^2 \in E_{11}\frac{P_1}{P_{1m}} + \alpha\pi A_{11}E_{20})$$

$$\frac{dE_{20}}{dt} = -\frac{\sigma}{2 \in}(\alpha\pi A_{11}E_{11} + 8\alpha^2 \in \frac{P_1}{P_{1m}} E_{20})$$

5. Heat, Salt and Nanoparticles concentration transport

The thermal Nusselt number, solutal concentration Nusselt number and nanoparticle concentration Nusselt number are given as

$$N(t) = 1 - 2\pi B_{02}(t)$$

$$N_c(t) = 1 - 2\pi D_{02}(t) + N_{ct}(1 - 2\pi B_{02}(t))$$

$$N_p(t) = 1 + 2\pi(B_{02}(t) - C_{02}(t))$$

6. Results and Discussion

6.1 Linear Stability Analysis

The effects of various parameters on the stability of the system for stationary convection and oscillatory convection were studied in a similar way as done by the author in [12]. It was found that the destabilizing behaviour of Soret parameter in absence of magnetic field [14] does not persist here in presence of magnetic field and converts to the stabilizing agent, whereas Dufour parameter still has a stabilizing behaviour as it was observed in absence of magnetic field [14].

Parameters D_a, $\in$, Ln and Q have been proved to be stabilizing agent in oscillatory convection whereas Le and N_a proved to advance the onset of convection. The dual behaviour of λ and σ was observed and Soret parameter proved to be a stabilizing agent whereas in absence of magnetic field its dual nature was observed [14]. Dufour parameter has been proved to have its dual effect on oscillatory convection whereas its dual behaviour does not persist in absence of magnetic field [14].

6.2 Nonlinear Stability Analysis

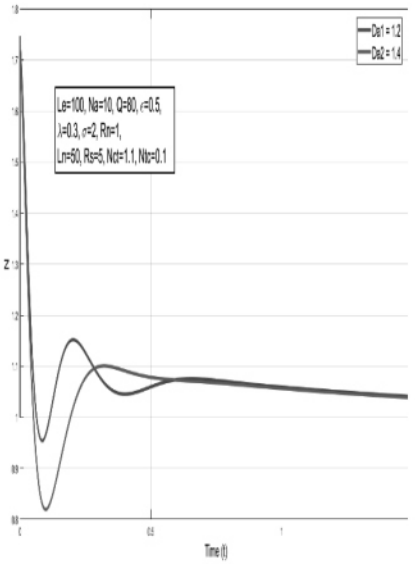
Fig. 2,3,4 depict the transient nature of thermal Nusselt number $N(t)$, concentration Nusselt number $N_c(t)$ and solute Nusselt number $N_p(t)$ respectively with respect to time in the unsteady state of motion. It is evident from all plots that in an initial state the transfer rate of heat, salt and mass is very high and a stationary state is approached after vigorous oscillations. It is clear from fig.2 that none of the parameters except D_a, $\in$, Ln, N_{tc} and Q have a significant effect on heat transfer. Figure 3 states that the salt transfer is decreased on increase in each parameter except the Dufour parameter N_{tc}. Initially an increase in N_{tc} causes increase in salt transfer but after a certain time the effect gets reversed. Fig.2(i) and 3(i) states that on increasing magnetic field there is retardation in heat transfer as well as salt transfer.

7. Conclusion

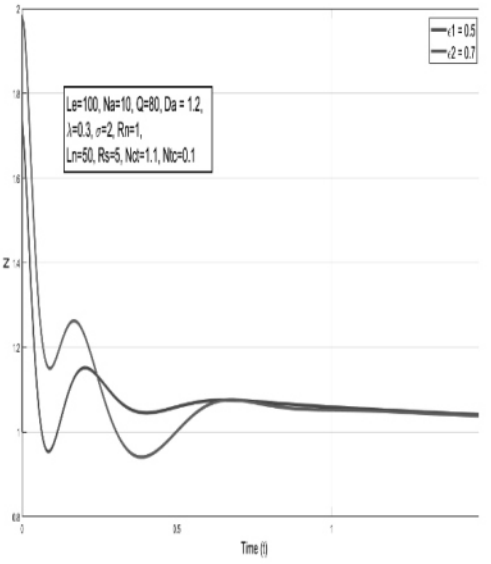
This paper presents an analytical study of linear and unsteady non-linear Rayleigh Benard triply diffusive magnetoconvection in a Maxwell nanofluid layer with Soret-Dufour effects. We reach at the following conclusions:

- For the stationary mode the Soret parameter, Dufour parameter and magnetic field have a stabilizing effect.
- For the oscillatory mode the Soret parameter which acts as destabilizing agent in absence of magnetic field has been proved to have stabilize the convection and the Dufour parameter which was observed to have stabilizing behaviour in absence of magnetic field [14], shows dual nature here in presence of magnetic field.
- The thermal Nusselt number here decreases with increase in magnetic field resulting in decrease of heat transport. This behaviour is same as was observed for Newtonian fluids [2].

