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VECTOR BASIS S-CORDIAL LABELING OF FRIENDSHIP GRAPH, FAN GRAPH, AND LILLY GRAPH

Abstract: Let G be a (p,q) graph. Let V be an inner product space with basis S. Let $\varphi : V(G) \to S$ be a map. For each xy assign the label $< x,y >$, where $< x,y >$ denotes the inner product of x and y. We say that φ is a vector basis S-cordial labeling if $|\varphi_x - \varphi_y| \leq 1$ and $|\gamma_i - \gamma_j| \leq 1$ where φ_x denotes the number of vertices labeled with the vector x and γ_i denotes the number of edges labeled with the scalar i. A graph with a vector basis S-cordial labeling is called a vector basis S-cordial graph. In this paper, we investigate the vector basis S-cordial labeling of certain graphs like friendship graph, fan graph, lilly graph, bistar graph, crown graph and armed crown graph where S = {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)} is a basis in R^4 .

Keywords: Friendship Graph, Fan Graph, Lilly Graph, Bistar Graph, and Crown Graph.

Mathematics Subject Classification (2020) No.: 05C78.

1. Introduction

In this paper, we consider only finite, simple and undirected graph

$G = (V(G), E(G))$ where $V(G)$ and $E(G)$ respectively, denote the vertex set and edge set of G. Note that $p = |V(G)|$ and $q = |E(G)|$ denote the number of vertices and edges of G respectively. The idea of graph labeling was first introduced by Rosa in 1967 [16]. Vertex odd graceful labeling has studied in [5]. Baskar Babujee and Shobana [3] have examined the prime and prime cordial labeling for some special graphs. Radio geometric mean labeling of some star like graphs have investigated in [8]. Parmar [18] proved that for the wheel, fan and friendship graphs are edge vertex prime.

The join $G_1 + G_2$ [6] of two graphs G_1 and G_2 is defined as the graph whose vertex set is $V(G_1) + V(G_2)$ and the edge set consists of these edges which are in G_1 and in G_2 and the edges contained by joining each vertex of G_1 to each vertex of G_2. The fan graph F_n [18] is a graph that is constructed by joining all vertices of a path P_n to a further vertex, called center. That is, $F_n = K_1 + P_n$. Amutha and Uma Devi [1] have explored the super graceful labeling for some families of fan graphs. Barasara [2] proved that the comb is an edge and total edge product cordial. For a dynamic survey on graph labeling, we refer to Gallian [6].

The friendship graph $C_3(n)$ [6] can be constructed by joining n copies of the cycle graph C_3 with a common vertex, which becomes a universal vertex for the graph. The corona $G_1 \odot G_2$ [6] of two graphs G_1 and G_2 is obtained by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 and joining each vertex of the i^{th} copy of G_2 to the i^{th} vertex of G_1.

The concept of cordial labeling was first introduced by I. Cahit [4]. Mitra and Bhoumik [11] have introduced the tribonacci cordial labeling of graphs. Parthiban and Sharma proved that the Lilly graph is a prime cordial graph in [13]. The Lilly graph $I_n, n \geq 2$ [13] can be constructed by two star graphs $2K_{1,n}, n \geq 2$ joining two paths $2P_n, n \geq 2$ with sharing a common vertex. That is, $I_n = 2K_{1,n} + 2P_n$. For the terminologies and different notations of graph theory, we refer the book of Harary [7] and of algebra; we refer the book of Herstein [9]. Sum divisor cordial labeling of theta graph was examined by Sugumaran and Rajesh in [17]. Difference cordial labeling for plus and hanging pyramid graphs have studied in [12]. Prajapati and A. Vantiya have proved that the triangular snake, double triangular snake, quadrilateral snake, double quadrilateral snake are SD-prime cordial in [14]. Kaneria

et al. [10] have investigated the balanced mean cordial labeling and graph operations.

We have introduced new labeling called vector basis *S*-cordial labeling in [15] and investigated the vector basis vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling behavior of some standard graphs like path, cycle, comb, star and complete graph. In this paper, we investigate the vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling of certain graphs like friendship graph, fan graph, lilly graph, bistar graph, crown graph and armed crown graph.

