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ON μ-PARACOMPACTNESS AND μ-LOCALNESS ON GT SPACES

Abstract. Firstly, we obtain some results on generalized paracompact spaces (X, μ) as applications of generalized preopen sets in X. Thereafter, we also obtain some characteristics of local finiteness of collections of subsets of a GT space (X, μ) with respect to a topology on X.

Keywords and phrases: μ-preopen set, μ-locally finiteness, μ-open refinement, μ-paracompact space, $\mathscr{T}_\mu$-local finiteness.

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1. Introduction

There are several generalizations of open sets of topological spaces $(X, \mathscr{T})$ e.g., semi-open sets [9, Levine], pre-open sets [10, Mashhour et al.] which are also called locally dense sets by Corson and Michael [2]. Császár [8] introduced and studied γ-open sets as a common generalization of such generalized sets of open sets in X. The γ-open sets finally leads to introduce and study the generalized topology on X.

Let $\exp(X)$ be the power set of the nonempty set X. A subcollection μ of $\exp(X)$ is called a generalized topology [7] on X if $\emptyset \in \mu$ and the union of arbitrary number of elements of μ is again a member of μ. A generalized topological space [7] is a nonempty set X endowed with a generalized topology μ and it is denoted by (X, μ). We write GT (resp. GT space) to denote the generalized topology μ on X (resp. generalized topological space (X, μ)). An element of μ is called a μ-open set of (X, μ). The complement of a μ-open set is called a μ-closed set of (X, μ). A GT μ on X is called quasi-topology [4] if μ is closed under finite intersection. A generalized topological space (X, μ) is called strong [6] (also called μ-space by Noiri [14]) if $X \in \mu$. For brevity, we retain the term μ-space due to Noiri [14] to mean the strongly generalized topological space (X, μ) as well.

For a subset A of a GT space X, the intersection of all μ-closed sets containing A is the generalized closure [5] of A and is denoted by $c_\mu(A)$. Also for a subset A of a GT space X, the union of all μ-open sets contained in A is the generalized interior [5] of A and is denoted by $i_\mu(A)$. It is easy to see that a subset A of X is μ-open (resp. μ-closed) if and only if $A = i_\mu(A)$ (resp. $A = c_\mu(A)$). Also for a subset A of X, we have $c_\mu(A) = X - i_\mu(X - A)$.

Throughout the paper, N denotes the set of natural numbers and the elements of N are denoted by n, m, l, k etc. We also write 'nbd' to mean 'neighbourhood' of a point $x \in X$.

2. μ-paracompact Spaces

Firstly, we recall below some concepts and results for follow easily the rest of our present work easily.

Definition 2.1 (Császár [5]). A subset A of X is called μ-preopen if $A \subset i_\mu(c_\mu(A))$.

It is easy to see that $A \subset X$ is μ-preopen if and only if there exists a μ-open set G such that $A \subset G \subset c_\mu(A)$.

Definition 2.2 (Sarsak [16]). A subset A of X is called μ-regularly closed if $A = c_\mu(i_\mu(A))$. The complement of a μ-regularly closed set is called a μ-regularly open set. So a subset A of a GT space is μ-regularly open if $A = i_\mu(c_\mu(A))$.

We see that $i_\mu(c_\mu(G))$ is μ-regularly open in X if G is μ-open in X.

We agree to write 'μ-open collection' and 'μ-preopen collection' to denote a collection consisting of nonempty μ-open sets and μ-preopen sets respectively of a GT space. According to existing convention, a 'μ-open cover' of X is a μ-open collection $\mathscr{C}$ of X such that $\bigcup_{U \in \mathscr{C}} U = X$. We see that if X is not μ-open in a GT space (X, μ), then the union of even all μ-open sets of X is not equal to X i.e., in such case, there exist no μ-open cover of X. So such a definition of μ-open covers of GT spaces become void. It instigates us to change the existing ideas of covering of GT spaces as follows.