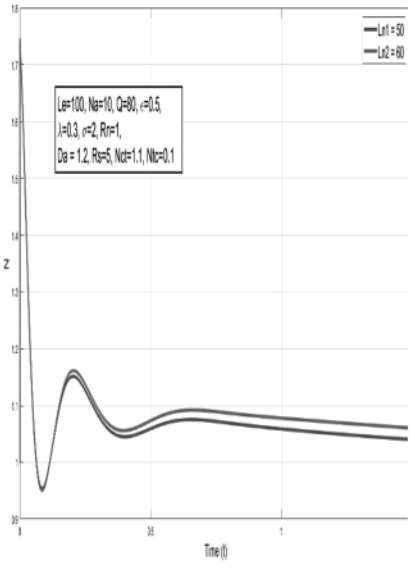
- The time-effect on transfer of heat, salt and nanoparticle concentration is found to be oscillatory when t is very small. But all the three Nusselt numbers approach a steady value when t has a very large value.



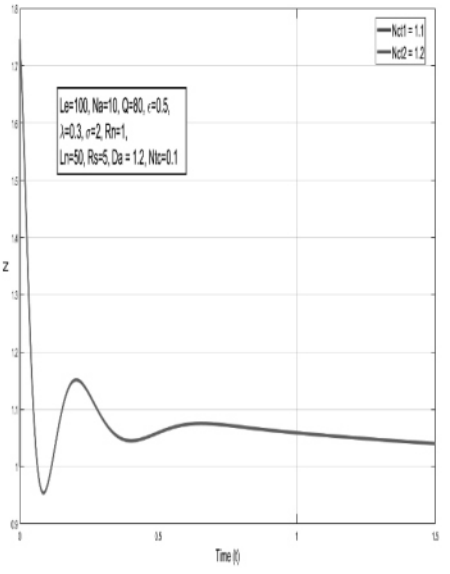
(a)



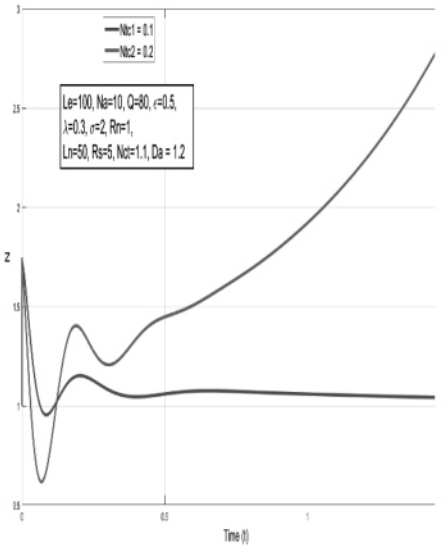
(b)



