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COEFFICIENT ESTIMATES FOR A CERTAIN SUBCLASS OF ANALYTIC AND BI UNIVALENT FUNCTIONS DEFINED USING AN OPERATOR ASSOCIATED WITH GENERALIZED MITTAG-LEFFLER FUNCTION

ABSTRACT. This paper presents a newly defined subclass of analytic and bi-univalent functions within the unit disc $\Delta = \{z \in \mathbb{C} : |z| < 1\}$. The subclass is constructed using an operator associated with the generalized Mittag-Leffler function. Furthermore, the initial coefficient bounds for functions in this subclass are estimated.

Key words : Taylor-Maclaurin series expansion, Analytic function, Univalent function, Bi-univalent function, coefficient bounds, Mittag-Leffler function
Mathematics Subject Classification: 30C45, 30C80

1. INTRODUCTION

Let $\mathcal{A}$ be the class of analytic functions defined on an open unit disc $\Delta = \{z \in \mathbb{C} : |z| < 1\}$ and which are in the form

$$h(z) = z + \sum_{j=2}^{\infty} a_j z^j \quad (z \in \Delta). \tag{1.1}$$

Further, S is the set of all univalent functions in $\mathcal{A}$. Here, the Koebe One-Quarter theorem [5] plays an important role and it says that the image of Δ under $h \in S$ contains a disc of radius $1/4$.
Clearly, for each function $h \in S$, h^{-1} exists and it is defined as

$$h^{-1}(h(z)) = z, \quad z \in \Delta$$

and

$$h(h^{-1}(w)) = w, \quad |w| < r_0, r_0(h) \geq 1/4$$

where

$$g(w) = h^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots . \tag{1.2}$$

If both functions $h \in S$ and h^{-1} are univalent in Δ, then h is said to be bi-univalent in Δ . Here we denote the set of all such bi-univalent functions by Σ. The study of subclasses of bi-univalent functions and their coefficient bounds is an interesting topic of research and it was first studied by Lewin [7] and he proved that $|a_2| < 1.51$. Later, Netanyahu [9] showed that $max|a_2| = 4/3$. Further, Brannan and Clunie [3] conjectured that, for $h \in \sum, |a_2| \leq \sqrt{2}$. Brannan and Taha [4] introduced very interesting two subclasses of the class bi-univalent functions Σ, the class of startlike functions $S^*(\beta)$ and the class $\mathcal{K}(\beta)$ of convex functions of order $\beta(\beta \in [0, 1))$ in Δ (see [9]). Later, $S^*_{\sum}(\beta)$ (the class of bi-starlike function of order β) and $\mathcal{K}_{\sum}(\beta)$ (the class of bi-convex functions of order β) were introduced on Δ. Many researchers worked on these classes and obtained initial coefficient bounds and later on many congruent subclasses

were introduced by mathematicians and obtained the coefficient bounds for diversified subclasses of bi-univalent functions(see[1], [10], [6]).

In 1903, Gosta Mittag-Leffler [8] introduced the following function

$$E_\alpha(z) = \sum_{j=0}^{\infty} \frac{z^j}{\Gamma(1+j\alpha)} \tag{1.3}$$

where $\alpha \geq 0$, $z \in \mathbb{C}$ and $\Gamma(s)$ is a gamma function.
Moreover, Srivastava and Tomovski [12] introduced the function

$$E^{\gamma,k}_{\alpha,\beta}(z) = \sum_{j=0}^{\infty} \frac{(\gamma)_{jk} z^j}{\Gamma(\beta+j\alpha)j!} \tag{1.4}$$

where $\gamma, \beta, \alpha \in \mathbb{C}; Re(k) > 0;\ Re(\alpha) > max\{0, Re(k)-1\}$ and $(\gamma)_j$ is Pochhammer symbol defined as

$$(\gamma)_j = \frac{\Gamma(j+\gamma)}{\Gamma(\gamma)}$$

and its value is 1 if $j = 0$ and its value is $\gamma(\gamma+1)\cdots(\gamma+j-1)$ if $j \in \mathbb{N}$.
Using operator (1.4), Attiya [2] defined the operator $\mathcal{H}^{\gamma}_{\alpha,\beta,k}(h) : \mathcal{A} \to \mathcal{A}$ by

