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EXACT SOLUTION FOR FLOW THROUGH POROUS MEDIUM OF A ROTATING VARIABLY INCLINED MHD FLUID BY MAGNETOGRAPH TRANSFORMATION

Abstract. An analytical study of the motion of a steady, homogenous, incompressible, plane rotating MHD fluid flow through a porous medium for exact solutions is carried out. The velocity vector of the fluid particle is thought to be variably inclined to the magnetic field vector at every point. The flow of fluid is governed by non-linear partial differential equations. These governing equations are converted into a system of linear partial differential equations by means of transformation technique known as magnetograph transformation. The two components of the magnetic field in the physical plane and two independent variables are switched around using the magnetograph transformation. Further, the flow equations have been derived using the Legendre transform of the magnetic flux function. Finally, several examples have been used to apply and illustrate the developed theory and exact solutions have been determined. The expressions for the components of velocity vector, components of magnetic field vector, magnetic lines and pressure distribution are obtained and analyzed graphically.

Keywords: MHD, Exact Solution, Magnetograph Transformation, Magnetic Flux Function, Legendre Transform Function, Porous Medium.

Mathematics Subject Classification (2020) No.: 35F05, 35Q30, 35Q35.

1. Introduction

The governing equations for the flow of non-Newtonian fluids give rise to

systems of non-linear partial differential equations; these equations have no general solution. The several approaches used to solve these equations and their applications have received excellent coverage from Ames [1]. Hodograph transformations, as employed by Martin [2] in fluid mechanics, are a class of transformations that change variables from the physical plane to the velocity plane.

The magnetograph transformation- a method for accurately solving non-linear partial differential equations- which govern the steady flow of a homogeneous, incompressible, viscous fluid with finite electrical conductivity in a porous medium in a rotating reference frame-is the subject of the current study. It is common practice to solve non-linear partial differential equations using transformation techniques. The magnetograph is a curve formed by the extremities of the magnetic field vectors when they are extended from a given point. An equivalent linear system is produced by using the magnetograph transformation to switch the roles of the independent and dependent variables. In other words, the transformations that are used to switch the roles of the two independent variables in the physical plane and the two components of the magnetic field are known as magnetograph transformations.

The governing non-linear equations are transformed into a linear form that may be solved by using the magnetograph transformation. Using magnetograph transformation, several researchers have studied MHD fluid flow and discovered precise answers. In order to investigate orthogonal MHD flow, S. N. Singh [3] invented and used magnetograph transformation. Researchers Venkateshappa, Siddabasappa, and Rudraswamy [26] as well as C. S. Bagewadi and Siddabasappa [4], looked on rotating MHD ow that was variably inclined in the magnetograph plane. Exact solutions were found by M. Kumar and S. Sil [5] after studying aligned MHD flow in the magnetograph plane.

The study of fluid flow in a rotating frame is important for many technical applications that are directly affected by the coriolis force created by the earth's rotation. Examples of these applications include spin coating, the creation and use of computer disks, rotational viscometers, centrifugal machinery, the pumping of liquid metals at high melting points, the growth of crystals from molten silicon, turbo-machinery etc. The coriolis force is shown to have a significant impact when compared to the viscous and inertial forces in the equations of motion.

The coriolis force has a major impact on the hydromagnetic flow in the liquid core of the earth, which is essential to the mean geomagnetic field [6]. Because of its role in solar physics and its relationship to the formation of sunspots and the solar cycle, the theory of rotating fluid is also significant. Several studies with rotating fluid have been carried out [9, 11, 10, 12, 7, 8, 13, 26]. Many works have been conducted on various types of flows for both non-MHD and MHD.

In the study of soil percolation in hydrology, the petroleum industry, agricultural engineering, and many other significant fields, the flow of a viscous fluid through a porous material is crucial. Numerous authors [17, 19, 14, 20, 21, 23, 22, 16, 15, 24, 25, 18, 28, 29] have investigated fluid flows across porous media and discovered an exact solution.

The objective of this research is to analyze the motion of a rotating, steady, homogenous, incompressible, variably inclined MHD plane flow through a porous medium in order to obtain exact solutions. The fluid flow equation is described by nonlinear partial differential equations. The magnetograph transformation helps the nonlinear partial differential equations turn into a system of linear partial differential equations. Two independent variables and the two components of the magnetic field in the physical plane have been swapped out using the magnetograph transformation. Moreover, the magnetic flux function's Legendre transform function has been utilized to illustrate the flow equations. Finally, a few examples have been used to clarify the proposed theory and exact solutions have been found.

The expressions for the pressure distribution, magnetic lines, velocity vector components and magnetic field vector components are obtained and graphically examined. We first consider the appropriate steady flow equations in a rotating frame of reference, which includes coriolis force and centrifugal force with non-uniform angular velocity. Using a Legendre transform of the magnetic flux function and rewriting all of the equations in terms of this transformed function, the exact solutions are found by switching the dependent and independent variables in the magnetograph plane. Examples are considered to point out the usefulness of the method. The geometry of streamlines and magnetic lines are discussed. The general solution for angular velocity is also found with the variation of pressure and angular velocity is discussed by plotting various graphs for some different form of suitable examples.

2. Basic Equations

The fundamental equations that regulate the steady flow of a homogeneous incompressible viscous fluid with finite electrical conductivity in a porous medium in the presence of a magnetic field in a rotating reference frame are

$$\nabla \cdot \mathbf{V} = 0 \ , (Continuity\ equation) \tag{1}$$

$$\rho((\mathbf{V}\cdot\nabla) + 2\boldsymbol{\Omega} \times \mathbf{V} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})) = -\nabla P + \eta\nabla^2\mathbf{V} + \mu\mathbf{Q} \times \mathbf{H} - \frac{\eta}{\kappa}\mathbf{V} \ , (Momentum\ Equation) \tag{2}$$

$$\nabla \times (\mathbf{V} \times \mathbf{H}) = \nabla \times ({}_{\gamma \mathbf{H}} \nabla \times \mathbf{H}), \quad (\textit{Diffusion equation}) \tag{3}$$

$$\nabla \cdot \mathbf{H} = 0, \quad (\textit{Solenoidal equation}) \tag{4}$$

where $\mathbf{V}$ = velocity field vector, P = fluid pressure, $\mathbf{H}$ = magnetic field vector, $\mathbf{Q}$ = current density, μ = magnetic permeability, σ = electrical conductivity of the fluid, ρ = the constant fluid field density, $\mathbf{\Omega}$ = angular velocity, η = coefficient of viscosity, κ = permeability of the medium, $\mathbf{r}$ = radius vector and $\gamma\mathbf{H}$ = magnetic viscosity, $\mathbf{\Omega} \times (\mathbf{\Omega} \times \tilde{\mathbf{r}})$ = centripital acceleration, $2\mathbf{\Omega} \times \mathbf{V}$ = coriolis acceleration.

On introducing the function

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \quad (\textit{vorticity function}) \tag{5}$$

$$Q = \frac{\partial \tilde{H}_2}{\partial x} - \frac{\partial \tilde{H}_1}{\partial y}, \quad (\textit{Current density function}) \tag{6}$$

$$B = \frac{1}{2}\rho V^2 + P' + \frac{1}{2}|\mathbf{\Omega} \times \mathbf{r}|^2, \quad (\textit{Bernoulli function}) \tag{7}$$

where $V^2 = \tilde{u}^2 + \tilde{v}^2$, P' is the reduced pressure and $P' = P - \frac{1}{2}|\mathbf{\Omega} \times \mathbf{r}|^2$ and the last term being the centrifugal contribution of the pressure. The above system reduces to

$$\frac{\partial \tilde{v}}{\partial x} + \frac{\partial \tilde{u}}{\partial y} = 0, \tag{8}$$

$$\frac{\partial B}{\partial x} + \eta \frac{\partial \omega}{\partial y} - 2\rho\tilde{v}\Omega - \rho\omega\tilde{v} + \eta\tilde{H}_2 Q + \frac{\eta}{\kappa}\tilde{u} = 0, \tag{9}$$

$$\frac{\partial B}{\partial y} - \eta \frac{\partial \omega}{\partial y} - 2\rho\tilde{v}\Omega - \rho\omega\tilde{v} + \eta\tilde{H}_1 Q + \frac{\eta}{\kappa}\tilde{v} = 0, \tag{10}$$

$$\tilde{u}\tilde{H}_2 - \tilde{v}\tilde{H}_1 = \gamma_H \frac{\partial \tilde{H}_2}{\partial x} - \frac{\partial \tilde{H}_1}{\partial y} + c, \tag{11}$$

$$\frac{\partial \tilde{H}_2}{\partial x} - \frac{\partial \tilde{H}_1}{\partial y} = 0, \tag{12}$$

$$Q(x, y) = \frac{\partial \tilde{H}_2}{\partial x} - \frac{\partial \tilde{H}_1}{\partial y}, \tag{13}$$

$$\omega(x, y) = \frac{\partial \tilde{v}}{\partial x} - \frac{\partial \tilde{u}}{\partial y}, \tag{14}$$

of seven partial differential equations in eight unknown functions $\tilde{u}, \tilde{v}, \tilde{H}_1, \tilde{H}_2, \Omega, \omega, Q$ and B which are functions of (x, y). In addition, c is an arbitrary integration constant that may be found using the diffusion equation (3). Martin [2] has successfully employed a first-order system similar to this one to investigate viscous non-MHD flows.