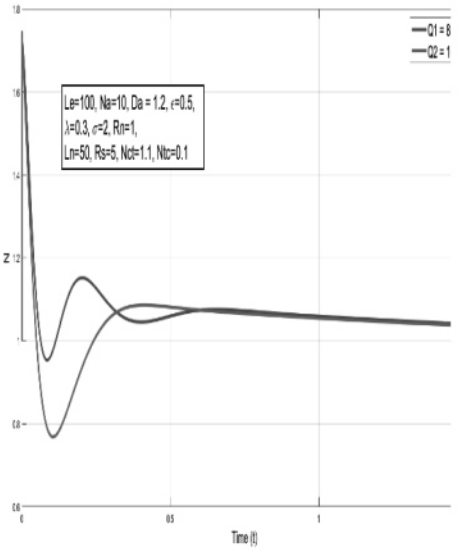
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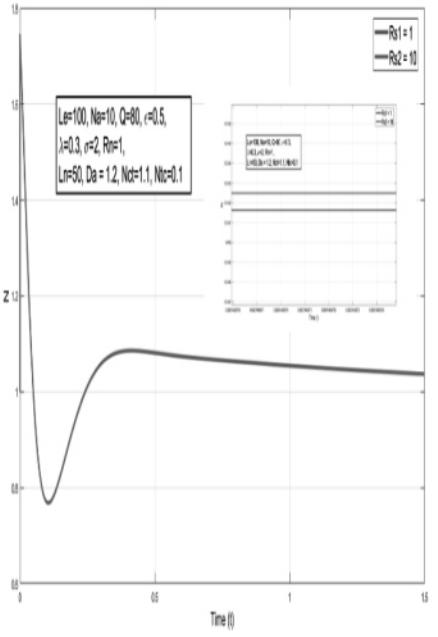
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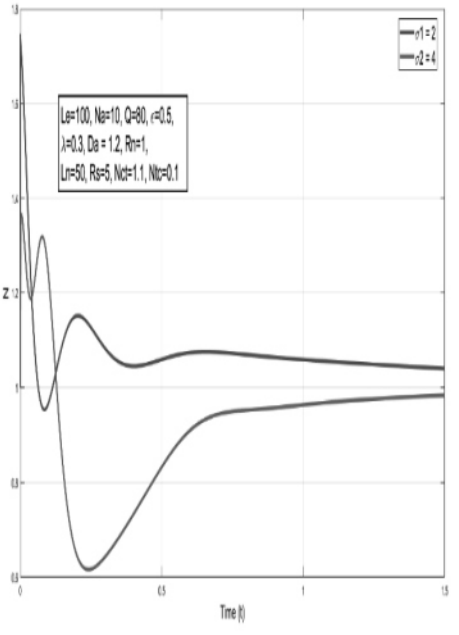
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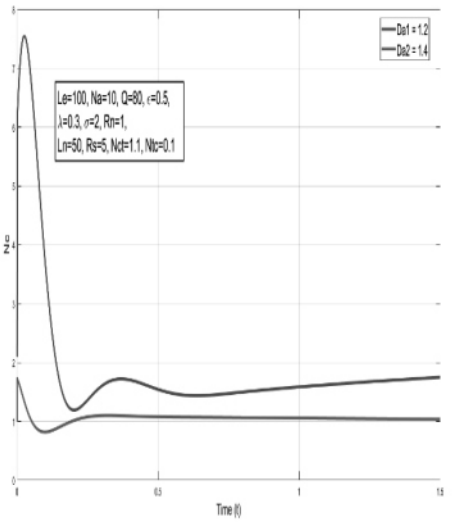


(g)

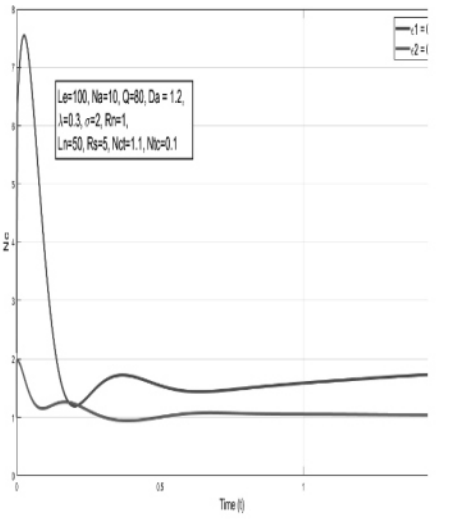


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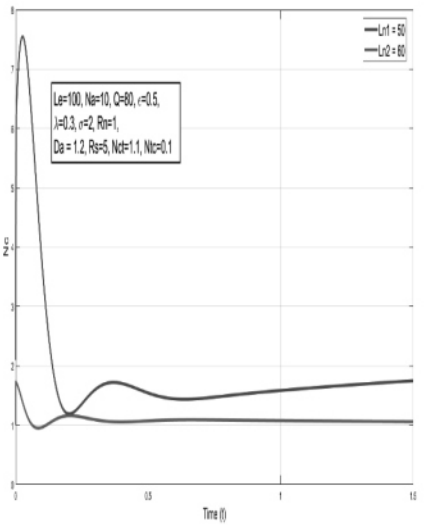
FIG.4 $N(t)$ **versus t for** (a) D_a (b) ε (c) Ln (d) N_{ct} (e) N_{tc} (f) Q (g) R_s (h) σ


Da1 = 1.2
Da2 = 1.4
Le=100, Na=10, Q=80, ε=0.5,
λ=0.3, σ=2, Rn=1,
Ln=50, Rs=5, Nct=1.1, Nbc=0.1
Nu
Time (t)