We write $X_\mu = \bigcup_{G \in \mu} G$. It is then clear that X_μ is μ-open in X and $X_\mu = X$ if X is a μ-space. A collection $\mathscr{C}$ of nonempty subsets of X is called a cover of X [13] if $\bigcup_{G \in \mathscr{C}} G = X_\mu$. A μ-open collection (resp. μ-preopen collection) $\mathscr{C}$ of X is said to be a μ-open (resp. μ-preopen) cover of X if $\bigcup_{G \in \mathscr{C}} G = X_\mu$.

Let $\mathscr{U}$ and $\mathscr{V}$ be two covers of X. The cover $\mathscr{V}$ is called a refinement [**?**, p. 144] of the cover $\mathscr{U}$ if for each $V \in \mathscr{V}$, there exists a $U \in \mathscr{U}$ such that $V \subset U$. Let $\mathscr{U}$ be a μ-open cover of X. If $\mathscr{V}$ is a cover of X by μ-open sets of X, then $\mathscr{V}$ is called a μ-open refinement of $\mathscr{U}$ [1]. In the same fashion, if $\mathscr{V}$ is a cover of X by μ-preopen sets of X, then $\mathscr{V}$ is called a μ-preopen refinement [13] of $\mathscr{U}$.

Again let (X, μ) be a GT space such that $X \notin \mu$ and $\mathscr{C}$ be a collection of subsets of X. Then for any $x \in X - X_\mu$, there exist no μ-open set U such that $x \in U$ and so there arise no question of existing a μ-open set U in X such that U intersects finitely many members of $\mathscr{C}$. Hence in the present study, we consider slightly modified versions of the notions like μ-locally finiteness (Definition 2.3), μ-paracompactness (Definition 2.5) in contrary

to the concern notions introduced by Deb Ray and Bhowmick [3], and Arar [1].

Definition 2.3 (Mukharjee and Roy [13]). A collection $\mathscr{U}$ of subsets of X is called μ-locally finite if for each $x \in X_\mu$, there exists a μ-open set U with $x \in U$ meeting only finitely many members of $\mathscr{U}$.

Example 2.4. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{E} \subset \mathbb{N}$ be the set of all even natural numbers. We define $\mu = \{\emptyset\} \cup \{G \mid G \text{ is an infinite subset of } \mathbb{E}\}$. Obviously, μ is a GT on $\mathbb{N}$ such that $\mathbb{N} \notin \mu$. In the GT space $(\mathbb{N}, \mu)$, we consider the collection $\mathscr{C} = \{\{2,4,6\}, \{4,6,8\}, \{6,8,10\}, \ldots\}$. We see that for any $x \in \mathbb{E}$, a μ-open set U with $x \in U$ may intersects finitely many members of $\mathscr{C}$. So $\mathscr{C}$ is a μ-locally finite collection in the GT space $(\mathbb{N}, \mu)$.

We observe that for any $A \in \mathscr{C}$, $\mathbb{N} - \mathbb{E} \subset c_\mu(A)$ and $c_\mu(A) = \{2,4,6\} \cup \{\mathbb{N} - \mathbb{E}\}$ when $A = \{2,4,6\}$.

Definition 2.5 (Mukharjee and Roy [13]). A GT space X is called μ-paracompact if each μ-open cover of X has a μ-locally finite μ-open refinement.

Definition 2.6 (Mukharjee and Roy [13]). A μ-preopen set G in a GT space X is called capped by a μ-open set if there exists a μ-open set W such that $G \subset W \subset U \cap V$ whenever U, V are μ-open sets in X containing G.

Definition 2.7 (Min [11]). A GT space (X, μ) is called μ-Hausdorff if for any pair of distinct $x, y \in X_\mu$ there exist μ-open sets U, V in X such that $x \in U, y \in V$ and $U \cap V = \emptyset$.