Let $\alpha = \alpha(x, y)$ be the variable angle such that $\alpha(x, y) \neq 0$ for every (x, y) in the region of flow. Equation (11) yields

$$\tilde{u}\tilde{H}_2 - \tilde{v}\tilde{H}_1 = UH \sin \alpha = c + \gamma HQ, \tag{15}$$

$$\tilde{u}\tilde{H}_1 + \tilde{v}\tilde{H}_2 = UH \cos \alpha = (c + \gamma_H Q) \cot \alpha, \tag{16}$$

where $H = \sqrt{(\tilde{H}_1^2 + \tilde{H}_2^2)}$. Considering these as two linear algebraic equations in the unknown's u and v, we solve (15) and (16) in terms of $\tilde{H}_1, \tilde{H}_2$, and α.

$$\tilde{u} = (c + \gamma_H Q)\left(\frac{\tilde{H}_2 + \tilde{H}_1 \cot \alpha}{\tilde{H}_1^2 + \tilde{H}_2^2}\right), \tag{17}$$

$$\tilde{v} = (c + \gamma_H Q)\left(\frac{\tilde{H}_2 \cot \alpha - \tilde{H}_1}{\tilde{H}_1^2 + \tilde{H}_2^2}\right), \tag{18}$$

we can eliminate $\tilde{u}$ and v from the system (8)-(14) by using equations (17) and (18) and then obtaining a system of equations to be solved for $\tilde{H}_1, \tilde{H}_2, \Omega, \omega, B, Q$ and α as functions of x and y, this approach leads to the study of system (8)-(14) in the

magnetograph plane. By using (17)-(18) and removing u and v from the system of (8)-(14) we get the system of six partial differential equations as under,

$$\frac{\partial \tilde{H}_1}{\partial x} + \frac{\partial \tilde{H}_2}{\partial y} = 0\,, \tag{19}$$

$$\eta \frac{\partial \Omega}{\partial x} - \rho(\omega + 2\Omega)\left[(c + \gamma_H Q)\left(\frac{\tilde{H}_1 \cot\alpha + \tilde{H}_2}{\tilde{H}_1^2 + \tilde{H}_2^2}\right)\right]$$

$$+\eta Q\tilde{H}_2 + \frac{\eta}{\kappa}(c + \gamma_H Q)\left(\frac{\tilde{H}_2 + \tilde{H}_1 \cot\alpha}{\tilde{H}_1^2 + \tilde{H}_2^2}\right) = -\frac{\partial B}{\partial x}\,, \tag{20}$$

$$\eta \frac{\partial \Omega}{\partial y} - \rho(\omega + 2\Omega)\left[(c + \gamma_H Q)\left(\frac{\tilde{H}_2 + \tilde{H}_1 \cot\alpha}{\tilde{H}_1^2 + \tilde{H}_2^2}\right)\right]$$

$$+\eta Q\tilde{H}_1 + \frac{\eta}{\kappa}(c + \gamma_H Q)\left(\frac{\tilde{H}_2 \cot\alpha - \tilde{H}_1}{\tilde{H}_1^2 + \tilde{H}_2^2}\right) = \frac{\partial B}{\partial y}\,, \tag{21}$$

$$(c + \gamma_H Q)\left[\tilde{H}_1 \frac{\partial \cot\alpha}{\partial x} + \tilde{H}_2 \frac{\partial \cot\alpha}{\partial y}\right.$$

$$+\left(\frac{\partial \tilde{H}_2}{\partial x} - \frac{\partial \tilde{H}_1}{\partial y}\right)\left(\frac{\tilde{H}_1^2 - \tilde{H}_2^2 - 2\tilde{H}_1\tilde{H}_2 \cot\alpha}{\tilde{H}_1^2 + \tilde{H}_2^2}\right)$$

$$\left.+\left(\frac{\partial \tilde{H}_2}{\partial y} - \frac{\partial \tilde{H}_1}{\partial x}\right)\left(\frac{\tilde{H}_1^2 \cot\alpha - \tilde{H}_2^2 \cot\alpha + 2\tilde{H}_1\tilde{H}_2 \cot\alpha}{\tilde{H}_1^2 + \tilde{H}_2^2}\right)\right]$$

$$+\gamma_H\left[(\tilde{H}_2 + \tilde{H}_1 \cot\alpha)\frac{\partial Q}{\partial x} + (\tilde{H}_2 + \tilde{H}_1 \cot\alpha)\frac{\partial Q}{\partial y}\right] = 0\,, \tag{22}$$

$$\frac{\partial \tilde{H}_2}{\partial x} - \frac{\partial \tilde{H}_1}{\partial y} = Q\,, \tag{23}$$

$$\frac{\partial\left[(c+\gamma_H Q)\left(\frac{\tilde{H}_2\cot\alpha-\tilde{H}_1}{\tilde{H}_1^2+\tilde{H}_2^2}\right)\right]}{\partial x}-\frac{\partial\left[(c+\gamma_H Q)\left(\frac{\tilde{H}_2+\tilde{H}_1\cot\alpha}{\tilde{H}_1^2+\tilde{H}_2^2}\right)\right]}{\partial y}=\omega, \tag{24}$$

3. Magnetograph transformations

As mentioned in the equations of flow $\tilde{H}_1=\tilde{H}_1(x,y)$, $\tilde{H}_2=\tilde{H}_2(x,y)$ the Jacobian

$$J(x,y)=\frac{\partial\tilde{H}_2}{\partial x}\frac{\partial\tilde{H}_2}{\partial y}-\frac{\partial\tilde{H}_1}{\partial y}\frac{\partial\tilde{H}_2}{\partial x}\neq 0 \tag{25}$$

Let x and y be functions of $\tilde{H}_1$ and $\tilde{H}_2$, that is, $x=x(\tilde{H}_1,\tilde{H}^2)$, $y=y(\tilde{H}_1,\tilde{H}^2)$.

Given these assumptions, we may have the following relations:

$$\frac{\partial\tilde{H}_1}{\partial x}=J\frac{\partial y}{\partial\tilde{H}_2},\frac{\partial\tilde{H}_2}{\partial x}=-J\frac{\partial y}{\partial\tilde{H}_1},\frac{\partial\tilde{H}_1}{\partial y}=-J\frac{\partial x}{\partial\tilde{H}_2},\frac{\partial\tilde{H}_2}{\partial y}=J\frac{\partial y}{\partial\tilde{H}_1} \tag{26}$$

Further,

$$J(x,y)=\frac{\partial(\tilde{H}_1,\tilde{H}_2)}{\partial(x,y)}=\left[\frac{\partial(x,y)}{\partial(\tilde{H}_1,\tilde{H}_2)}\right]^{-1}=j(\tilde{H}_1,\tilde{H}_2),$$

$$\frac{\partial f}{\partial x}=j\frac{\partial(f,y)}{\partial(\tilde{H}_1,\tilde{H}_2)},\frac{\partial f}{\partial y}=j\frac{\partial(x,f)}{\partial(\tilde{H}_1,\tilde{H}_2)}, \tag{27}$$

where $f(\tilde{H}_1,\tilde{H}_2)$ is transformed function of continuously differentiable function of f in the $\tilde{H}_1\tilde{H}_2$-plane.