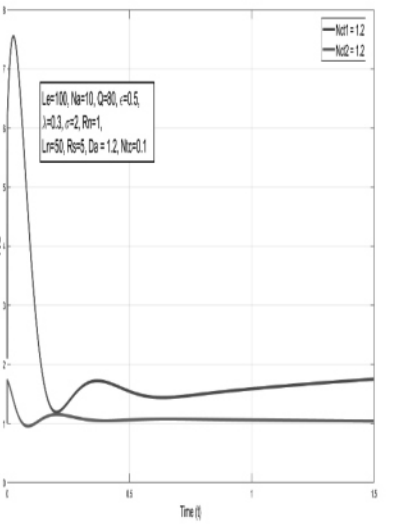
(a)


Le=100, Na=10, Q=80, Da = 1.2,
λ=0.3, σ=2, Rn=1,
Ln=50, Rs=5, Nct=1.1, Nbc=0.1
Nu
Time (t)

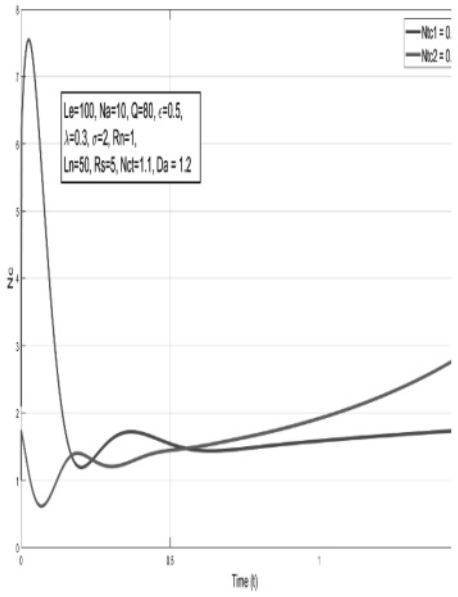
(b)


Ln1 = 50
Ln2 = 60
Le=100, Na=10, Q=80, ε=0.5,
λ=0.3, σ=2, Rn=1,
Da = 1.2, Rs=5, Nct=1.1, Nbc=0.1
Nu
Time (t)

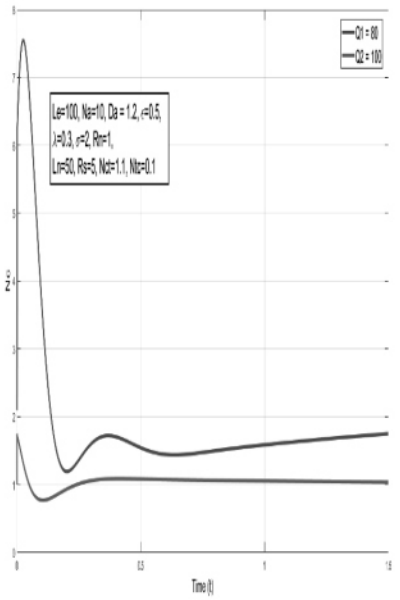
(c)


Nct1 = 1.2
Nct2 = 1.2
Le=100, Na=10, Q=80, ε=0.5,
λ=0.3, σ=2, Rn=1,
Ln=50, Rs=5, Da = 1.2, Nbc=0.1
Nu
Time (t)

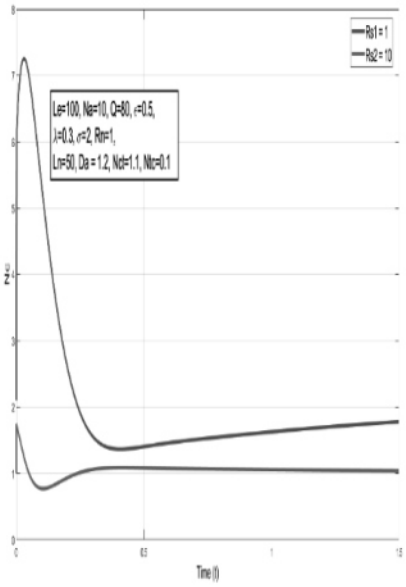
(d)



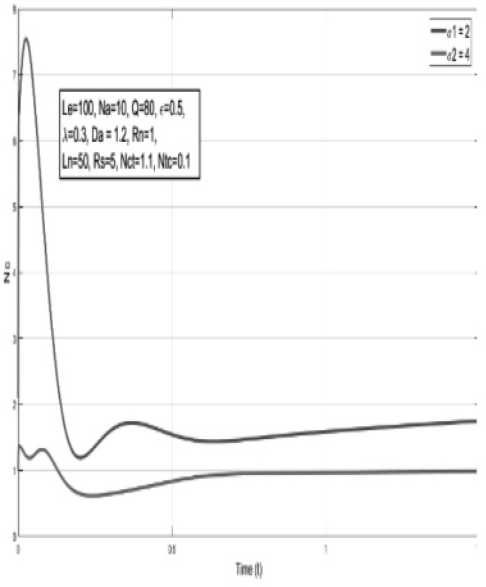
(e)