Definition 2.8 (Min [12]). A GT space (X, μ) is called μ-regular if for any $x \in X_\mu$ and any μ-closed set F in X not containing x there exist μ-open sets U, V in X such that $x \in U, F \cap X_\mu \subset V$ and $U \cap V = \emptyset$.

It is shown that the GT space X is μ-regular if and only if for any $x \in X_\mu$ and any μ-open set U with $x \in U$ there exists a μ-open set V such that $x \in V \subset c_\mu(V) \cap X_\mu \subset U$.

Lemma 2.9. *If F is a μ-closed set in a GT space X, then*

(i) $F = (F \cap X_\mu) \cup (X - X_\mu)$,
(ii) $X - F = X_\mu - F$ *and* $X_\mu - F$ *is μ-open in X.*

Proof. Both the results follows easily from the facts that $X = X_\mu \cup (X - X_\mu)$ and $X - X_\mu \subset F$. □

Theorem 2.10. *A GT space X is μ-regular if and only if for any $x \in X_\mu$ and any μ-closed set F in X with $x \notin F \cap X_\mu$ there exist μ-open sets U, V in X such that $x \in U, F \cap X_\mu \subset V$ and $U \cap V = \emptyset$.*

Proof. Necessity: Suppose that $x \in X_\mu$ and F is a μ-closed set in X such that $x \notin F \cap X_\mu$. As $x \in X_\mu$, $x \notin X - X_\mu$. Hence we find that $x \notin F$ by

Lemma 2.9(i). By the μ-regularity of X, we obtain μ-open sets U, V in X such that $x \in U, F \cap X_\mu \subset V$ and $U \cap V = \emptyset$.

Sufficiency: Let $x \in X_\mu$ and F be a μ-closed set in X not containing x. Then obviously, we have $x \notin F \cap X_\mu$. Hence we obtain μ-open sets U, V in X such that $x \in U, F \cap X_\mu \subset V$ and $U \cap V = \emptyset$. □

Theorem 2.11 (Mukharjee and Roy [13])**.** *Let μ-preopen sets in a GT space X are capped by μ-open sets of X. The GT space X is μ-paracompact if and only if each μ-open covers of X has a μ-locally finite μ-preopen refinement.*

Theorem 2.12. *Let μ-preopen sets in a generalized topological space X are capped by μ-open sets in X. The generalized topological space X is μ-paracompact if each μ-preopen cover of X has a μ-locally finite μ-preopen refinement.*

Proof. The results follows by Theorem 2.11 from the fact that a μ-open cover of X is also a μ-preopen cover of X. □

Theorem 2.13. *Let X be a μ-Hausdorff μ-paracompact space and μ-preopen sets in X are capped by μ-open sets in X. Then for $x \in X$ and a μ-closed set E with $x \notin E$, there exist a subset G of X and a μ-preopen set H in X such that $x \in G, E \cap X_\mu \subset H$ and $pc_\mu(G) \cap H = \emptyset$.*