4. Flow Equations in Magnetograph Plane

Applying the aforementioned transformation relations to the system of equations (19)-(24) in the magnetograph plane, or $(\tilde{H}_1,\tilde{H}_2)$ plane, for the first order

partial derivatives results in the transformed system of partial differential equations being

$$\frac{\partial x}{\partial \tilde{H}_1} + \frac{\partial y}{\partial \tilde{H}_2} = 0\,, \tag{28}$$

$$\eta j \frac{\partial(x,\omega)}{\partial(\tilde{H}_1,\tilde{H}_2)} + \frac{\partial y}{\partial \tilde{H}_2} - \rho(\omega + 2\Omega)\left[(c+\gamma_H Q)\left(\frac{\tilde{H}_2\cot\alpha - \tilde{H}_1}{\tilde{H}_1^2 + \tilde{H}_2^2}\right)\right]$$

$$+\eta Q\tilde{H}_2 + \frac{\eta}{\kappa}(c+\gamma_H Q)\left(\frac{\tilde{H}_2 + \tilde{H}_1\cot\alpha}{\tilde{H}_1^2 + \tilde{H}_2^2}\right) = -j\frac{\partial(B,y)}{\partial(\tilde{H}_1,\tilde{H}_2)}\,, \tag{29}$$

$$\eta j \frac{\partial(\omega,y)}{\partial(\tilde{H}_1,\tilde{H}_2)} - \rho(\omega+2\Omega)\left[(c+\gamma_H Q)\left(\frac{\tilde{H}_2 + \tilde{H}_2\cot\alpha}{\tilde{H}_1^2 + \tilde{H}_2^2}\right)\right]$$

$$+\eta Q\tilde{H}_1 - \frac{\eta}{\kappa}(c+\gamma_H Q)\left(\frac{\tilde{H}_2\cot\alpha - \tilde{H}_1}{\tilde{H}_1^2 + \tilde{H}_2^2}\right) = \frac{\partial(x,B)}{\partial(\tilde{H}_1,\tilde{H}_2)}\,, \tag{30}$$

$$j\left(\frac{\partial x}{\partial \tilde{H}_2} - \frac{\partial y}{\partial \tilde{H}_1}\right) = Q\,, \tag{31}$$

$$j\left[\frac{\partial\left(\left[(c+\gamma_H Q)\left(\frac{\tilde{H}_2\cot\alpha-\tilde{H}_1}{\tilde{H}_1^2+\tilde{H}_2^2}\right)\right],y\right)}{\partial(\tilde{H}_1,\tilde{H}_2)} - \frac{\partial\left(x,\left[(c+\gamma_H Q)\left(\frac{\tilde{H}_2+\tilde{H}_1\cot\alpha}{\tilde{H}_1^2+\tilde{H}_2^2}\right)\right]\right)}{\partial(\tilde{H}_1,\tilde{H}_2)}\right] = \omega\,, \tag{32}$$

$$\frac{\partial x}{\partial \tilde{H}_1}\left[(c+\gamma_H Q)\left\{H_1^2\cot\alpha - H_2^2\cot\alpha + 2\tilde{H}_1\tilde{H}_2\cot + \tilde{H}_2 H^2\frac{\partial\cot\alpha}{\partial\tilde{H}_1}\right\}\right.$$

$$\left.+H^2\gamma_H\frac{\partial Q}{\partial\tilde{H}_1}(\tilde{H}_2\cot\alpha - \tilde{H}_1)\right]$$

$$+\frac{\partial x}{\partial\tilde{H}_2}\left[(c+\gamma_H Q)\left\{\tilde{H}_1^2 - \tilde{H}_2^2 - 2\tilde{H}_1\tilde{H}_2\cot + \tilde{H}_2 H^2\frac{\partial\cot\alpha}{\partial\tilde{H}_1}\right\}\right.$$

$$+H^2\gamma_H \frac{\partial Q}{\partial \tilde{H}_1}(\tilde{H}_1 - \tilde{H}_2 \cot\alpha)\Bigg]$$

$$+\frac{\partial y}{\partial \tilde{H}_2}\Bigg[(c + \gamma_H Q)\left\{\tilde{H}_2^2 - \tilde{H}_1^2 + 2\tilde{H}_1\tilde{H}_2 \cot\alpha + \tilde{H}_1 H^2 \frac{\partial \cot\alpha}{\partial \tilde{H}_2}\right\}$$

$$+H^2\gamma_H \frac{\partial Q}{\partial \tilde{H}_2}(\tilde{H}_1 \cot\alpha + \tilde{H}_2)\Bigg]$$

$$+\frac{\partial y}{\partial \tilde{H}_1}\Bigg[(c + \gamma_H Q)\left\{H_2^2 \cot\alpha - H_1^2 \cot\alpha + 2\tilde{H}_1\tilde{H}_2 + \tilde{H}_1 H^2 \frac{\partial \cot\alpha}{\partial \tilde{H}_1}\right\}$$

$$+H^2\gamma_H \frac{\partial Q}{\partial \tilde{H}_1}(\tilde{H}_1 \cot\alpha - \tilde{H}_2)\Bigg] = 0. \tag{33}$$

5. Legendre Transform of Magnetic Flux Function

The solenoidal equation (19) verified the existence of the magnetic flux function $\phi(x, y)$ and is such that

$$d\phi = -\tilde{H}_2 dx + \tilde{H}_1 dy \quad \text{or} \quad \frac{\partial\phi}{\partial x} = -\tilde{H}_2, \frac{\partial\phi}{\partial y} = \tilde{H}_1, \tag{34}$$

Similarly, for the magnetic flux function $\phi(x, y)$, equation (28) verified the existence of the function $L(\tilde{H}_1, \tilde{H}_2)$, also known as Legendre's transform function. It is such that

$$dL = -y d\tilde{H}_1 + x d\tilde{H}_2 \quad \text{or} \quad \frac{\partial L}{\partial \tilde{H}_1} = -y, \frac{\partial L}{\partial \tilde{H}_2} = x, \tag{35}$$

and these two equation are connected by $L(\tilde{H}_1, \tilde{H}_2) = \tilde{H}_2 x - \tilde{H}_1 y + \phi(x, y)$.

Introducing $L(\tilde{H}_1, \tilde{H}_2)$ into the system of equations (28)-(33) it follows that equation (28) is identically satisfied with *j* given by (27) and the system is substituted by

$$\eta j \frac{\partial(\frac{\partial L}{\partial \tilde{H}_2}, \omega)}{\partial(\tilde{H}_1, \tilde{H}_2)} + \frac{\partial y}{\partial \tilde{H}_2} - \rho(\omega + 2\Omega)\left[(c + \gamma_H Q)\left(\frac{\tilde{H}_2 \cot\alpha - \tilde{H}_1}{\tilde{H}_1^2 + \tilde{H}_2^2}\right)\right]$$

$$+\eta J\tilde{H}_2 + \frac{\eta}{\kappa}(c + \gamma_H)\frac{\tilde{H}_2 + \tilde{H}_1 \cot\alpha}{\tilde{H}_1^2 + \tilde{H}_2^2} = j\frac{\partial B, \frac{\partial L}{\partial \tilde{H}_1}}{\partial(\tilde{H}_1, \tilde{H}_2)} \tag{36}$$

$$\eta j \frac{\partial(\omega, \frac{\partial L}{\partial \tilde{H}_1})}{\partial(\tilde{H}_1, \tilde{H}_2)} - \rho(\omega + 2\Omega)\left[(c + \gamma_H Q)\left(\frac{\tilde{H}_2 + \tilde{H}_1 \cot\alpha}{\tilde{H}_1^2 + \tilde{H}_2^2}\right)\right]$$

$$+\eta Q\tilde{H}_1 - \frac{\eta}{\kappa}(c + \gamma_H Q)\left(\frac{\tilde{H}_2 \cot\alpha - \tilde{H}_1}{\tilde{H}_1^2 + \tilde{H}_2^2}\right) = \frac{\partial(\frac{\partial L}{\partial \tilde{H}_2}, B)}{\partial(\tilde{H}_1, \tilde{H}_2)} \tag{37}$$