(f)

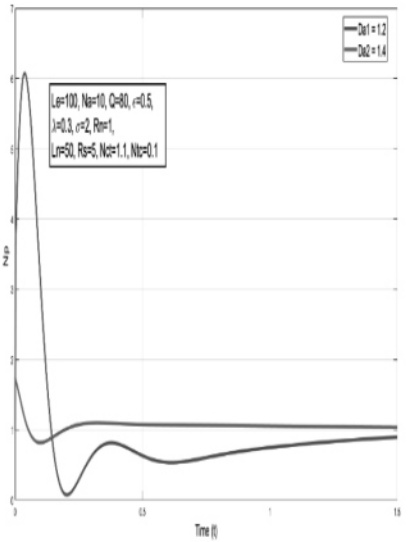


(g)

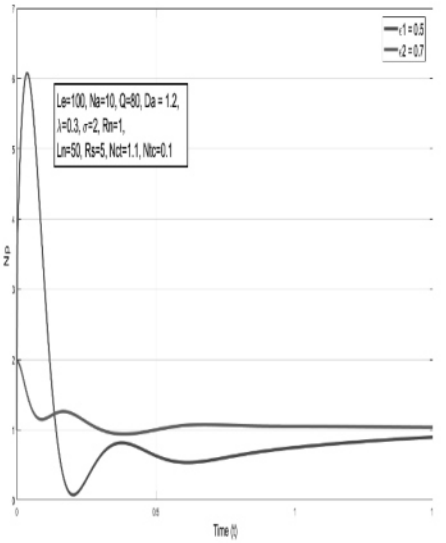


(h)

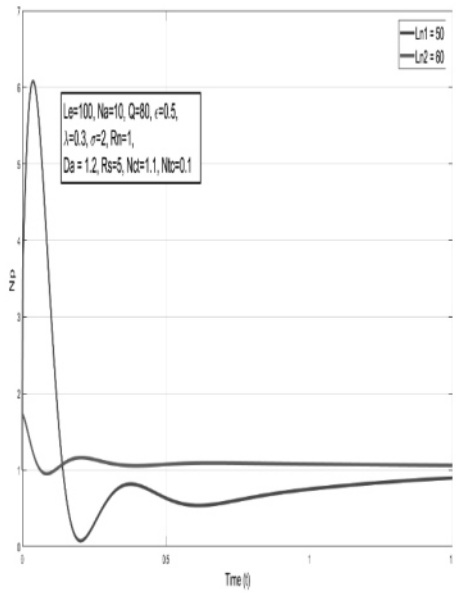
FIG.5 $N_c(t)$ **versus t for** (a) D_a (b) $\mathcal{E}$ (c) Ln (d) N_{ct} (e) N_{tc} (f) Q (g) R_s (h) σ


Da1 = 1.2
Da2 = 1.4
Le=100, Na=10, Q=80, τ=0.5,
λ=0.3, σ=2, Rn=1,
Ln=50, Rs=5, Nct=1.1, Nltc=0.1
NP
Time (t)

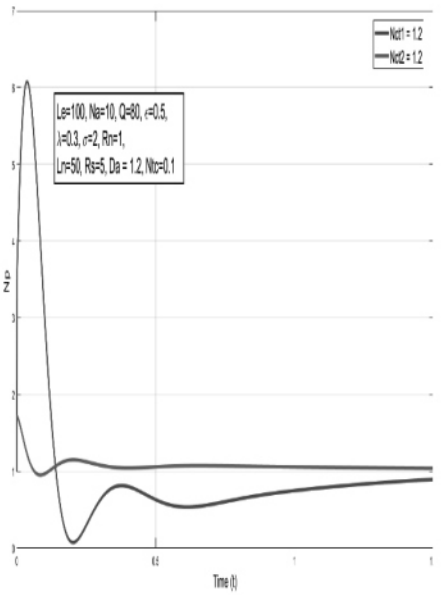
(a)


