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NUMBER OF FUZZY SUBGROUPS OF $D_8 \times C_2$

Abstract

One such rudimentary issue in fuzzy theory of groups is to count abundant fuzzy subgroups (FSGs). Several investigators have noteworthy arguments in contemporary times for the boom of the subject matter. It is always infinite when we try to find out the number of FSGs independently. To study the equivalence of FSG, some researchers have used the concept of an equivalence relation in fuzzy set (FS). The issues of reckoning an abundant distinct FSGs of a finite group can always be obtained corresponding to the equivalence relation chosen by us. The total count of FSG of a specific group be different and will depend on the choice of the equivalence relation chosen. For computing the number of FSGs in our case, we have followed the approach similar to Sulaiman & Ahmad [10]. The sole aim of this paper is to follow the approach somewhat similar to that in the cited paper for enumerating the number of FSG of $D_8 \times C_2$. For reckoning the number in this particular case, we will make use of the lattice subgroups graphic.

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1 Introduction

Zadeh [1] propounded the conception of FS. The literature on FS theory and its application has been proliferating rapidly across many disciplines of human knowledge. In 1971, Azriel Rosenfeld [2] initiated FS in the domain of group theory and coined the notion of FSGs of the group. Afterwards, the research in the corresponding branch grew. To make a dent in reckoning the abundant FSGs of the group covered by the equivalence relation. In this regard, the work of Sulaiman & Ahmad [10] is of worth. Our sole purpose in this article is to enumerate abundant FSGs of $D_8 \times C_2$ following an approach similar to that of Sulaiman and others. Recently, Dhiraj Kumar and M. K. Singh [15,16] calculated the aggregates of FSGs of D_8, Q_8, and Q_{12} using this relation. In this work, we have reckoned the aggregates of FSGs of $D_8 \times C_2$.

We have divided this paper into four sections. We've provided an introduction in the first section. Section two covers a preliminary outline by Sulaiman & Ahmad. Section three discussed the effects of locating aggregates of FSGs of $D_8 \times C_2$. At last, we have discussed the conclusion and future research.

2. Preliminaries

Definition 2.1 ([1]). Assume that $W \neq \emptyset$. Suppose $P: W \to [0,1]$ is a mapping designated as FS of W.

Definition 2.2 ([2]). Any FS μ of W is referred to as a FSG of W if, for all $\alpha, \beta \in W$, the succeeding axioms are fulfilled:

(i) $\mu(\alpha\beta) \geq min\{\mu(\alpha), \mu(\beta)\}\ \forall\ \alpha, \beta \in W$

(ii) $\mu(\alpha^{-1}) \geq \mu(\alpha)\ \forall\ \alpha \in W$

Theorem 2.1. [9] A FS μ of M is a FSG of W iff $\exists$ a sequence of subgroups of M, $N_1(\mu) \subset N_2(\mu) \subset \cdots\cdots\cdots \subset N_p(\mu) = W$ such that μ can be expressed as:

$$\mu(x) = \begin{cases} \theta_1 & x \in N_1(\mu) \\ \theta_2 & x \in N_2(\mu) - N_1(\mu) \\ \vdots & \\ \theta_p & x \in N_p(\mu) - N_{p-1}(\mu) \end{cases}$$

Definition 2.3. [9] Suppose that μ, γ be FSG of W of the form

$$\mu(x) = \begin{cases} \theta_1 & x \in P_1 \\ \theta_2 & x \in P_2 - P_1 \\ \vdots & \\ \theta_n & x \in P_n - P_{n-1} \end{cases}$$

and

$$\gamma(x) = \begin{cases} \delta_1 & x \in M_1 \\ \delta_2 & x \in M_2 - M_1 \\ \vdots & \\ \delta_m & x \in M_m - M_{m-1} \end{cases}$$

Now, one can say that μ is equivalent to γ and we denote it as $\mu \sim \gamma$, if:

(i) $m = n$

(ii) $P_i(\mu) = M_i(\gamma),\ \forall\ i \in \{1,2,\ldots,n\}$

3. Number of FSGs of $D_8 \times C_2$

We have $D_8 = < a, b: a^4 = b^2 = 1, ab = ba^{-1} >$

Clearly $D_8 = \{1, a, a^2, a^3, b, ab, a^2b, a^3b\}$

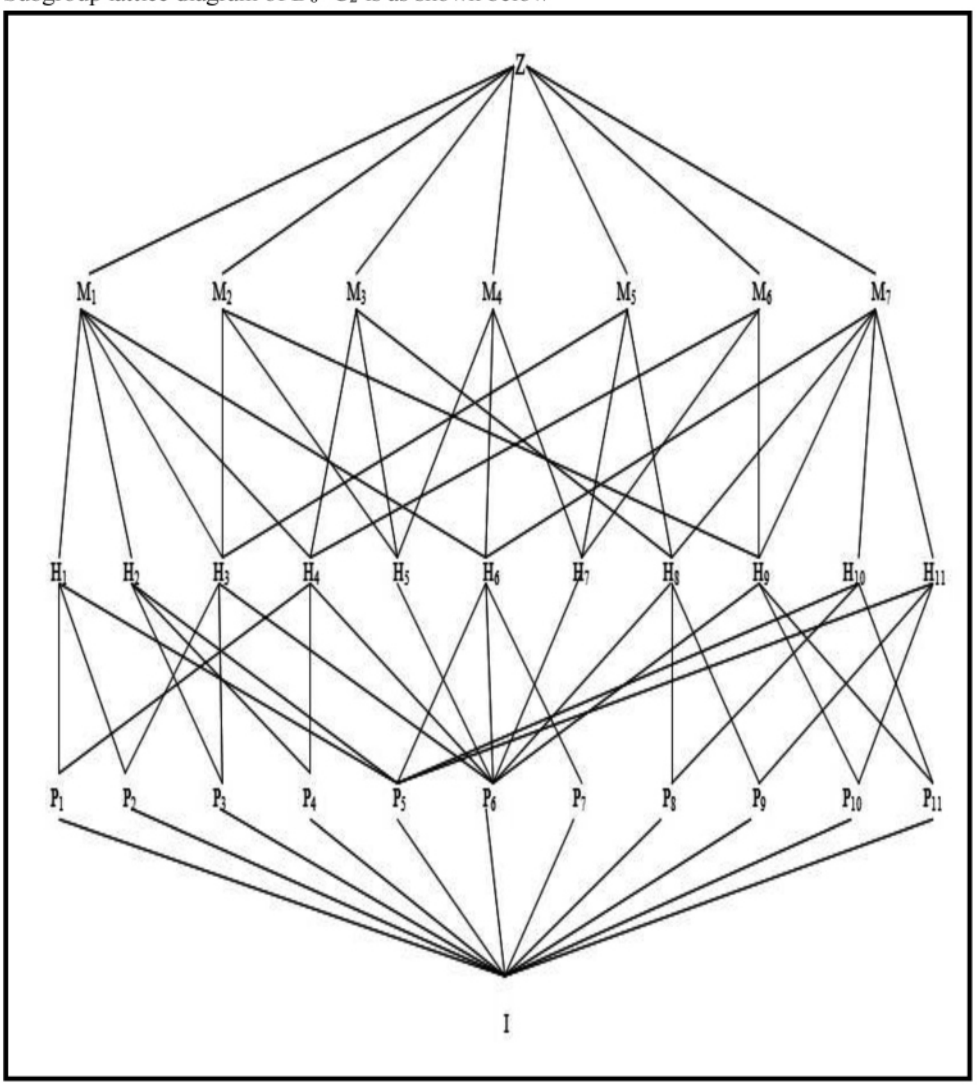
and $C_2 = \{1, x\}$ where $x^2 = 1$

Then, we have

$D_8 \times C_2 = \{(1,1), (a,1), (a^2,1), (a^3,1), (b,1), (ab,1), (a^2b,1), (a^3b,1),$

$(1,x), (a,x), (a^2,x), (a^3,x), (b,x), (ab,x), (a^2b,x), (a^3b,x)\}$

Subgroup lattice diagram of $D_8 \times C_2$ is as shown below



We have 31 subgroups of $D_8 \times C_2$; those are $I = \{(1,1)\}$, $P_1 = \{(1,1), (b,1)\}$

$P_2 = \{(1,1),(a^2b,x)\}$ $P_3 = \{(1,1),(b,x)\}$ $P_4 = \{(1,1),(a^2b,1)\}$

$P_5=\{(1,1),(a^2,x)\}$ $P_6=\{(1,1),(a^2,1)\}P_7=\{(1,1),(1,x)\}$ $P_8=\{(1,1),(ab,x)\}$ $P_9=\{(1,1),(a^3b,x)\}$ $P_{10}=\{(1,1),(ab,1)\}$ $P_{11}=\{(1,1),(a^3b,1)\}$