$$j\left[\frac{\partial\left(\left[(c + \gamma_H Q)\left(\frac{\tilde{H}_2 \cot\alpha - \tilde{H}_1}{\tilde{H}_1^2 + \tilde{H}_2^2}\right)\right], \frac{\partial L}{\partial \tilde{H}_1}\right)}{\partial(\tilde{H}_1, \tilde{H}_2)} - \frac{\partial\left(\frac{\partial L}{\partial \tilde{H}_2}, \left[(c + \gamma_H Q)\left(\frac{\tilde{H}_2 + \tilde{H}_1 \cot\alpha}{\tilde{H}_1^2 + \tilde{H}_2^2}\right)\right]\right)}{\partial(\tilde{H}_1, \tilde{H}_2)}\right] = \omega, \tag{38}$$

$$j\left(\frac{\partial^2 L}{\partial \tilde{H}_2^2} - \frac{\partial^2 L}{\partial \tilde{H}_1^2}\right) = Q, \tag{39}$$

$$\begin{aligned}
&\frac{\partial^2 L}{\partial \tilde{H}_2^2}(c + \gamma_H Q)\left\{\tilde{H}_2^2 - \tilde{H}_1^2 + 2\tilde{H}_1\tilde{H}_2 \cot + \tilde{H}_2 H^2 \frac{\partial \cot\alpha}{\partial \tilde{H}_1}\right\} \\
&\quad + H^2\gamma_H \frac{\partial Q}{\partial \tilde{H}_1}(\tilde{H}_1 - \tilde{H}_2 \cot\alpha)\Big] \\
&\quad - \frac{\partial^2 L}{\partial \tilde{H}_1^2}(c + \gamma_H Q)\left\{\tilde{H}_1^2 - \tilde{H}_2^2 + 2\tilde{H}_1\tilde{H}_2 \cot + \tilde{H}_2 H^2 \frac{\partial \cot\alpha}{\partial \tilde{H}_1}\right\} \\
&\quad + H^2\gamma_H \frac{\partial Q}{\partial \tilde{H}_1}(\tilde{H}_2 \cot\alpha) + \tilde{H}_1\Big] \\
&\quad + \frac{\partial^2 y}{\partial \tilde{H}_1 \partial \tilde{H}_2}(c + \gamma_H Q)H^2\left(\frac{\partial \cot\alpha}{\partial \tilde{H}_2} - \tilde{H}_1 \frac{\partial \cot\alpha}{\partial \tilde{H}_1}\right) \\
&\quad + H^2\gamma_H\left\{\frac{\partial Q}{\partial \tilde{H}_2}(\tilde{H}_2 \cot\alpha - \tilde{H}_1) - \frac{\partial Q}{\partial \tilde{H}_1}(\tilde{H}_1 \cot\alpha + \tilde{H}_2)\right\} \\
&\quad + (c + \gamma_H Q)(2\tilde{H}_1^2 \cot\alpha - \tilde{H}_2^2 \cot\alpha + 4\tilde{H}_1\tilde{H}_2 + \tilde{H}_1 H^2)\Big] = 0
\end{aligned} \tag{40}$$

$$j = \left[\frac{\partial^2 L}{\partial \tilde{H}_1^2} \frac{\partial^2 L}{\partial \tilde{H}_2{}^2} - \left(\frac{\partial^2 L}{\partial \tilde{H}_1 \partial \tilde{H}_2} \right)^2 \right]^{-1}, \tag{41}$$

for the seven functions $L(\tilde{H}_1\tilde{H}_2)$, $B(\tilde{H}_1\tilde{H}_2)$, $\omega(\tilde{H}_1\tilde{H}_2)$, $j(\tilde{H}_1\tilde{H}_2)$, $\alpha(\tilde{H}_1\tilde{H}_2)$, $J(\tilde{H}_1\tilde{H}_2)$ and $\Omega(\tilde{H}_1\tilde{H}_2)$.

Introducing polar co-ordinates $(H, \theta)\tilde{H}_1 = H\cos\theta$ and $\tilde{H}_2 = H\sin\theta$

$$\frac{\partial(F, G)}{\partial(\tilde{H}_1, \tilde{H}_2)} = \frac{1}{H} \frac{\partial(F^*, G^*)}{\partial(\tilde{H}_1, \tilde{H}_2)},$$

$$\frac{\partial}{\partial \tilde{H}_1} = \cos\theta \frac{\partial}{\partial H} - \frac{\sin\theta}{H} \frac{\partial}{\partial \theta},$$

$$\frac{\partial}{\partial \tilde{H}_2} = \sin\theta \frac{\partial}{\partial H} + \frac{\cos\theta}{q} \frac{\partial}{\partial \theta}$$

where $F(\tilde{H}_1, \tilde{H}_2) = F^*(H, \theta)$; $G(\tilde{H}_1\tilde{H}_2) = G^*(H, \theta)$ are continuously differentiable functions in (H, θ) coordinates, the equation (40) takes the form

$$\frac{\partial^2 L^*}{\partial H_2^2} \left[(c + \gamma_H J) H \frac{\partial \cot\alpha^*}{\partial \theta} + \gamma_H \cot\alpha^* \frac{\partial Q}{\partial \theta} \right]$$

$$+ \left(\frac{1}{H^2} \frac{\partial 2 L^*}{\partial \theta^2} + \frac{1}{H^2} \frac{\partial L^*}{\partial \theta} \right) \left[H\gamma_H \frac{\partial j}{\partial H} - (c + \gamma_H Q) \right]$$

$$+ \left(\frac{1}{H} \frac{\partial^2 L^*}{\partial H \partial \theta} - \frac{1}{H^*} \frac{\partial L^*}{\partial \theta} \right) \left[(c + \gamma_H Q) \left(2\cot\alpha^* - H \frac{\partial \cot\alpha^*}{\partial H} \right) - \cot\alpha^* H \gamma_H \frac{\partial Q^*}{\partial \theta} \right] = 0 . \tag{42}$$

6. Applications

Example 1: Let

$$L(\tilde{H}_1\tilde{H}_2) = N_1 \tan^{-1}\left(\frac{\tilde{H}_2}{\tilde{H}_1}\right) + N_2, \alpha(\tilde{H}_1, \tilde{H}_2) = \cot^{-1}(N_3H_1^2 + N_3H_2^2) \qquad (43)$$

form a set of solution of the partial differential equation (40) when $N_1 \neq 0, N_2$ and N_3 are arbitrary constants. As N_3 is arbitrary, there are two cases of the solution which may considered by (43).

(i) If $N_3 \neq 0$ i.e., variably inclined flows and

(ii) If $N_3 = 0$ i.e., crossed flows.

When (i) $N_3 \neq 0$.

Using (43) in (35) we have

$$\tilde{H}_1(x, y) = \frac{N_1 x}{r^2}; \ \tilde{H}_2(x, y) = \frac{N_1 y}{r^2}, \quad r^2 = x^2 + y^2. \qquad (44)$$

This represents radial flow and magnetic field profile is thus the arc of a rectangular hyperbola, using (44) we obtain

$$\tilde{u} = \frac{c}{N_1 r^2}(yr^2 + N_3N_1^2x), \ \tilde{v} = \frac{c}{N_1 r^2}(N_3N_1^2y - xr^2),$$

$$\omega(x, y) = \frac{-2c}{N_1}, \qquad Q = 0, \qquad \alpha(x, y) = \cot^{-1}\left(\frac{N_1^2N_3}{r^2}\right) \qquad (45)$$

With the help of (45) and integrability condition on B i.e.,

$$\frac{\partial^2 B}{\partial x \partial y} = \frac{\partial^2 B}{\partial y \partial x}$$

from equations (9) and (10) we get angular velocity

$$\left[y(x^2 + y^2) + N_3N_1^2x\right]\frac{\partial \Omega}{\partial x} - \left[x(x^2 + y^2) - N_3N_1^2y\right]\frac{\partial \Omega}{\partial y} - \frac{\eta}{k\rho}(x^2 + y^2) = 0 \qquad (46)$$