2. Vector basis *S*-cordial labeling

Definition 2.1: Let G be a (p,q) graph. Let V be an inner product space with basis S. Let $\varphi : V(G) \to S$ be a map. For each xy assign the label $< x,y >$, where $< x,y >$ denotes the inner product of x and y. We say that φ is a vector basis S-cordial labeling if $| \varphi_x - \varphi_y | \leq 1$ and $| \gamma_i - \gamma_j | \leq 1$ where φ_x denotes the number of vertices labeled with the vector x and γ_i denotes the number of edges labeled with the scalar i. A graph with a vector basis S-cordial labeling is called a vector basis S- cordial graph.

Theorem 2.2: [15] The set S = {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)} is a basis for R^4 over R.

Theorem 2.3: [15] A graph G is vector basis {(1,0),(0,1)}-cordial if and only if G is a cordial graph.

Theorem 2.4: [15] The path P_n is a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial graph for all $n \geq 1$.

Theorem 2.5: [15] The cycle C_n is a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial if and only if $n \equiv 1,2,3 \pmod 4$.

In this paper, we consider the inner product space Rn and the standard inner product $< x,y > = x_1y_1 + x_2y_2 + \cdots + x_ny_n$ where $x = (x_1, x_2, \ldots, x_n)$, $y = (y_1, y_2, \ldots, y_n)$, $x_i, y_i \in R$.

3. Main Results

In this section, we consider the basis S = {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}.

Theorem 3.1: The friendship graph $C_3(n)$ is a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial if and only if $n \equiv 0,1,2 \pmod 4$.

Proof: The friendship graph $C_3(n)$ is a planar, undirected graph with $2n+1$ vertices and $3n$ edges. Let $V(C_3(n)) = \{u, u_i \mid 1 \le i \le 2n\}$ and $E(C_3(n)) = \{uu_{2i-1}, uu_{2i}, u_{2i-1}u_{2i} \mid 1 \le i \le n\}$ respectively be the vertex and edge sets of $C_3(n)$.Then $|V(C_3(n))| = p = 2n+1$ and $|E(C_3(n))| = q = 3n$. There are four case arises.

Case (i): $n \equiv 0 \pmod 4$

Let $n = 4k$. Then, $p = 2n+1 = 8k+1$. Next, we assign the vector (1,1,1,1) to the vertex u. Assign the vector (1,1,1,1) to the vertices $u_1, u_2, \ldots, u_{2k}$. We assign the vector (1,1,1,0) to the next vertices $u_{2k+1}, u_{2k+2}, \ldots, u_{4k}$. Then assign the vector (1,1,0,0) to the next vertices $u_{4k+1}, u_{4k+2}, \ldots, u_{6k}$. Also assign the vector (1,0,0,0) to the remaining vertices $u_{6k+1}, u_{6k+2}, \ldots, u_{8k}$.

Case (ii): $n \equiv 1 \pmod 4$

Let $n = 4k+1$. Then, $p = 8k+3$. Now, we assign the vector (1,1,1,1) to the vertex u. So assign the vector (1,1,1,1) to the vertices $u_1, u_2, \ldots, u_{2k}$. Assign the vector (1,1,1,0) to the next vertices $u_{2k+1}, u_{2k+2}, \ldots, u_{4k}$. We assign the vector (1,1,0,0) to the next vertices $u_{4k+1}, u_{4k+2}, \ldots, u_{6k}$. Further, assign the vector (1,1,1,0) to the vertex u_{6k+1}. Assign the vector (1,0,0,0) to the vertex u_{6k+2}. Then assign the vector (1,1,0,0) to the vertex u_{6k+3}. Finally, assign the vector (1,0,0,0) to the remaining $2k-1$ vertices $u_{6k+4}, u_{6k+5}, \ldots, u_{8k+3}$.

Case (iii): $n \equiv 2 \pmod 4$

Let $n = 4k+2$. Then, $p = 8k+5$. Also, we assign the vector (1,1,1,1) to the vertex u. Assign the vector (1,1,1,1) to the vertices $u_1, u_2, \ldots, u_{2k+1}$. We assign the vector (1,1,1,0) to the next vertices u_{2k+2}, $u_{2k+3}, \ldots, u_{4k+2}$. Then assign the vector (1,1,0,0) to the vertices $u_{4k+3}, u_{4k+4}, \ldots, u_{6k+3}$. Moreover, assign the vector (1,0,0,0) to the remaining $2k$ vertices $u_{6k+4}, u_{6k+5}, \ldots, u_{8k+4}$.