Proof. Let E be a μ-closed set in X and $x \in X_\mu$ such that $x \notin E$. Since $E = (E \cap X_\mu) \cup (X - X_\mu)$ by Lemma 2.9, $x \in X_\mu$ whenever $x \in X$ and $x \notin E$. For each $y \in E \cap X_\mu$, there exist μ-open sets U_y, V_y in X such that $x \in U_y, y \in V_y$ and $U_y \cap V_y = \emptyset$ as $x \neq y$ and X is μ-Hausdorff. We write $\mathscr{V} = \{V_y \mid y \in X_\mu \cap E\}$. Then $\mathscr{U} = \mathscr{V} \cup \{X - E\}$ is a μ-open cover of X. By Theorem 2.11 on μ-paracompactness of X, we obtain a μ-locally finite μ-preopen refinement $\mathscr{G}$ of $\mathscr{U}$. Putting $H = \bigcup\{U \mid U \in \mathscr{G}, U \cap (E \cap X_\mu) \neq \emptyset\}$, we see that $E \cap X_\mu \subset H$. Since $\mathscr{G}$ is μ-locally finite, there exists a μ-open set W in X with $x \in W$ intersecting finitely many members of $\mathscr{G}$. Let the finite collection of members of $\mathscr{G}$ intersecting W be $\mathscr{G}_m$. As $\mathscr{G}$ is a μ-preopen refinement of $\mathscr{U}$, there may exists finitely many $W_k \in \mathscr{G}_m, k \in \{1, 2, \ldots, n\}$ such that $W_k \subset V_{y_k}, V_{y_k} \in \mathscr{V}$ and $V_{y_k} \cap (E \cap X_\mu) \neq \emptyset$ for each $k \in \{1, 2, \ldots, n\}$. We put $G = W \cap (\bigcap_{k=1}^{n} U_{y_k})$. To show $G \cap H = \emptyset$, we suppose that $z \in G \cap H$. Then $z \in W$ and $z \in U_{y_k}$ for all $k \in \{1, 2, \ldots, n\}$. Also $z \in U$ for some $U \in \mathscr{G}$ with $U \cap (E \cap X_\mu) \neq \emptyset$. So this U is equal to W_k for some $k \in \{1, 2, \ldots, n\}$. Hence we get $U_{y_k} \cap V_{y_k} \neq \emptyset$ which gives a contradiction to $U_{y_k} \cap V_{y_k} = \emptyset$. As H is μ-preopen and $G \cap H = \emptyset$, we have $pc_\mu(G) \cap H = \emptyset$. □

Remark 2.14. Let E be a μ-closed set in X. We see that for $x \in X_\mu$, $x \notin E \cap X_\mu$ whenever $x \notin E$. Hence Theorem 2.13 also hold if we choose $x \notin E$ instead of $x \notin E \cap X_\mu$ in the statement of the theorem.

Theorem 2.15. *Let X be a μ-Hausdorff μ-paracompact space. If $x \in X$ and E is μ-closed in X with $x \notin E$, then there exist a subset G of X and a μ-open set H in X such that $x \in G, X_\mu \cap E \subset H$ and $c_\mu(G) \cap H = \emptyset$.*

Proof. Similar to that of Theorem 2.13. □

Deb Roy and Bhowmick [3], defined γ_μ-closure operator on a GT space (X, μ) and showed that the γ_μ-closure operator gave rise to a topology μ^* on X. It is also seen here that $\mu = \mu^*$ if μ is topology on X. We note that if μ is a GT on X, there exist a topology $\mathscr{T}$, namely $\exp(X)$ such that $\mu \subset \mathscr{T}$.

Noiri and Roy [15] initiated the study of unification of generalized open sets on topological spaces and two placed below.

Definition 2.16 (cf. Noiri and Roy [15])**.** Let μ be a GT on a topological space $(X, \mathscr{T})$. The topological space $(X, \mathscr{T})$ is called $\mathscr{T}_\mu$-regular if for each $x \in X_\mu$ and each closed set F on X with $x \notin F$ there exist disjoint μ-open sets U, V such that $x \in U, F \subset V$.

Henceforth we write, X is $\mathscr{T}_\mu$-regular to mean $\mathscr{T}_\mu$-regularity of the topological space $(X, \mathscr{T})$ with respect to a GT μ on X as defined in Definition 2.16.

Theorem 2.17 (cf. Noiri and Roy [15])**.** *Let μ be a GT on a topological space $(X, \mathscr{T})$. The topological space $(X, \mathscr{T})$ is $\mathscr{T}_\mu$-regular if and only if for each $x \in X$ and each $\mathscr{T}$-open set U with $x \in U$, there exists a μ-open set V such that $x \in V \subset c_\mu(V) \cap X_\mu \subset U$.*

The observations recorded above due to Deb Roy and Bhowmick [3] and Noiri and Roy [15] instigated us to consider a topology $\mathscr{T}$ on a GT space (X, μ) to get some characterizations on the μ-local finiteness.