The Lagrange form of solution of this equation is

$$\Omega = -N_3 N_1^2 \tan^{-1}\frac{y}{x} + C, \quad \text{where} \quad N_1^2 N_3 = \frac{\eta}{\rho k}, \tag{47}$$

the streamlines are given by $(x^2 + y^2) + N_1 N_3 \tan^{-1}\frac{y}{x} = constant$, the magnetic flux function is

$$\tan^{-1}\frac{y}{x} = constant$$

and from (9) and (10) we have

$$B(x,y) = \left(4\rho c^2 N_3 + \frac{\eta c}{kN_1} + \frac{2\eta c}{\kappa} N_3 N_1 - \frac{\eta c y}{kN_1}\right)\tan^{-1}\frac{y}{x} + \frac{\rho c^2}{N_1^2}(x^2 + y^2)$$

$$+\frac{\eta c x^2}{kN_1} + \frac{\eta c}{\kappa} + N_3 N_1 \left(\tan^{-1}\frac{y}{x}\right)^2$$

$$+\frac{\rho c^2}{N_1^2}(x^2 + y^2)\tan^{-1}\frac{y}{x} - \frac{\eta c}{\kappa} N_3 N_1 \ln(x^2 + y^2) + constant, \tag{48}$$

and hence the pressure

$$P(x,y) = B - \frac{1}{2}\rho V^2,$$

is

$$P(x,y) = \left(4\rho c^2 N_3 + \frac{\eta c}{kN_1} + \frac{2\eta c}{\kappa} N_3 N_1 - \frac{\eta c y}{kN_1}\right)\tan^{-1}\frac{y}{x} + \frac{\rho c^2}{N_1^2}(x^2 + y^2)P(x,y)$$

$$+\frac{\eta c x^2}{kN_1} + \frac{\eta c}{\kappa} + N_3 N_1 \left(\tan^{-1}\frac{y}{x}\right)^2 + \frac{\rho c^2}{N_1^2}(x^2 + y^2)\left(\tan^{-1}\frac{y}{x}\right) - \frac{\eta c}{\kappa} N_3 N_1 \ln(x^2 + y^2)$$

$$-\frac{1}{2}\frac{\rho c^2}{N_1^2}(x^2 + y^2) - \frac{1}{2}\frac{\rho c^2 N_3 N_1}{2(x^2 + y^2)} + constant \tag{49}$$

and

(ii) for $N_3 = 0$ i.e., crossed flows, the value of u, v, α calculate similarly by putting $N_3 = 0$ in equation (6.1).

By putting $N_3 = 0$ in equation (6.4) we get

$$\Omega = C_1 - \frac{\eta}{\rho k} \ln y$$

Again, B and P can be calculated by putting $N_3 = 0$ in equations (6.6) and (6.7) respectively.

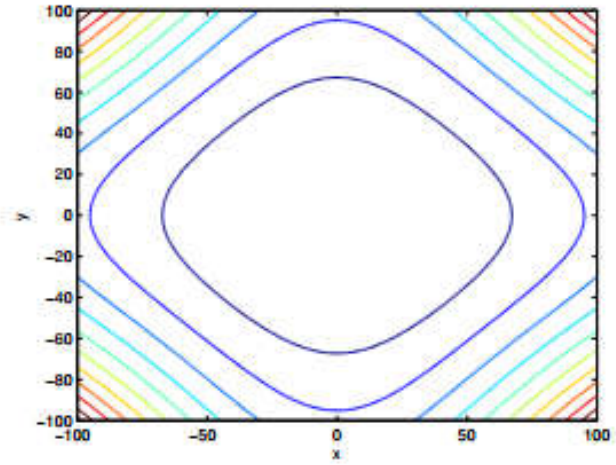


Figure 1: Streamlines for example 1

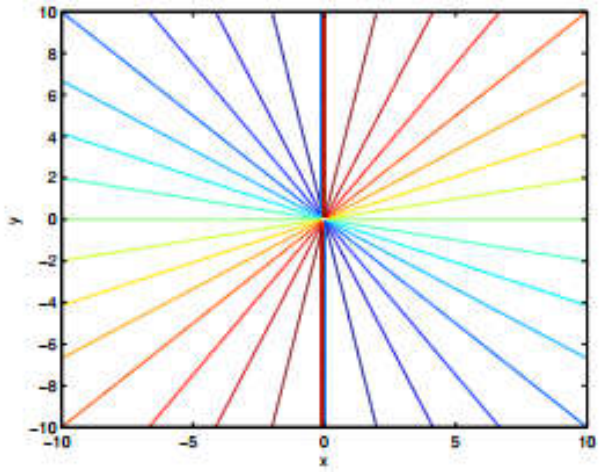


Figure 2: Magnetic lines for example 1

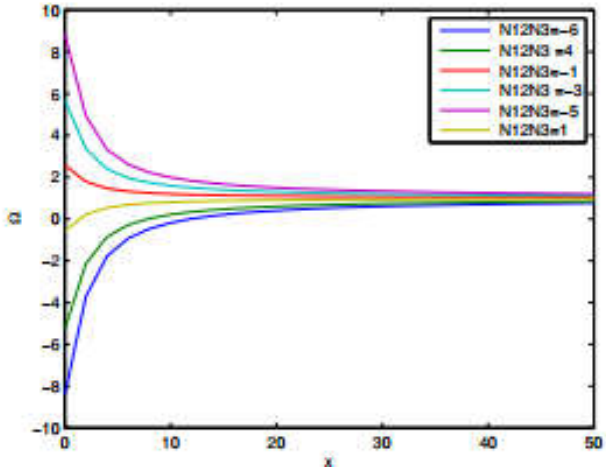


Figure 3: Variation of angular velocity versus x for example 1

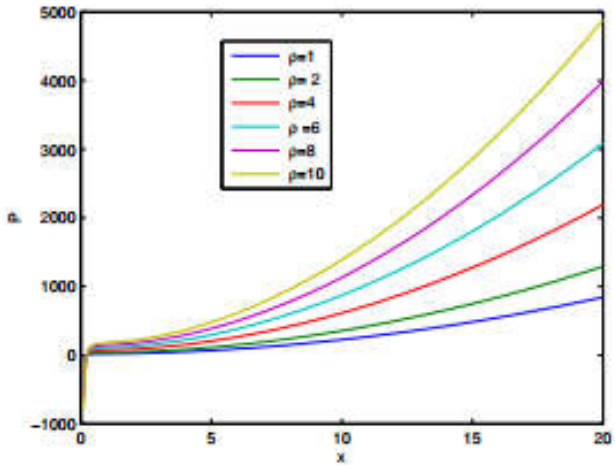


Figure 4: Variation of pressure versus x at $y = 2$ for density variation example 1

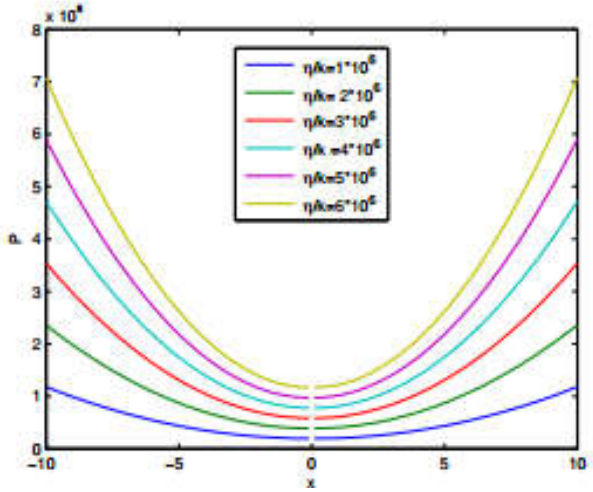


Figure 5: Variation of pressure versus x at $y = 2$ for porosity variation for example1

Example 2: Another solution of equation (5.7) is

$$L(\tilde{H}_1, \tilde{H}_2) = M_1(H_2^2 + H_1^2) + M_2, \ \alpha(\tilde{H}_1\tilde{H}_2) = \cot^{-1}(M_3H_1^2 + M_3H_2^2 + M_4) \tag{50}$$

Where $M_1 \neq 0, M_2, M_3$ and M_4 are arbitrary constants.