Case (iv): $n \equiv 3 \ (mod\ 4)$

Let $n = 4k+3$. Then $p = 8k+7$ and $q = 12k+9$. If we assign vector (1,1,1,1) to the vertex u and we have to assign (1,1,1,1) to the $2k+1$ vertices, then $\gamma_4 = 2k + 1 + \frac{2k+1}{2} < 2k+1+k+1 < 3k+2$, a contradiction. But $\gamma_4 = 3k+1$ or $\gamma_4 = k+1$ according as the vertex u receive the vector (1,1,1,1) or not. This is a contradiction since the size of $C_3(n)$ is $12k+9$.

Clearly the above labeling pattern provides a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling for the friendship graph $C_3(n)$ if $n \equiv 0,1,2 \pmod 4$.

Theorem 3.2: The fan graph F_n is a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial if and only if $n \equiv 0 \pmod 4$.

Proof: Let $V(F_n) = \{u, u_i \mid 1 \le i \le n\}$ and $E(F_n) = \{uu_i \mid 1 \le i \le n\} \bigcup \{u_i u_{i+1} \mid 1 \le i \le n-1\}$ respectively be the vertex and edge sets of F_n. Then $\left|V(F_n)\right| = p = n+1$ and $E\left|(F_n)\right| = q = 2n-1$. There are four cases arises.

Case (i): $n \equiv 0 \pmod 4$

Let $n = 4k$. Then, $p = 4k+1$. Next, we assign the vector (1,1,1,1) to the vertex u. Assign the vector (1,1,1,1) to the first k vertices $u_1, u_2, \ldots, u_k$. Then, assign

the vector (1,1,1,0) to the next k vertices $u_{k+1}, u_{k+2}, \ldots, u_{2k}$. Also, assign the vector (1,1,0,0) to the next k vertices $u_{2k+1}, u_{2k+2}, \ldots, u_{3k}$. Moreover, assign the vector (1,0,0,0) to the remaining k vertices $u_{3k+1}, u_{3k+2}, \ldots, u_{4k}$.

Case (ii): $n \equiv 1 \pmod 4$

Let $n = 4k+1$. Then $p = n+1 = 4k+2 = (k+1)+(k+1)+k+k$ and $q = 2n - 1 = 8k+1 = (2k+1)+2k+2k+2k$. Clearly, $\gamma_4 = 2k-1$ or $\gamma_4 = k$ according as the vertex u receive the vector (1,1,1,1) or not. This is a contradiction since the size of F_n is $8k+1$.

Case (iii): $n \equiv 2 \pmod 4$

Let $n = 4k+2$. Then $p = 4k+3 = (k+1)+(k+1)+(k+1)+k$ and $q = 8k+3 = (2k+1)+(2k+1)+(2k+1)+2k$. Thus, $\gamma_4 = 2k-1$ or $\gamma_4 = k$ according as the vertex u receive the vector (1,1,1,1) or not. We get a contradiction since the size of F_n is $8k+3$.

Case (iv): $n \equiv 3 \pmod 4$

Let $n = 4k+3$. Then $p = 4k+4 = (k+1)+(k+1)+(k+1)+(k+1)$ and $q = 8k+5 = (2k+2)+(2k+1)+(2k+1)+(2k+1)$. Hence, $\gamma_4 = 2k-1$ or $\gamma_4 = k$ according as the vertex u receive the vector (1,1,1,1) or not. We get a contradiction since the size of F_n is $8k+5$.

Clearly the above labeling method provides a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling for the fan graph F_n if $n \equiv 0 \pmod 4$.

Example 3.3: The following Figure 1 illustrates the vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling fan graph F_4.