$$\mathcal{H}^{\gamma}_{\alpha,\beta,k}(h)(z) = \mathcal{Q}^{\gamma}_{\alpha,\beta,k}(z) * h(z) \quad (z \in \Delta) \tag{1.5}$$

where

$$\mathcal{Q}^{\gamma}_{\alpha,\beta,k}(z) = \frac{\Gamma(\beta+\alpha)}{(\gamma)_k}\left(E^{\gamma,k}_{\alpha,\beta}(z) - \frac{1}{\Gamma(\beta)}\right) \quad (z \in \Delta),$$

where $\gamma, \beta, \alpha \in \mathbb{C}; Re(k) > 0; Re(\alpha) > max\{0, Re(k)-1\}; Re(\alpha) = 0$ when $Re(k) = 1$, $\beta \neq 0$ and the symbol(*) denotes the convolution (or Hadamard product).
It can be easily observed that

$$\mathcal{H}^{\gamma}_{\alpha,\beta,k}(h)(z) = z + \sum_{j=2}^{\infty} \frac{\Gamma(jk+\gamma)\Gamma(\beta+\alpha)}{\Gamma(k+\gamma)\Gamma(\beta+j\alpha)j!} a_j z^j \tag{1.6}$$

and

$$z\left(\mathcal{H}^{\gamma}_{\alpha,\beta,k}(h)(z)\right)' = \left(\frac{\gamma+k}{k}\right)\left(\mathcal{H}^{\gamma+1}_{\alpha,\beta,k}(h)(z)\right) - \frac{\gamma}{k}\left(\mathcal{H}^{\gamma}_{\alpha,\beta,k}(h)(z)\right). \tag{1.7}$$

Motivate by the work (especially [13]) and using the operator defined in (1.5), we introduce a new subclass $\mathcal{N}^{\gamma,u,v}_{\alpha,\beta,k}(\lambda)$ of analytic function class $\mathcal{A}$.

Definition 1.1. Let $h \in \mathcal{A}$, g be an extension of h^{-1} on Δ and $u, v : \Delta \to \mathbb{C}$ be functions such that $min\{\mathcal{R}(u(z)), \mathcal{R}(v(z))\} > 0$ $(z \in \Delta)$ with $u(0) = v(0) = 1$. Then we say that $h \in \mathcal{N}^{\gamma,u,v}_{\alpha,\beta,k}(\lambda)$ if following conditions are satisfied.

$$h \in \Sigma, \quad \frac{z\left(\mathcal{H}^{\gamma}_{\alpha,\beta,k}h(z)\right)'}{\lambda z\left(\mathcal{H}^{\gamma}_{\alpha,\beta,k}h(z)\right)' + (1-\lambda)\mathcal{H}^{\gamma}_{\alpha,\beta,k}h(z)} \in u(\Delta) \quad (\lambda \in [0,1); z \in \Delta) \tag{1.8}$$

and

$$\frac{w\left(\mathcal{H}^{\gamma}_{\alpha,\beta,k}g(w)\right)'}{\lambda w\left(\mathcal{H}^{\gamma}_{\alpha,\beta,k}g(w)\right)' + (1-\lambda)\mathcal{H}^{\gamma}_{\alpha,\beta,k}g(w)} \in v(\Delta) \quad (\lambda \in [0,1); w \in \Delta) \tag{1.9}$$

where $\gamma, \beta, \alpha \in \mathbb{C}; Re(k) > 0; Re(\alpha) > max\{0, Re(k) - 1\}; Re(\alpha) = 0$ when $Re(k) = 1$ and $\beta \neq 0$.

For particular values of γ, α, k and for different choices of functions u and v, the class $\mathcal{N}^{\gamma,u,v}_{\alpha,\beta,k}(\lambda)$ reduces to many well known classes of holomorphic and bi-univalent functions as follows.

Remark 1.2. (1) For $\gamma = 1, \alpha = 0$ and $k = 1$, the class $\mathcal{N}^{\gamma,u,v}_{\alpha,\beta,k}(\lambda)$ reduces to the class $\mathcal{M}^{u,v}_{\Sigma}(\lambda)$ studied by Yamini J [13].