We have

$$\frac{dL}{d\tilde{H}_1} = -y, \frac{dL}{d\tilde{H}_2} = x,$$

We examine the case where M_3 and M_4 are arbitrary constants. When flows are variably inclined, $M_3 \neq 0$, i.e. The resulting flows are crossed if $M_3 = M_4 = 0$ and constantly inclined if $M_3 = 0, M_4 \neq 0$. Now consider the case when $M_3 \neq 0, M_4 \neq 0$. Using (49) in (35) we obtain

$$x = 2M_1\tilde{H}_2, y = -2M_1\tilde{H}_1$$

and therefore

$$\tilde{H}_1 = \frac{-y}{2M_1}, \quad \tilde{H}_2 = \frac{-x}{2M_2}, \tag{51}$$

This indicates that the radial distance from the central axis directly affects the magnetic field $H = \frac{-r}{2M_1}$.

The changing angle between the velocity and magnetic fields in the physical plane is given by

$$\alpha(x, y) = \cot^{-1}\left[\frac{M_3 r^2}{4M_1^2} + M_4\right]$$

and hence vorticity, current density and velocity components are given by

$$\omega = \left(\frac{M_3}{M_1}c + \frac{\gamma_H}{M_1}\right), \quad Q = \frac{1}{M_1}$$

$$\tilde{u} = \left(c + \frac{\gamma_H}{M_1}\right) \left[\frac{2M_1(x - M_4 y)}{(x^2 + y^2)} - \frac{M_3 y}{2M_1}\right];$$

$$\tilde{v} = \left(c + \frac{\gamma_H}{M_1} \right) \left[\frac{2M_1(x - M_4 y)}{(x^2 + y^2)} - \frac{M_3 y}{2M_1} \right]. \tag{52}$$

It is to be noted that velocity of the fluid is infinite when $r = 0$ i.e., when $H = 0$. And fluid velocity is zero when the radial distance is infinite and so the velocity of the fluid decreases as the r-increases. From (48) and integrability condition on B equations (9) and (10) yields the angular velocity Ω as

$$[M_3 x(x^2 + y^2) + 4M_1^2(y + M_4 x)] \frac{\partial \Omega}{\partial y} + [4M_1^2(x - M_4 y - M_3 x)(x^2 + y^2)] \frac{\partial \Omega}{\partial x} + \frac{\eta M_3}{k\rho} (x^2 + y^2) = 0. \tag{53}$$

The solution to this problem in Lagrange form is

$$\Omega = 4M_1^2 \tan^{-1} \frac{y}{x} - 2M_1^2 M_4 \ln(x^2 + y^2) - (x^2 + y^2) \frac{M_3 + 2}{2},$$

where
$$\frac{4M_1^2}{M_3} = \frac{\eta}{\rho k}, \tag{54}$$

The streamlines are provided by

$$8M_1^2 \tan^{-1} \frac{y}{x} - 8M_1^2 M_4 \ln(x^2 + y^2) + M_3(x^2 + y^2) = constant,$$

the magnetic flux function is

$$x^2 + y^2 = constant$$

and (54) yield the energy function B as

$$B(x, y) = \rho\omega \left(c + \frac{\gamma_H}{M_1} \right) \left[-4M_1 \tan^{-1} \frac{y}{x} + 2M_1 M_4 \ln(x^2 + y^2) + \frac{M_3}{2M_1} (x^2 + y^2) \right]$$

$$+2\rho\left(c+\frac{\gamma_H}{M_1}\right)\left[8\frac{M_1^3}{M_3}\left(\tan^{-1}\frac{y}{x}\right)^2-8\frac{M_1^3M_4}{M_3}\tan^{-1}\frac{y}{x}\ln(x^2+y^2)+2M_1xy\right.$$

$$\left.-2M_1(x^2+y^2)\tan^{-1}\frac{y}{x}+2\frac{M_1^3M_4}{M_3}(\ln(x^2+y^2))^2+M_1M_4(x^2+y^2)\right]$$

$$-\frac{\eta}{4M_1^2}(x^2+y^2)+\frac{\eta M_3}{kM_1}c+\frac{\gamma_H}{M_1}xy+constant\,. \tag{55}$$

And pressure is

$$P(x,y)=\rho\omega\left(c+\frac{\gamma_H}{M_1}\right)\left[-4M_1\tan^{-1}\frac{y}{x}+2M_1M_4\ln(x^2+y^2)+\frac{M_3}{2M_1}(x^2+y^2)\right]$$

$$+2\rho\left(c+\frac{\gamma_H}{M_1}\right)\left[8\frac{M_1^3}{M_3}\left(\tan^{-1}\frac{y}{x}\right)^2-8\frac{M_1^3M_4}{M_3}\tan^{-1}\frac{y}{x}\ln(x^2+y^2)+2M_1xy\right.$$

$$\left.-2M_1(x^2+y^2)\tan^{-1}\frac{y}{x}+2\frac{M_1^3M_4}{M_3}(\ln(x^2+y^2))^2+M_1M_4(x^2+y^2)\right]$$

$$-\frac{\eta}{4M_1^2}(x^2+y^2)+\frac{\eta M_3}{kM_1}\left(c+\frac{\gamma_H}{M_1}\right)xy$$

$$-\frac{1}{2}\rho\left(c+\frac{\gamma_H}{M_1}\right)^2\left[\frac{4M(1+M_4^2)}{(x^2+y^2)}+\frac{M_3}{4M_2}(x^2+y^2)+2M_3M_4\right]+P_0\,.$$