Definition 2.18. A collection $\mathscr{U}$ of subsets of X is called $\mathscr{T}_\mu$-locally finite if for each $x \in X_\mu$, there exists a $\mathscr{T}$-open set U with $x \in U$ meeting only finitely many members of $\mathscr{U}$.

A $\mathscr{T}$-open set U in $(X, \mathscr{T})$ with $x \in U$ always exists as there have a μ-open set G in (X, μ) such that $x \in G$ and $\mu \subset \mathscr{T}$.

Theorem 2.19. *Let X be $\mathscr{T}_\mu$-regular. If $\{U_\alpha \mid \alpha \in A\}$ is a $\mathscr{T}_\mu$-locally finite cover of X then $\{c_\mu(U_\alpha) \cap X_\mu \mid \alpha \in A\}$ is a μ-locally finite cover of X.*

Proof. Since $\{U_\alpha \mid \alpha \in A\}$ is $\mathscr{T}_\mu$-locally finite, for each $x \in X_\mu$ there exists a $\mathscr{T}$-open set U in X containing x such that $U_\alpha \cap G \neq \emptyset$ for finitely many $\alpha \in A$. Let $U_{\alpha_k} \cap G \neq \emptyset$ for $k \in \{1, 2, \ldots, m\}, m \in \mathbb{N}$. It means that $U_\alpha \cap G = \emptyset$ whenever $\alpha \in A - \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$. By Theorem 2.17 there exists a μ-open set H such that $x \in H \subset c_\mu(H) \cap X_\mu \subset G$. As $H \subset G$, $U_\alpha \cap H = \emptyset$ whenever $\alpha \in A - \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$. Thus we have $c_\mu(U_\alpha) \cap H = \emptyset$ whenever $\alpha \in A - \{\alpha_1, \alpha_2, \ldots, \alpha_m\}$. So $\{c_\mu(U_\alpha) \mid \alpha \in A\}$ is μ-locally finite. Since $c_\mu(U_\alpha) \cap X_\mu \subset c_\mu(U_\alpha)$ for each $\alpha \in A$, $\{c_\mu(U_\alpha) \cap X_\mu \mid \alpha \in A\}$ is also

μ-locally finite. Again for any $x \in X_\mu$, there exists an $\alpha \in A$ such that $x \in U_\alpha \subset c_\mu(U_\alpha)$. Hence $\{c_\mu(U_\alpha) \cap X_\mu \mid \alpha \in A\}$ is a cover of X. □

Theorem 2.20. *Let X be a GT space. If $\mathscr{U} = \{U_\alpha \mid \alpha \in A\}$ is a $\mathscr{T}_\mu$-locally finite μ-open cover of X then $\bigcup_{\alpha\in A} c_\mu(U_\alpha) = c_\mu\left(\bigcup_{\alpha\in A} U_\alpha\right)$.*

Proof. We only need to show $\bigcup_{\alpha\in A} c_\mu(U_\alpha) \supset c_\mu\left(\bigcup_{\alpha\in A} U_\alpha\right)$. Suppose that $x \in c_\mu\left(\bigcup_{\alpha\in A} U_\alpha\right)$. If $x \in X - X_\mu$, then there exist no μ-open set in X containing x. Hence $x \in c_\mu(U_\alpha)$ for each $\alpha \in A$. So in this case, $\bigcup_{\alpha\in A} c_\mu(U_\alpha) \supset c_\mu\left(\bigcup_{\alpha\in A} U_\alpha\right)$ hold. Now we suppose that $x \in X_\mu$. By $\mathscr{T}_\mu$-local finiteness of $\mathscr{U}$, there exists a $\mathscr{T}$-open nbd U of x such that U intersects finitely many members $U_{\alpha_1}, U_{\alpha_2}, \ldots, U_{\alpha_n}$ of $\mathscr{U}$. As $x \in c_\mu\left(\bigcup_{\alpha\in A} U_\alpha\right)$, each μ-open nbd V of x intersects $\bigcup_{\alpha\in A} U_\alpha$. Since $\mu \subset \mathscr{T}$, V is also a $\mathscr{T}$-open set which in turns implies that $U \cap V$ is a $\mathscr{T}$-open nbd of x. Since $\mathscr{U}$ is a cover of X, $(U \cap V) \cap (\bigcup_{i=1}^{n} U_{\alpha_i}) \neq \emptyset$ which gives $V \cap (\bigcup_{i=1}^{n} U_{\alpha_i}) \neq \emptyset$ since $U \cap V \subset V$. It means that $x \in c_\mu\left(\bigcup_{i=1}^{n} U_{\alpha_i}\right) = c_\mu(U_{\alpha_1}) \cup c_\mu(U_{\alpha_2}) \cup \ldots \cup c_\mu(U_{\alpha_n}) \subset \bigcup_{\alpha\in A} c_\mu(U_\alpha)$. □