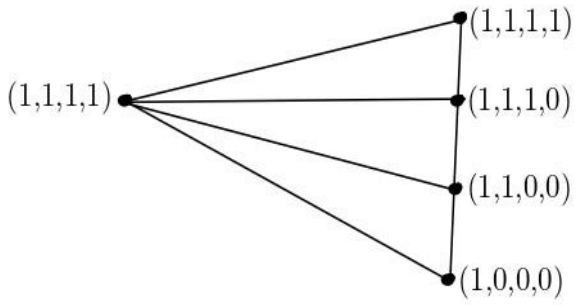


Figure 1
Vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling of F_4.

Theorem 3.4: The Lilly graph I_n is a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial graph for all $n \geq 2$.

Proof: Consider the Lilly graph I_n, $n \geq 2$. Let $V(I_n) = \{u, u_i, v_i \mid 1 \leq i \leq n-1\} \cup \{x_i, y_i \mid 1 \leq i \leq n\}$ and $E(I_n) = \{ux_i, uy_i \mid 1 \leq i \leq n\} \cup \{uv_1, uu_1, u_iu_{i+1},, v_iv_{i+1} \mid 1 \leq i \leq n-2\}$ respectively be the vertex and edge sets of I_n. Then $|V(I_n)| = p = 4n-1$ and $|E(I_n)| = q = 4n-2$.

First, we assign the vector (1,1,1,1) to the vertex u. Assign the vector (1,1,1,1) to the vertices $u_1, u_2, \ldots, u_{n-1}$. Then, assign the vector (1,1,1,0) to the vertices $v_1, v_2, \ldots, v_{n-1}$. Also, assign the vector (1,1,0,0) to the vertices $x_1, x_2, \ldots, x_n$. Moreover, assign the vector (1,0,0,0) to the vertices $y_1, y_2, \ldots, y_n$.

Hence, the above labeling technique provides a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}- cordial labeling for the Lilly graph I_n.

Example 3.5: The following Figure 2 illustrates the vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling of Lilly graph I_4.

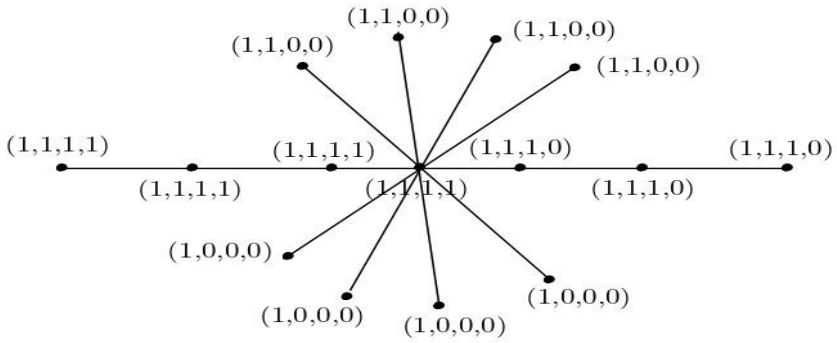


Figure 2
Vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling of I_4.

Theorem 3.6: The crown graph $C_n \odot K_1$ is a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial if and only if n is odd.

Proof: Consider the crown graph $C_n \odot K_1$. Let C_n be the cycle $u_1u_2 \ldots u_nu_1$. Let $V(C_n \odot K_1) = V(C_n) \cup \{v_i \mid 1 \le i \le n\}$ and $E(C_n \odot K_1) = E(C) \cup \{u_iv_i \mid 1 \le i \le n\}$ respectively be the vertex and edge sets of $C_n \odot K_1$. Then $| V(C_n \odot K_1) | = p = 2n$ and $| E(C_n \odot K_1) | = q = 2n$. We have considered the two cases.

Case (i): $p \equiv 0 \ (mod\ 4)$

Let $p = 4k$. To get the edge label 4, the vector (1,1,1,1) should be assigned to the consecutive vertices of $C_n \odot K_1$. As the size of $C_n \odot K_1$ is $2n = p = 4k$, the maximum edges with label 4 is $k - 1$, a contradiction arises.

Case (ii): $p \equiv 2 \ (mod\ 4)$

Let $p = 4k + 2$. Then, assign the vector in the following order $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n$. We assign the vector (1,1,1,1) to the first $k + 1$ vertices $u_1, u_2, \ldots, u_{k+1}$. Also, assign the vector (1,1,1,0) to the next k vertices

$u_{k+2}, u_{k+3}, \ldots, u_n$. We assign the vector (1,1,0,0) to the $k+1$ vertices $v_1, v_2, \ldots, v_{k+1}$. Moreover, assign the vector (1,0,0,0) to the next k vertices $v_{k+2}, v_{k+3}, \cdots, v_n$.