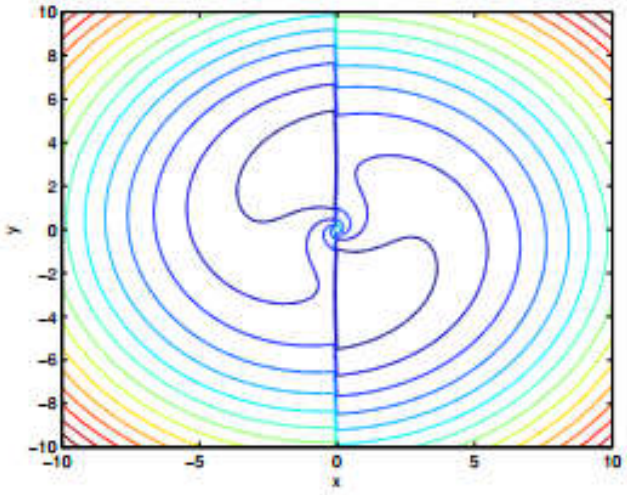


Figure 6: Streamlines for example 2

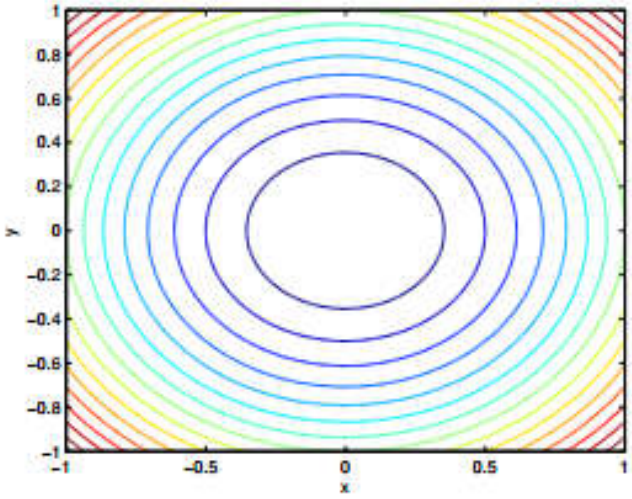


Figure 7: Magnetic lines for example 2

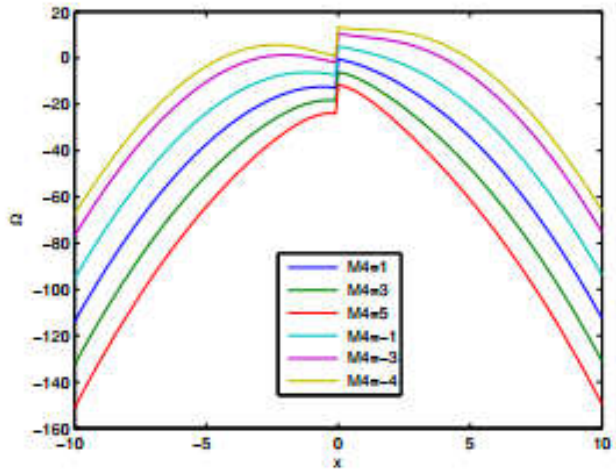


Figure 8: Variation of angular velocity versus x for example 2

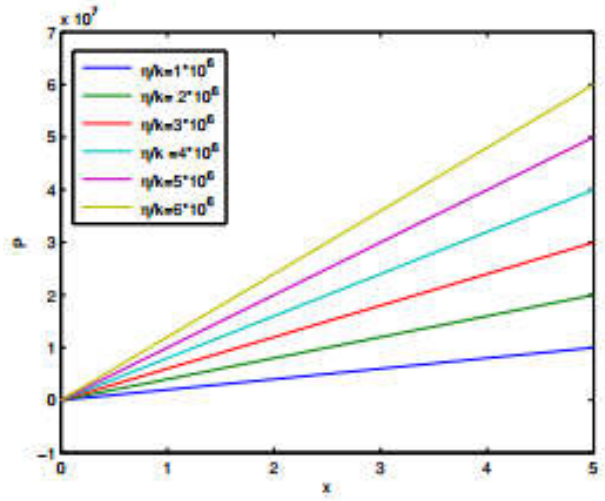


Figure 9: Variation of pressure versus x at $y = 2$ for $\frac{\eta}{\kappa}$ variation example 2

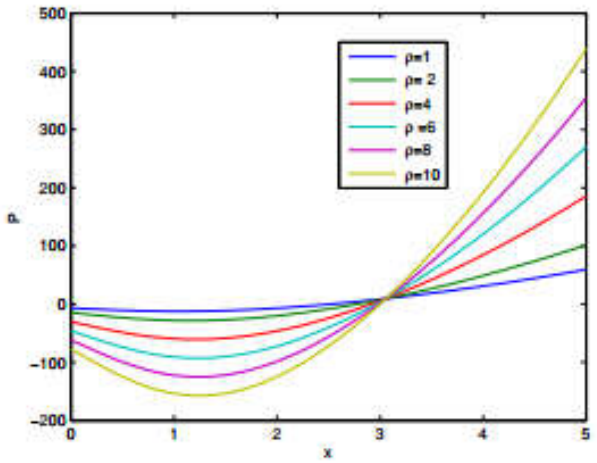


Figure 10: Variation of pressure versus x at $y = 2$ for $\kappa\,\eta$ variation example 2

Example 3: Consider

$$L^*(H,\theta) = A\theta + B\ln H + D \tag{57}$$

In (H,θ) coordinates, where D is an arbitrary constant and A and B are real values that are not zero. Applying this in (42) we have

$$B\frac{\partial}{\partial\theta}\cot\alpha - AH\frac{\partial}{\partial H}\cot\alpha + 2A\cot\alpha + 2B = 0$$

This has solution

$$\cot\alpha = -\frac{B}{A} + M_1H_2, M_1 = arbitrary\ constant$$

and

$$L^*(H,\theta) = A\theta + B\ln H + D,$$

and

$$\alpha^* = \cot^{-1}\left(-\frac{B}{A} + M_1H^2\right),$$

forms a solution set of the partial differential equation (42). If $M_1 = 0$ the flows are constantly inclined with

$$\alpha^* = \cot^{-1}\left(-\frac{B}{A}\right), \tag{58}$$

and when $M_1 \neq 0$, the flows are variably inclined, we have

$$\tilde{H}_1(x, y) = \frac{Ax - By}{r^2}\,; \qquad \tilde{H}_2(x, y) = \frac{Bx + Ay}{r^2}, \qquad r^2 = x^2 + y^2. \tag{59}$$

$$\tilde{u} = c\left\{\frac{y}{A} + \frac{(M_1(Ax - By))}{r^2}\right\}, \quad \tilde{v} = c\left\{-\frac{x}{A} + \frac{(M_1(Ay + Bx))}{r^2}\right\},$$

$$\omega(x, y) = \frac{-2c}{A}, \qquad Q = 0.$$

Now integrability condition for B yields

$$\{y(x^2 + y^2) + M_1A(Ax - By)\}\frac{\partial\Omega}{\partial x} - \{AM_1(Bx + Ay) - x(x^2 + y^2)\}\frac{\partial\Omega}{\partial y} - \frac{\eta}{k\rho}(x^2 + y^2) = 0. \tag{60}$$

The Lagrange form of solution of this equation is

$$\Omega = M_1A^2 \tan^{-1}\frac{y}{x} + M_1AB\ln(x^2 + y^2), \text{ where } A^2M_1 = \frac{\eta}{\rho k}. \tag{61}$$

the streamlines are given by

$$M_1A^2 \tan^{-1}\frac{y}{x} + M_1AB\ln(x^2 + y^2) + (x^2 + y^2) = constant$$

and the magnetic flux function is

$$A\tan^{-1}\frac{y}{x} + B\ln r = constant.$$

Now equation (9), (10) and Ω gives us the energy function

$$B(x,y) = \rho\frac{c^2}{A}\left\{-(x^2+y^2)+\frac{M_1}{2}(A+B)\ln(x^2+y^2)+M_1\tan^{-1}\frac{x}{y}\right\}$$

$$+2\rho c\left\{\frac{2}{3}M_1Axy+\frac{2}{3}M_1A(x^2+y^2)\tan^{-1}\frac{y}{x}+\frac{m_1^2A^2}{2}(1+A)\left(\tan^{-1}\frac{y}{x}\right)\right.$$

$$-\left(\frac{M_1B}{2}+\frac{M_1^2A^2B}{2}\right)\ln(x^2+y^2)\tan^{-1}\frac{y}{x}$$

$$\left.-\frac{M_1B}{2}(x^2+y^2)\ln(x^2+y^2)+\frac{M_1B}{2}(x^2+y^2)+\frac{M_1^2A^2B}{2}(\ln(x^2+y^2))^2\right\}$$

$$+\frac{\eta c}{\kappa}\left(-2\frac{xy}{A}+2M_1B\tan^{-1}\frac{x}{y}\right)+P. \tag{62}$$

And hence, the pressure function is

$$P(x,y) = \rho\frac{c^2}{A}\left\{-(x^2+y^2)+\frac{M_1}{2}(A+B)\ln(x^2+y^2)+M_1\tan^{-1}\frac{x}{y}\right\}$$

$$+2\rho c\left\{\frac{2}{3}M_1Axy+\frac{2}{3}M_1A(x^2+y^2)\tan^{-1}\frac{y}{x}+\frac{m_1^2A^2}{2}(1+A)\left(\tan^{-1}\frac{y}{x}\right)\right.$$

$$-\left(\frac{M_1B}{2}+\frac{M_1^2A^2B}{2}\right)\ln(x^2+y^2)\tan^{-1}\frac{y}{x}$$

$$\left.-\frac{M_1B}{2}(x^2+y^2)\ln(x^2+y^2)+\frac{M_1B}{2}(x^2+y^2)+\frac{M_1^2A^2B}{2}(\ln(x^2+y^2))^2\right\}$$

$$+\frac{\eta c}{\kappa}\left(-2\frac{xy}{A}+2M_1B\tan^{-1}\frac{x}{y}\right)-\frac{1}{2}\rho c\left[\frac{(x^2+y^2)}{A^2}+M_1^2\frac{A^2+B^2}{(x^2+y^2)}-2\frac{M_1B}{A^2}\right]+P_0. \tag{63}$$