(2) For $\gamma = 1, \alpha = 0, k = 1$ and by setting $u(z) = \left(\frac{1+z}{1-z}\right)^{\eta}$ and $v(z) = \left(\frac{1-z}{1+z}\right)^{\eta}$ (where $\eta \in (0, 1]; z \in \Delta$), the class $\mathcal{N}^{\gamma,u,v}_{\alpha,\beta,k}(\lambda)$ reduces to the class $SS^{*}_{\Sigma}(\eta, \lambda)$ introduced by Murugusundaramoorthy et al [11].

(3) For $\gamma = 1, \alpha = 0, k = 1$ and by setting $u(z) = \frac{1+(1-2\eta)z}{1-z}$ and $v(z) = \frac{1-(1-2\eta)z}{1+z}$ (where $\eta \in [0, 1); z \in \Delta$), the class $\mathcal{N}^{\gamma,u,v}_{\alpha,\beta,k}(\lambda)$ reduces to the class $S^{*}_{\Sigma}(\eta, \lambda)$ introduced by Murugusundaramoorthy et al [11].

(4) By setting $\lambda = 0$ in the classes $SS^{*}_{\Sigma}(\eta, \lambda)$ and $S^{*}_{\Sigma}(\eta, \lambda)$, we get the classes introduced by Brannan and Taha [4].

2. COEFFICIENT BOUNDS FOR THE FUNCTION CLASS $\mathcal{N}^{\gamma,u,v}_{\alpha,\beta,k}(\lambda)$

In this section, I derive initial coefficient bounds of the functions from the class $\mathcal{M}^{\gamma,u,v}_{\alpha,\beta,k}(\lambda)$.

Theorem 2.1. *If $h \in \Sigma$ given by (1.1) and it is a member of the class $\mathcal{N}^{\gamma,u,v}_{\alpha,\beta,k}(\lambda)$, then*

(2.1)

$$|a_2| \leq min \left\{ \frac{\sqrt{2\left(|u'(0)|^2 + |v'(0)|^2\right)}}{1-\lambda} \cdot \left|\frac{\Gamma(k+\gamma)}{\Gamma(2k+\gamma)}\right| \left|\frac{\Gamma(\beta+2\alpha)}{\Gamma(\beta+\alpha)}\right|, \sqrt{\frac{|u''(0)| + |v''(0)|}{2|\tau|}} \right\}$$

and

(2.2)

$$|a_3| \leq \frac{3(|u''(0)| + |v''(0)|)}{4(1-\lambda)} \cdot \left|\frac{\Gamma(k+\gamma)}{\Gamma(3k+\gamma)}\right| \cdot \left|\frac{\Gamma(\beta+3\alpha)}{\Gamma(\beta+\alpha)}\right| + min \left\{ \frac{2(|u'(0)|^2 + |v'(0)|^2)}{(1-\lambda)^2} \cdot \left|\frac{\Gamma(k+\gamma)}{\Gamma(2k+\gamma)}\right|^2 \left|\frac{\Gamma(\beta+2\alpha)}{\Gamma(\beta+\alpha)}\right|^2, \frac{|u''(0)| + |v''(0)|}{2|\tau|} \right\}$$

where

$$\tau = \left(\frac{\lambda^2 - 1}{2}\right) \cdot \left(\frac{\Gamma(2k+\gamma)}{\Gamma(k+\gamma)}\right)^2 \cdot \left(\frac{\Gamma(\beta+\alpha)}{\Gamma(\beta+2\alpha)}\right)^2 + \frac{2(1-\lambda)}{3} \cdot \frac{\Gamma(3k+\gamma)}{\Gamma(k+\gamma)} \cdot \frac{\Gamma(\beta+\alpha)}{\Gamma(\beta+3\alpha)}$$

and $\gamma, \beta, \alpha \in \mathbb{C}; Re(k) > 0; Re(\alpha) > max\{0, Re(k) - 1\}; Re(\alpha) = 0$ when $Re(k) = 1$ and $\beta \neq 0$.