Thus, the above labeling technique provides a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling for the crown graph $C_n \odot K_1$.

Theorem 3.7: The armed crown graph AC_n is a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial if and only if $n \equiv 1,2,3 \pmod 4$.

Proof: The armed crown graph AC_n is the graph obtained from the cycle $u_1u_2 \ldots u_nu_1$ with $V(ACn) = V(C_n) \cup \{v_i, w_i \mid 1 \le i \le n\}$ and $E(AC_n) + E(C_n) \cup \{u_iv_i, v_iw_i \mid 1 \le i \le n\}$ respectively be the vertex and edge sets of AC_n. Then $\left|V(AC_n)\right| = p = 3n$ and $\left|E(AC_n)\right| = q = 3n$. We have considered the four cases.

Case (i): $p \equiv 0 \pmod 4$

Let $p = 4k$. To get the edge label 4, the vector (1,1,1,1) should be assigned to the consecutive vertices of the graph. AC_n As the size of AC_n is $3n = p = 4k$, the maximum edges with label 4 is $k-1$, a contradiction.

Case (ii): $p \equiv 1 \pmod 4$

Let $p = 4k+1$. Then, assign the vector in the following order $u_1, u_2, \ldots u_n$, $v_1, w_1, v_2, w_2, \ldots v_n, w_n$. We assign the vector (1,1,1,1) to the first $k+1$ vertices. Next, assign the vector (1,1,1,0) to the next k vertices. Also assign the vector (1,1,0,0) to the next k vertices. Further, assign the vector (1,0,0,0) to the remaining k vertices.

Case (iii): $p \equiv 2 \pmod 4$

Let $p = 4k+2$. Then, assign the vector in the following order $u_1, u_2, \ldots u_n$,

$v_1, w_1, v_2, w_2, \ldots v_n, w_n$. Now, assign the vector (1,1,1,1) to the first $k+1$ vertices. So assign the vector (1,1,1,0) to the next k vertices. Next, assign the vector (1,1,0,0) to the next $k+1$ vertices. Finally, assign the vector (1,0,0,0) to the remaining k vertices.

Case (iv): $p \equiv 3 \pmod 4$

Let $p = 4k+3$. Then, assign the vector in the following order $u_1, u_2, \ldots u_n$, $v_1, w_1, v_2, w_2, \ldots v_n, w_n$. Also, assign the vector (1,1,1,1) to the first $k+1$ vertices. Assign the vector (1,1,1,0) to the next k vertices. Then, assign the vector (1,1,0,0) to the next $k+1$ vertices. Moreover, assign the vector (1,0,0,0) to the remaining $k+1$ vertices.

Therefore, the above labeling method provides a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling for the armed crown graph $A C_n$.

Theorem 3.8: The bistar graph $B_{n,n}$ is a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial graph for all n.

Proof: Let $V(B_{n,n}) = \{u, u, u_i, v_i \mid 1 \le i \le n\}$ and $E(B_{n,n}) = \{uv, uu_i, vv_i \mid 1 \le i \le n\}$ respectively be the vertex and edge sets of $B_{n,n}$. Note that $\left|V(Bn,n)\right| = p = 2n+2$ and $\left|E(B_{n,n})\right| = q = 2n+1$. We have considered the two cases.

Case (i): $p \equiv 0 \pmod 4$

Let $p = 4k$. Next, we assign the vector (1,1,1,1) to the vertices u and v. Assign the vector (1,1,1,1) to the vertices $u_1, u_2, \ldots, u_{k-2}$. Then, assign the vector (1,1,1,0) to the vertices $u_{k-1}, u_k, \ldots, u_{2k-2}$. Also, assign the vector (1,1,0,0) to the next k vertices $v_1, v_2, \ldots, v_k$. Moreover, assign the vector (1,0,0,0) to the remaining k vertices.