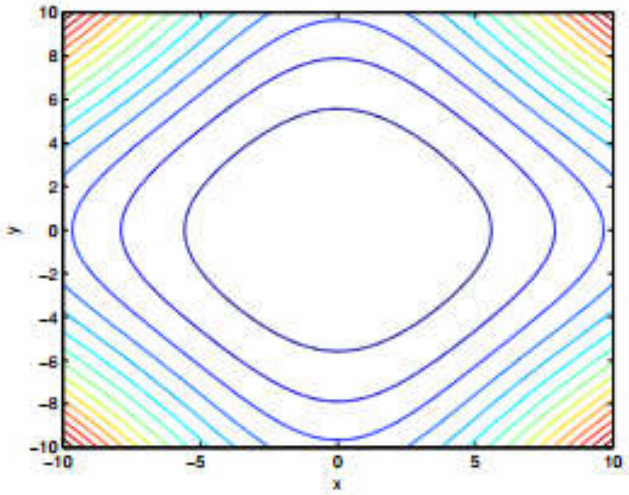


Figure 11: Streamline for example 3

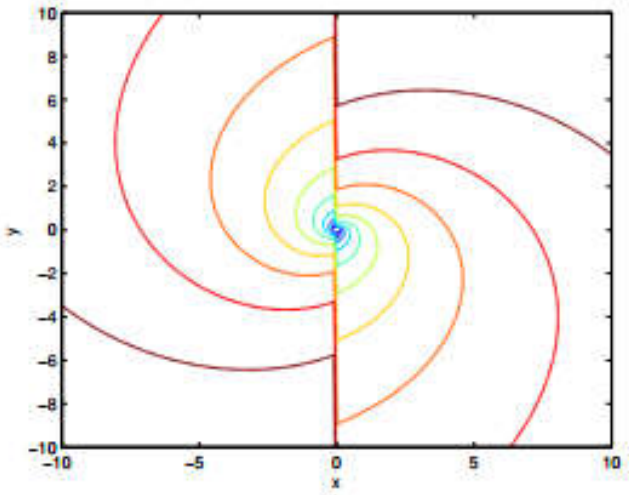


Figure 12: Magnetic line for example 3

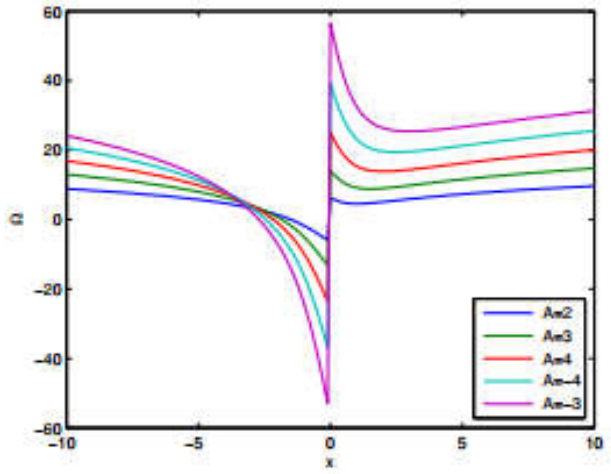


Figure 13: Variation of angular velocity verses x for example 3

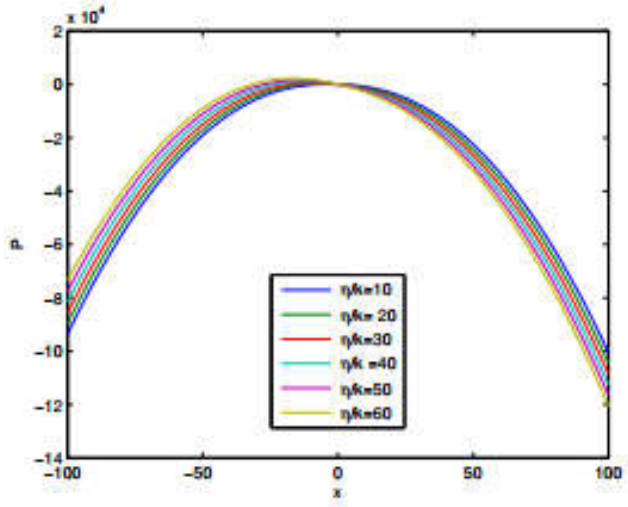


Figure 14: Variation of pressure verses x at $y = 2$ for $\frac{\eta}{\kappa}$ variation for example 3

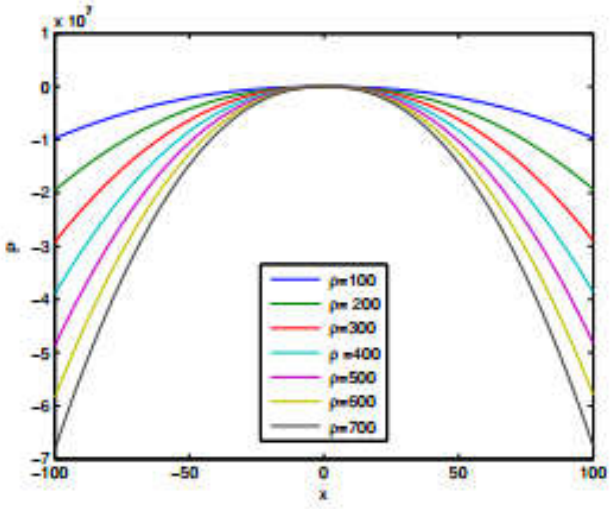


Figure 15: Variation of pressure verses x at $y = 2$ for fluid density variation for example 3

7. Conclusion

In this work, an approach has been carried out where magnetograph transformation method has been applied for the exact solution of the equations governing the flow of a homogeneous, incompressible viscous fluid through porous media of a variably inclined rotating MHD with finite electrical conductivity. We have utilized magnetograph transformation in this problem to reformulate the governing non-linear equation into linear once. Three different forms of Legendre transform function of the magnetic flux function have been considered as examples to illustrate the technique of solving for the exact solution. The expressions for streamlines, magnetic lines, angular velocity and pressure distribution are found out in each case. The main results are listed below:

- In example 1 the streamlines are given by $(x^2 + y^2) + N_1 N_3 \tan^{-1}\frac{y}{x} = constant$ and magnetic lines are given by $\tan^{-1}\frac{y}{x} = constant$.

- In example 2 streamlines and magnetic lines are given by $8M_1^2 \tan^{-1}\frac{y}{x} - 8M_1^2 M_4 \ln(x^2 + y^2) + M_3(x^2 + y^2) = constant$ and $x^2 + y^2 = constant$ respectively.

- In example 3, the streamlines are given by $M_1A^2 \tan^{-1}\frac{y}{x} + M_1AB \ln(x^2 + y^2) + (x^2 + y^2) = constant$ and magnetic lines are given by $A \tan^{-1}\frac{y}{x} + B \ln r = constant$.

- Also for example 1 components of velocity are independent of permeability of porous medium and angular velocity of rotating frame. The vorticity function is constant and current density is zero. Pressure depends on angular velocity, permeability of the medium and the fluid density.

- In example 2 for the form of Legendre transform function we find that the magnetic field varies with the radial distance from central axis. Current density function is constant and verticity function containing magnetic viscosity term is also a constant. The components of velocity depends on magnetic viscosity and current density function. Also, velocity of the fluid decreases with radial distance. Magnetic viscosity, current function, angular velocity, permeability of the medium and fluid density affects the pressure function.

- For the form of Legendre transform function considered in example 3 verticity function is constant, components of velocity does not involve permeability of medium and angular velocity. Pressure depends on angular velocity, permeability of medium and fluid density.

- Angular velocity depends on permeability of porous medium for all examples.

- In example 1 angular velocity for positive $N_1^2N_3$ decreases with x and for negative increases x becoming almost constant beyond $x = 7$ for both cases. For the form of Legendre transform function represents radial flow and magnetic field profile is arc of a rectangular hyperbola.

- In example 2 angular velocity is found to increase with x in the beginning and shoots up at $x = 100$ and decreases afterward in Figure 4.

- In example 3 angular velocity is found to decrease with x in the beginning shoot up at $x = 0$ and shows varying trend there afterwards (Figure 8). In example 1 (Figure 2) pressure increases at constant $\frac{\eta}{\kappa}$ for different fluid of different densities. For different $\frac{\eta}{\kappa}$ values at constant fluid density ρ the

pressure shows parabolic variations with x and is almost symmetric about $x = 100$.

- In examples 2 (Figure 5) Pressure varies linearly with x for different values of $\frac{\eta}{\kappa}$ at constant fluid density. For fluid of different densities at constant $\frac{\eta}{\kappa}$ pressure declines initially and increases rapidly with large x values.

- In example 3 pressure has a inverted parabolic variation (Figure 8 and 9) with x for different $\frac{\eta}{\kappa}$ at constant density ρ as well as different fluid density at constant $\frac{\eta}{\kappa}$ which are symmetric about $x = 0$.

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