$H_1=\{(1,1),(a^2,x),(b,1),((a^2b,x)\}$

$H_2=\{(1,1),(a^2,x),(b,x),((a^2b,1)\}$

$H_3=\{(1,1),(a^2,1),(b,x),((a^2b,x)\}$

$H_4=\{(1,1),(a^2,1),(b,1),((a^2b,1)\}$

$H_5=\{(1,1),(a,x),(a^2,1),((a^3,x)\}$

$H_6=\{(1,1),(a^2,1),(1,x),((a^2,x)\}$

$H_7=\{(1,1),(a,1),(a^2,1),((a^3,1)\}$

$H_8=\{(1,1),(a^2,1),(ab,x),((a^3b,x)\}$

$H_9=\{(1,1),(a^2,1),(ab,1),((a^3b,1)\}$ $H_{10}=\{(1,1),(a^2,x),(ab,x),((a^3b,1)\}$

$H_{11}=\{(1,1),(a^2,x),(ab,1),((a^3b,x)\}$

$M_1=\{(1,1),(a^2,1),(b,1),(a^2b,1),(1,x),(a^2,x),(b,x),(a^2b,x)\}$
$M_2=\{(1,1),(a,x),(a^2,1),(a^3,x),(b,x),(ab,1),(a^2b,x),(a^3b,1)\}$

$M_3=\{(1,1),(a,x),(a^2,1),(a^3,x),(b,1),(ab,x),(a^2b,1),(a^3b,x)\}$

$M_4=\{(1,1),(a,1),(a^2,1),(a^3,1),(1,x),(a,x),(a^2,x),(a^3,x)\}$

$M_5=\{(1,1),(a,1),(a^2,1),(a^3,1),(b,x),(ab,x),(a^2b,x),(a^3b,x)\}$

$M_6=\{(1,1),(a,1),(a^2,1),(a^3,1),(b,1),(ab,1),(a^2b,1),(a^3b,1)\}$

$M_7=\{(1,1),(a^2,1),(ab,1),(a^3b,1),(1,x),(a^2,x),(ab,x),(a^3b,x)\}$ and $Z=D_8\times C_2$ itself.

We have the following chains

(1) $D_8 \times C_2 = Z, M_1 < Z, M_2 < Z, M_3 < Z, M_4 < Z, M_5 < Z, M_6 < Z, M_7 < Z$

Total chains=8

(2) $H_1 < M_1 < Z, H_1 < Z, H_2 < M_1 < Z, H_2 < Z, H_3 < M_1 < Z, H_3 < Z, H_3 < M_2 < Z, H_3 < M_5 < Z, H_4 < M_1 < Z, H_4 < Z, H_4 < M_3 < Z, H_4 < M_6 < Z, H_5 < M_2 < Z, H_5 < Z, H_5 < M_3 < Z, H_5 < M_4 < Z, H_6 < M_1 < Z, H_6 < Z, H_6 < M_4 < Z, H_6 < M_7 < Z, H_7 < M_4 < Z, H_7 <$

$Z, H_7 < M_5 < Z, H_7 < M_6 < Z, H_8 < M_5 < Z, H_8 < Z, H_8 < M_3 < Z, H_8 < M_7 < Z, H_9 < M_2 < Z, H_9 < Z, H_9 < M_6 < Z, H_9 < M_7 < Z, H_{10} < M_7 < Z, H_{10} < Z, H_{11} < M_7 < Z, H_{11} < Z$

Total chains = 36

(3) (i)$P_1 < H_1 < M_1 < Z, P_1 < H_1 < Z, P_1 < M_1 < Z, P_1 < Z, P_1 < H_4 < M_1 < Z, P_1 < H_4 < M_3 < Z, P_1 < H_4 < M_6 < Z, P_1 < H_4 < Z, P_1 < M_3 < Z, P_1 < M_6 < Z$

(ii) $P_2 < H_1 < M_1 < Z, P_2 < H_1 < Z, P_2 < M_1 < Z, P_2 < Z, P_2 < H_3 < M_1 < Z, P_2 < H_3 < Z, P_2 < H_3 < M_2 < Z, P_2 < M_2 < Z, P_2 << H_3 < M_5 < Z, P_2 < M_5 < Z$

(iii) $P_3 < H_2 < M_1 < Z, P_3 < H_2 < Z, P_3 < M_1 < Z, P_3 < Z, P_3 < H_3 < M_1 < Z, P_3 < H_3 < Z, P_3 < H_3 < M_2 < Z, P_3 < M_2 < Z, P_3 < H_3 < M_5 < Z, P_3 < M_5 < Z$

(iv) $P_4 < H_2 < M_1 < Z, P_4 < H_2 < Z, P_4 < M_1 < Z, P_4 < Z, P_4 < H_4 < M_1 < Z, P_4 < H_4 < Z, P_4 < H_4 < M_3 < Z, P_4 < M_3 < Z, P_4 < H_4 < M_6 < Z, P_4 < M_6 < Z$