Proof. Let $h \in \mathcal{N}^{\gamma,u,v}_{\alpha,\beta,k}(\lambda)$. Then from (1.8) and (1.9), we get

$$\frac{z\left(\mathcal{H}^{\gamma}_{\alpha,\beta,k}h(z)\right)'}{\lambda z\left(\mathcal{H}^{\gamma}_{\alpha,\beta,k}h(z)\right)' + (1-\lambda)\mathcal{H}^{\gamma}_{\alpha,\beta,k}h(z)} = u(z) \quad (\lambda \in [0, 1); z \in \Delta) \tag{2.3}$$

and

$$\frac{w\left(\mathcal{H}^{\gamma}_{\alpha,\beta,k}g(w)\right)'}{\lambda w\left(\mathcal{H}^{\gamma}_{\alpha,\beta,k}g(w)\right)'+(1-\lambda)\mathcal{H}^{\gamma}_{\alpha,\beta,k}g(w)}=v(w)\quad(\lambda\in[0,1);w\in\Delta) \tag{2.4}$$

where u and v are functions which satisfy the conditions of Definition 1.1. So, Taylor-Maclaurin series of u and v can be written as follows.

$$u(z)=1+\phi_1 z+\phi_2 z^2+\cdots$$

and

$$v(w)=1+\psi_1 w+\psi_2 w^2+\cdots.$$

Now, equating the coefficients of $\frac{z\left(\mathcal{H}^{\gamma}_{\alpha,\beta,k}h(z)\right)'}{\lambda z\left(\mathcal{H}^{\gamma}_{\alpha,\beta,k}h(z)\right)'+(1-\lambda)\mathcal{H}^{\gamma}_{\alpha,\beta,k}h(z)}$ with coefficient of $u(z)$, we get the following equations.

$$(1-\lambda)\frac{\Gamma(2k+\gamma)}{\Gamma(k+\gamma)}.\frac{\Gamma(\beta+\alpha)}{\Gamma(\beta+2\alpha)}.\frac{a_2}{2!}=\phi_1 \tag{2.5}$$

and

(2.6)

$$(\lambda^2-1)\left(\frac{\Gamma(2k+\gamma)}{\Gamma(k+\gamma)}\right)^2\left(\frac{\Gamma(\beta+\alpha)}{\Gamma(\beta+2\alpha)}\right)^2\frac{a_2^2}{4}+2(1-\lambda)\frac{\Gamma(3k+\gamma)}{\Gamma(k+\gamma)}.\frac{\Gamma(\beta+\alpha)}{\Gamma(\beta+3\alpha)}.\frac{a_3}{3!}=\phi_2.$$

Now, equating the coefficients of $\frac{w\left(\mathcal{H}^{\gamma}_{\alpha,\beta,k}g(w)\right)'}{\lambda w\left(\mathcal{H}^{\gamma}_{\alpha,\beta,k}g(w)\right)'+(1-\lambda)\mathcal{H}^{\gamma}_{\alpha,\beta,k}g(w)}$ with coefficient of $v(w)$, we get the following equations.

$$(\lambda-1)\frac{\Gamma(2k+\gamma)}{\Gamma(k+\gamma)}.\frac{\Gamma(\beta+\alpha)}{\Gamma(\beta+2\alpha)}.\frac{a_2}{2!}=\psi_1 \tag{2.7}$$

and

(2.8)

$$\left(\frac{1-\lambda}{3}\right).\frac{\Gamma(3k+\gamma)}{\Gamma(k+\gamma)}.\frac{\Gamma(\beta+\alpha)}{\Gamma(\beta+3\alpha)}(2a_2^2-a_3)+\left(\frac{\lambda^2-1}{4}\right).\left(\frac{\Gamma(2k+\gamma)}{\Gamma(k+\gamma)}\right)^2.\left(\frac{\Gamma(\beta+\alpha)}{\Gamma(\beta+2\alpha)}\right)^2a_2^2=\psi_2.$$

From (2.5) and (2.7), we get

$$\phi_1=-\psi_1 \tag{2.9}$$

and

$$2(1-\lambda)^2\left(\frac{\Gamma(\gamma+2k)}{\Gamma(\gamma+k)}\right)^2\left(\frac{\Gamma(\alpha+\beta)}{\Gamma(2\alpha+\beta)}\right)^2\frac{a_2^2}{4}=\phi_1^2+\psi_1^2. \tag{2.10}$$