Case (ii): $p \equiv 2 \pmod 4$

Let $p = 4k + 2$. Now, we assign the vector (1,1,1,1) to the vertices u and v. Assign the vector (1,1,1,1) to the vertices $u_1, u_2, \ldots, u_{k-1}$. Next, assign the vector (1,1,1,0) to the vertices $u_k, u_{k+1}, \ldots, u_{2k-1}$. So, assign the vector (1,1,0,0) to the next k vertices $v_1, v_2, \ldots, v_k$. Further, assign the vector (1,0,0,0) to the remaining vertices.

Clearly the above labeling method provides a vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling for the bistar graph $B_{n,n}$.

4. Conclusion

Vector basis {(1,1,1,1), (1,1,1,0), (1,1,0,0), (1,0,0,0)}-cordial labeling behavior of certain standard graphs like friendship graph, fan graph, lilly graph, bistar graph, crown graph and armed crown graph have been investigated in this paper. The investigation of different kinds of families of graphs for existence of vector basis S-cordial labeling is an open problem.

REFERENCES

[1] S. Amutha and M. Uma Devi (2019): Super graceful labeling for some families of fan graphs, *Journal Comput. Math. Sci*., Vol. 10(8), pp. 1551-1562.

[2] C. M. Barasara (2018): Edge and total edge product cordial labeling of some new graphs, *Internat. Engin., Sci. Math*., Vol. 7(2), pp. 263-273.

[3] J. Baskar Babujee and L. Shobana (2010): Prime and prime cordial labeling for some special graphs, *Int. J. Contemp. Math. Sciences*, Vol. 5, pp. 2347-2356.

[4] I. Cahit (1987): Cordial Graphs: A weaker version of Graceful and Harmonious Graphs, *Ars Comb*., Vol. 23(3), pp. 201-207.

[5] S. N. Daoud (2019): Vertex odd graceful labeling, *Ars Combin*., Vol. 142, pp. 65-87.

[6] A. Gallian (2024): A Dynamic Survey of Graph Labeling, *Electronic Journal of Combinatorics*, Vol. 27, pp. 1-712.

[7] F. Harary (1988): Graph Theory, Narosa Publishing House, New Delhi.

[8] V. Hemalatha, V. Mohanaselvi, and K. Amuthavalli (2017): Radio geometric mean labeling of some star like graphs, *J. Informatics Math. Sci*., 9(3), pp. 969-977.

[9] I. N. Herstein (1991): Topics in Algebra, John Wiley & Sons, United States of America.

[10] V. J. Kaneria, M J Khoda, and H M Karavadiya (2016): Balanced mean cordial labeling and graph operations, *Int. J. Math. Appl*., Vol. 4(3-A), pp. 181-184.

[11] S. Mitra and S. Bhoumik (2022): Tribonacci cordial labeling of graphs, *Journal of Applied Mathematics and Physics*, Vol. 10(4), pp. 1394-1402.

[12] V. Mohan and A. Sugumaran (2019): Difference cordial labeling for plus and hanging pyramid graphs, *Internat. J. Res. Analytical Reviews*, Vol. 6(1), pp. 1106-1114.

[13] A. Parthiban and Vishally Sharma (2021): Some results on prime cordial labeling of Lilly graphs, *Journal of Physics*: *Conference Series*, 1831(012035), pp. 1-10.

[14] U. M. Prajapati and A. V. Vantiya (2019): SD-prime cordial labeling of some snake graphs, *JASC: J. Appl. Sci. Comput*., Vol. VI(IV), pp. 1857-1868.

[15] R. Ponraj and R. Jeya: Vector basis S-cordial labeling of graphs (Submitted by the Journal)

[16] A. Rosa (1967): On certain valuations of the vertices of a graph, Theory of Graphs, (*Internat. Symposium*, Rome, July 1966), Gordon and Breach, N. Y. and Dunod Paris, pp. 349-355.

[17] A. Sugumaran and K. Rajesh (2017): Sum divisor cordial labeling of theta graph, *Annals Pure Appl. Math.*, Vol. 14(2), pp. 313-320.

[18] Y. M. Parmar (2017): Edge vertex prime labeling for wheel, fan and friendship graph, *International Journal of Mathematics and Statistics Invention*, Vol. 5(8), pp. 23-29.

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