Corollary 2.21. *Let X be a GT space. If $\mathscr{U} = \{U_\alpha \mid \alpha \in A\}$ is a $\mathscr{T}_\mu$-locally finite μ-open cover of X then $\bigcup_{\alpha\in A}(c_\mu(U_\alpha) \cap X_\mu) = c_\mu\left(\bigcup_{\alpha\in A} U_\alpha\right) \cap X_\mu$.*

Proof. Since $\bigcup_{\alpha\in A}(c_\mu(U_\alpha) \cap X_\mu) = \left(\bigcup_{\alpha\in A} c_\mu(U_\alpha)\right) \cap X_\mu$, the result follows by Theorem 2.20. □

Lemma 2.22. *Let μ be a GT on the topological space $(X, \mathscr{T})$ and $\mu \subset T$. Then $\mathscr{T}_\mu$-regularity of X implies the μ-regularity of the GT space (X, μ).*

Proof. Easy to proof and hence omitted. □

Theorem 2.23. *Let μ be a GT on the topological space $(X, \mathscr{T})$ such that $\mu \subset \mathscr{T}$ and let X be a $\mathscr{T}_\mu$-regular space. Then each μ-open cover $\mathscr{G}$ of X has a μ-locally finite refinement $\mathscr{F} = \{F_\gamma \cap X_\mu \mid \gamma \in \Gamma\}$ of $\mathscr{U}$ where $\{F_\gamma \mid \gamma \in \Gamma\}$ is a collection of μ-closed sets in X if each μ-open cover of X has a $\mathscr{T}_\mu$-locally finite refinement.*

Proof. Let $\mathscr{G} = \{G_\gamma \mid \gamma \in \Gamma\}$ be a μ-open cover of X. For each $x \in X_\mu$, there exists a $\gamma(x) \in \Gamma$ such that $x \in G_{\gamma(x)}$. As $\mathscr{T}_\mu$-regular space is μ-regular also by Lemma 2.22, there exists a μ-open set V_x such that $x \in V_x \subset c_\mu(V_x) \cap X_\mu \subset G_{\gamma(x)}$. The μ-open cover $\mathscr{V} = \{V_x \mid x \in X_\mu\}$ has a $\mathscr{T}_\mu$-locally finite refinement $\mathscr{U} = \{U_x \mid x \in X_\mu\}$. By Lemma 2.19, $\{c_\mu(U_x) \cap X_\mu \mid x \in X_\mu\}$ is a μ-locally finite cover of X. As $U_x \subset V_x$ for each $x \in X_\mu$, $c_\mu(U_x) \cap X_\mu \subset c_\mu(V_x) \cap X_\mu \subset G_{\gamma(x)}, \gamma(x) \in \Gamma$ for each $x \in X_\mu$. Hence $\{c_\mu(U_x) \cap X_\mu \mid x \in X_\mu\}$ is also a refinement of $\mathscr{G}$. □

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