Now from (2.6) and (2.8), we get

(2.11)

$$2(\lambda^2-1)^2\left(\frac{\Gamma(2k+\gamma)}{\Gamma(k+\gamma)}\right)^2\left(\frac{\Gamma(\beta+\alpha)}{\Gamma(\beta+2\alpha)}\right)^2\frac{a_2^2}{4}+\frac{2(1-\lambda)}{3}.\frac{\Gamma(3k+\gamma)}{\Gamma(k+\gamma)}.\frac{\Gamma(\beta+\alpha)}{\Gamma(\beta+3\alpha)}a_2^2=\phi_2+\psi_2.$$

From (2.10) and (2.11), we can easily obtain

$$a_2^2=\frac{2(\phi_1^2+\psi_1^2)}{(1-\lambda)^2}.\left(\frac{\Gamma(k+\gamma)}{\Gamma(2k+\gamma)}\right)^2\left(\frac{\Gamma(\beta+2\alpha)}{\Gamma(\beta+\alpha)}\right)^2 \tag{2.12}$$

and

$$a_2^2=\frac{(\phi_2+\psi_2)}{|\tau|} \tag{2.13}$$

where

$$\tau = \left(\frac{\lambda^2-1}{2}\right)\cdot\left(\frac{\Gamma(2k+\gamma)}{\Gamma(k+\gamma)}\right)^2\cdot\left(\frac{\Gamma(\beta+\alpha)}{\Gamma(\beta+2\alpha)}\right)^2+\frac{2(1-\lambda)}{3}\cdot\frac{\Gamma(3k+\gamma)}{\Gamma(k+\gamma)}\cdot\frac{\Gamma(\alpha+\beta)}{\Gamma(3\alpha+\beta)}.$$

By using the fact $u(z)\in u(\Delta)$ and $v(w)\in v(\Delta)$, we get

$$|a_2|^2 \le \frac{2\left(|u'(0)|+|v'(0)|\right)}{(1-\lambda)^2}\cdot\left|\frac{\Gamma(k+\gamma)}{\Gamma(2k+\gamma)}\right|^2\left|\frac{\Gamma(\beta+2\alpha)}{\Gamma(\beta+\alpha)}\right|^2 \tag{2.14}$$

and

$$|a_2|^2 \le \frac{|u''(0)|+|v''(0)|}{2|\tau|}. \tag{2.15}$$

Using these inequalities, we get the desire estimate on $|a_2|$ stated in (2.1).
Now to obtain the bound on $|a_3|$, we subtract (2.8) from (2.6) and we get,

$$\frac{2(1-\lambda)}{3}\cdot\frac{\Gamma(3k+\gamma)}{\Gamma(k+\gamma)}\cdot\frac{\Gamma(\beta+\alpha)}{\Gamma(\beta+3\alpha)}\cdot(a_3-a_2^2)=\phi_2-\psi_2. \tag{2.16}$$

Now, by substituting the values of a_2^2 from (2.12) and (2.13) in (2.16), we have

$$a_3=\frac{2(\phi_1^2+\psi_1^2)}{(1-\lambda)^2}\cdot\left(\frac{\Gamma(k+\gamma)}{\Gamma(2k+\gamma)}\right)^2\left(\frac{\Gamma(\beta+2\alpha)}{\Gamma(\beta+\alpha)}\right)^2+\frac{3(\phi_2-\psi_2)}{2(1-\lambda)}\cdot\frac{\Gamma(k+\gamma)}{\Gamma(3k+\gamma)}\cdot\frac{\Gamma(\beta+3\alpha)}{\Gamma(\beta+\alpha)} \tag{2.17}$$

and

$$a_3=\frac{(\phi_2+\psi_2)}{|\tau|}+\frac{3(\phi_2-\psi_2)}{2(1-\lambda)}\cdot\frac{\Gamma(k+\gamma)}{\Gamma(3k+\gamma)}\cdot\frac{\Gamma(\beta+3\alpha)}{\Gamma(\beta+\alpha)} \tag{2.18}$$

respectively.
By using the fact $u(z)\in u(\Delta)$ and $v(w)\in v(\Delta)$, we get