τ1 = 0.5
τ2 = 0.7
Le=100, Na=10, Q=80, Da = 1.2,
λ=0.3, σ=2, Rn=1,
Ln=50, Rs=5, Nct=1.1, Nltc=0.1
NP
Time (t)

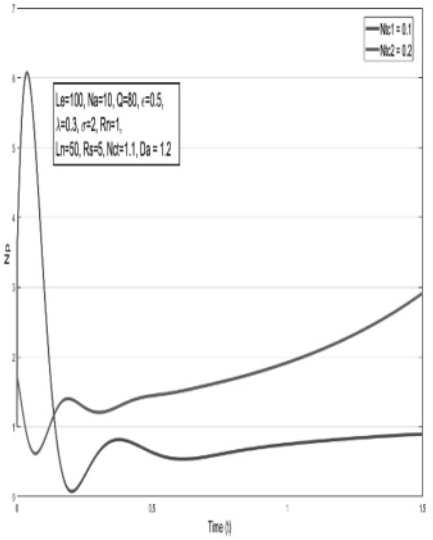
(b)


Ln1 = 50
Ln2 = 60
Le=100, Na=10, Q=80, τ=0.5,
λ=0.3, σ=2, Rn=1,
Da = 1.2, Rs=5, Nct=1.1, Nltc=0.1
NP
Time (t)

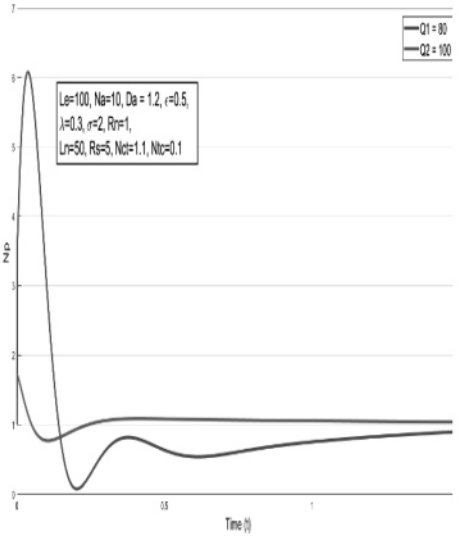
(c)


Nct1 = 1.2
Nct2 = 1.2
Le=100, Na=10, Q=80, τ=0.5,
λ=0.3, σ=2, Rn=1,
Ln=50, Rs=5, Da = 1.2, Nltc=0.1
NP
Time (t)

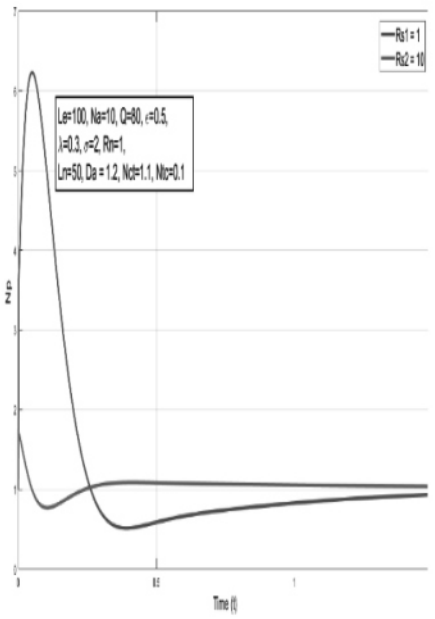
(d)



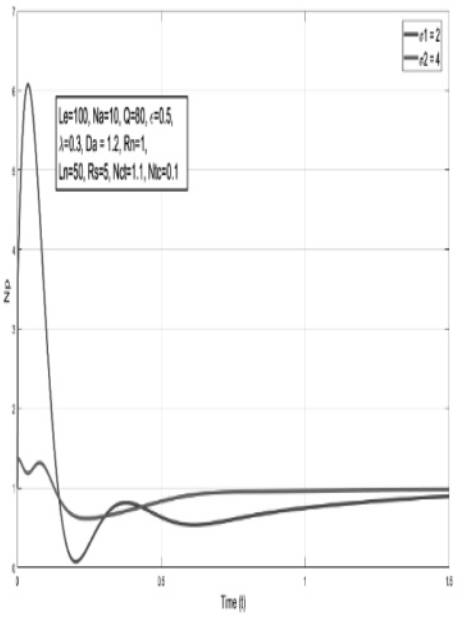
(e)



(f)



(g)



(h)

FIG.6 $N_p(t)$ **versus t for** (a) D_a (b) ε (c) Ln (d) N_{ct} (e) N_{tc} (f) Q (g) R_s (h) σ

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