(v) $P_5 < H_1 < M_1 < Z, P_5 < H_1 < Z, P_5 < M_1 < Z, P_5 < Z, P_5 < H_{11} < M_7 < Z, P_5 < H_{11} < Z, P_5 < H_6 < M_1 < Z, P_5 < H_6 < Z, P_5 < H_6 < M_4 < Z, P_5 < M_4 < Z, P_5 < H_6 < M_7 < Z, P_5 < M_7 < Z, P_5 < H_{10} < M_7 < Z, P_5 < H_{10} < Z, P_5 < H_2 < M_1 < Z, P_5 < H_2 < Z$

(vi) $P_6 < H_6 < M_4 < Z, P_6 < H_6 < Z, P_6 < M_4 < Z, P_6 < Z, P_6 < H_5 < M_2 < Z, P_6 < H_5 < Z, P_6 < M_2 < Z, , P_6 < H_5 < M_3 < Z, P_6 < M_3 < Z, P_6 < H_5 < M_4 < Z, P_6 < H_4 < M_1 < Z, P_6 < H_4 < Z, P_6 < M_1 < Z, P_6 < H_4 < M_3 < Z, P_6 < H_4 < M_6 < Z, P_6 < H_3 < M_1 < Z, P_6 < H_3 < Z, P_6 < H_3 < M_2 < Z, P_6 < H_3 < M_5 < Z, P_6 < H_7 < M_4 < Z, P_6 < H_7 < Z, P_6 < H_7 < M_5 < Z, P_6 < M_5 < Z, P_6 < H_7 < M_6 < Z, P_6 < M_6 < Z, P_6 < H_8 < M_5 < Z, , P_6 < H_8 < Z, P_6 < H_8 < M_7 < Z, P_6 < M_7 < Z, P_6 < H_8 < M_3 < Z, P_6 < H_9 < M_2 < Z, P_6 < H_9 < Z, P_6 < H_9 < M_6 < Z, P_6 < H_9 < M_7 < Z, P_6 < H_6 < M_1 < Z, P_6 < H_6 < M_7 < Z$

(vii) $P_7 < H_6 < M_1 < Z, P_7 < H_6 < Z, P_7 < M_1 < Z, P_7 < Z, P_7 < H_6 < M_4 < Z, P_7 < M_4 < Z, P_7 < H_6 < M_7 < Z, P_7 < M_7 < Z$

(viii) $P_8 < H_8 < M_3 < Z, P_8 < H_8 < Z, P_8 < M_3 < Z, P_8 < Z, P_8 < H_8 < M_5 < Z, P_8 < M_5 < Z, P_8 < H_8 < M_7 < Z, P_8 < M_7 < Z, P_8 < H_{10} < M_7 < Z, P_8 < H_{10} < Z$

(ix) $P_9 < H_{11} < M_7 < Z, P_9 < H_{11} < Z, P_9 < M_7 < Z, P_9 < Z, P_9 < H_8 < M_3 < Z, P_9 < M_3 < Z, P_9 < H_8 < M_7 < Z, P_9 < M_7 < Z, P_9 < H_8 < M_5 < Z, P_9 < M_5 < Z, P_9 < H_8 < Z$

(x) $P_{10} < H_{11} < M_7 < Z, P_{10} < H_{11} < Z, P_{10} < M_7 < Z, P_{10} < Z, P_{10} < H_9 < M_2 < Z, P_{10} < M_2 < Z, P_{10} < H_9 < Z, P_{10} < H_9 < M_6 < Z, P_{10} < M_6 < Z, P_{10} < H_9 < M_7 < Z, P_{10} < M_7 < Z$

(xi) $P_{11} < H_9 < M_2 < Z, P_{11} < H_9 < Z, P_{11} < M_2 < Z, P_{11} < Z, P_{11} < H_9 < M_6 < Z, P_{11} < M_6 < Z, P_{11} < H_9 < M_7 < Z, P_{11} < M_7 < Z, \ P_{11} < H_{10} < M_7 < Z, \ P_{11} < H_{10} < Z$

Total chains = 10 + 10 + 10 + 10 + 16 + 36 + 8 + 10 + 11 + 11 + 10 = 142

Total chains in (1) + (2) + (3) = 8 + 36 + 142 = 186

Hence, the number of FSGs corresponding to the above chains is 186.

(4) As a consequence of $P_1(\mu) = \{(1,1)\}$, we have 186 fuzzy subgroups.

Therefore, the overall number of FSGs of $D_8 \times C_2$ is $2 \times 186 = 372$

4. Conclusion and Future Research

We use the equivalence relation introduced by R. Sulaiman and Abd. Ghafur to obtain the number of fuzzy subgroups of $D_8 \times C_2$. This method can further be applied to count fuzzy subgroups of some more abelian and non-abelian groups.

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