$$|a_3|\le\frac{2(|u'(0)|^2+|v'(0)|^2)}{(1-\lambda)^2}\cdot\left|\frac{\Gamma(k+\gamma)}{\Gamma(2k+\gamma)}\right|^2\left|\frac{\Gamma(\beta+2\alpha)}{\Gamma(\beta+\alpha)}\right|^2+\frac{3(|u''(0)|+|v''(0)|)}{4(1-\lambda)}\cdot\left|\frac{\Gamma(k+\gamma)}{\Gamma(3k+\gamma)}\right|\cdot\left|\frac{\Gamma(\beta+3\alpha)}{\Gamma(\beta+\alpha)}\right| \tag{2.19}$$

and

$$|a_3|\le\frac{|u''(0)|+|v''(0)|}{2|\tau|}+\frac{3(|u''(0)|+|v''(0)|)}{4(1-\lambda)}\cdot\left|\frac{\Gamma(k+\gamma)}{\Gamma(3k+\gamma)}\right|\cdot\left|\frac{\Gamma(\beta+3\alpha)}{\Gamma(\beta+\alpha)}\right|. \tag{2.20}$$

Hence we get (2.2) and it completes the proof. □

3. COROLLARIES AND CONSEQUENCES

For particular values of γ,α,k and for different choices of functions u and v, in Theorem 2.1, we get many well known results as a corollaries of our result as follows.

By setting $\alpha=0,\gamma=1$ and $k=1$ in Theorem 2.1, we get the result obtained by Yamini J [13].

Corollary 3.1. *If $h\in\sum$ is a function given by (1.1), is in the class $\mathcal{N}_{\Sigma}^{u,v}(\lambda)$ ($\beta\in[0,1);\lambda\in[0,1)$) then*

$$|a_2|\le min\left\{\sqrt{\frac{|u'(0)|^2+|v'(0)|^2}{2(1-\lambda)^2}},\frac{\sqrt{|u''(0)|+|v''(0)|}}{2(1-\lambda)}\right\}$$

and

$$|a_3| \leq min\left\{\frac{|u''(0)|+|v''(0)|}{8(1-\lambda)}+\frac{|u'(0)|^2+|v'(0)|^2}{2(1-\lambda)^2}, \frac{(3-\lambda)|u''(0)|+(1+\lambda)|v''(0)|}{8(1-\lambda)^2}\right\}.$$

By setting $\alpha = 0, \gamma = 1, k = 1$ and by considering functions $u(z) = \left(\frac{1+z}{1-z}\right)^{\eta}$ and $v(z) = \left(\frac{1-z}{1+z}\right)^{\eta}$ $(\eta \in (0,1], z \in \Delta)$ in Theorem 2.1, we get the following corollary.

Corollary 3.2. *If $h \in SS^*_{\Sigma}(\eta, \lambda)$ is the function given by (1.1), then*

$$|a_2| \leq min\left\{\frac{2\eta}{1-\lambda}, \frac{\sqrt{2}\eta}{1-\lambda}\right\}$$

and

$$|a_3| \leq min\left\{\frac{2\eta^2}{(1-\lambda)^2}, \frac{\eta^2}{1-\lambda}+\frac{4\eta^2}{(1-\lambda)^2}\right\}.$$

If we set $\alpha = 0, \gamma = 1, k = 1$ and choose the functions $u(z) = \frac{1+(1-2\eta)z}{1-z}$ and $v(z) = \frac{1-(1-2\eta)z}{1+z}$ $(\eta \in [0,1), z \in \Delta)$ in Theorem 2.1, we get the following corollary.

Corollary 3.3. *If $h \in S^*_{\Sigma}(\eta, \lambda)$ is the function given by (1.1), then*

$$|a_2| \leq min\left\{\frac{2(1-\eta)}{1-\lambda}, \frac{\sqrt{2(1-\eta)}}{1-\lambda}\right\}$$

and

$$|a_3| \leq min\left\{\frac{2(1-\eta)}{(1-\lambda)^2}, \frac{1-\eta}{1-\lambda}+\frac{4(1-\eta)^2}{(1-\lambda)^2}\right\}.$$

4. Conflict of Interests

The authors confirm that there are no conflicts of interest related to the publication of this paper.

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