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A COMPARATIVE ANALYSIS OF SELJE TOPOLOGICAL SPACE WITH OTHER TOPOLOGICAL SPACES

Abstract: In recent years, numerous topologies have emerged, including the newly discovered Selje topology, which builds on micro and nano topologies. This paper offers a comparative analysis of Selje topology, emphasizing its real-world applications, particularly in analyzing dynamic systems such as climate change. The fundamental principles that link Selje, Micro and Nano topologies are discussed. The analysis demonstrates that Selje topology provides a more refined and flexible framework, allowing for greater precision in understanding complex, multifactorial systems. Key findings highlight Selje's ability to handle intricate interdependencies and scalability challenges more effectively than nano and micro topologies, making it especially valuable for studying large datasets and highly interconnected systems.

Keywords: Selje Topological Space, Micro Topology, Nano Topology, Scalability, Precision, Inclusion.

Mathematics Subject Classification: 54A05, 54B05.

1. Introduction

Topology, a branch of mathematics focused on studying properties of space that remain invariant under continuous deformations, has evolved significantly with the development of specialized topological structures. These structures have become essential in analyzing complex, multifactorial systems across diverse fields, such as engineering, medical sciences and, more recently, climate change analysis. Among

the notable advancements in topological spaces are nano topology, micro topology and the newly introduced Selje topology. Each of these topologies offers unique frameworks for examining spatial relationships, continuity and the interaction of critical variables within complex systems.

Nano topology [12] introduced by Lellis Thivagar in 2013, relies on lower and upper approximations, providing a binary classification system that identifies whether elements belong to a critical or non-critical set. This straightforward structure excels in isolating key spatial elements in relatively simple systems. However, nano topology struggles with more complex and interdependent systems, as it cannot fully capture the wide range of possible relationships between elements. This limitation becomes especially pronounced in systems where variables interact dynamically and change over time.

To overcome these limitations, micro topology [10] was developed by Sakkraiveeranan in 2019. Micro topology builds on the framework of nano topology by incorporating Levine's generalized closed sets, which allow for more flexible and detailed approximations. This extension provides a deeper exploration of open and closed sets, making micro topology better suited for dynamic systems with greater complexity. While this approach offers a more refined understanding of spatial relationships, it still encounters difficulties when handling highly multifactorial systems with overlapping interdependent variables.

In 2023, Selje topology [5] introduced by Jeyanthi and Selva Nandhini, emerged as a further refinement of nano and micro topologies. It was developed to address the challenges posed by complex systems where multiple variables interact in intricate ways. Selje topology builds on the strengths of its predecessors, incorporating Selje-open and Selje-closed sets that provide even finer approximations of spatial elements. This enhanced framework allows for better handling of set intersections and scalability, making it particularly effective in studying systems that involve intricate dependencies and relationships among multiple variables.

While climate change analysis represents a key application of Selje topology, its usefulness extends beyond this field. The topology's ability to manage multifactorial systems makes it suitable for other domains as well, including biological systems where gene interactions and cellular processes are interdependent. Similarly, in network analysis, Selje topology can provide insights into the intricate relationships within social or communication networks, where multiple layers of connection and influence must be considered. By offering a more refined and adaptable approach to spatial relationships, Selje topology demonstrates significant potential for analyzing dynamic, interconnected systems across various disciplines.

This paper presents a comparative analysis of nano, micro and Selje topologies, focusing on their respective strengths and limitations in addressing the complexities of climate change. By examining the foundational theorems of each topology and applying them to climate change impact analysis, this study aims to show how Selje topology offers a deeper, more flexible understanding of multifactorial processes. The analysis highlights the critical role of topological methods in detecting and analyzing the intricate patterns and relationships that define dynamic systems, emphasizing the practical utility of these frameworks in contemporary scientific research.

Preliminaries

Definition 2.1: *Let $\mathfrak{V}$ denote a non-empty finite set of objects referred to as the universe and let $\mathcal{R}$ represent an equivalence relation on $\mathfrak{V}$ known as the indiscernibility relation. Elements within the same equivalence class are considered indiscernible from each other. This pair, denoted as $(\mathfrak{V}, \mathcal{R})$, constitutes the approximation space.*

Let $\mathfrak{E}$ be a subset of $\mathfrak{V}$.

1. *The lower approximation of $\mathfrak{E}$ with respect to $\mathcal{R}$, denoted as $\mathcal{L}_{\mathcal{R}}(\mathfrak{E})$, consists of all objects that can definitively be classified as belonging to $\mathfrak{E}$ under the influence of $\mathcal{R}$. In mathematical terms, $\mathcal{L}_{\mathcal{R}}(\mathfrak{E}) = \cap\{\mathcal{R}(\mathcal{X}) : \mathcal{R}(\mathcal{X}) \subseteq \mathfrak{E}\}$ where $\mathcal{R}$ signifies the equivalence class determined by $\mathfrak{E}$.*

2. *The upper approximation of $\mathfrak{E}$ with respect to $\mathcal{R}$, denoted as $\mathcal{U}_{\mathcal{R}}(\mathfrak{E})$, comprises all objects that could potentially be classified as $\mathfrak{E}$ under the influence of $\mathcal{R}$. Mathematically, $\mathcal{U}_{\mathcal{R}}(\mathfrak{E}) = \cap\{\mathcal{R}(\mathcal{X}) : \mathcal{R}(\mathcal{X}) \cap \mathfrak{E} \neq \Phi\}$*

3. *The boundary region of $\mathfrak{E}$ with respect to $\mathcal{R}$, denoted as $\mathcal{B}_{\mathcal{R}}(\mathfrak{E})$, includes all objects that cannot be definitively classified as either belonging to $\mathfrak{E}$ or not belonging to $\mathfrak{E}$ under the influence of $\mathcal{R}$. In mathematical terms, $\mathcal{B}_{\mathcal{R}}(\mathfrak{E}) = \mathcal{U}_{\mathcal{R}}(\mathfrak{E}) - \mathcal{L}_{\mathcal{R}}(\mathfrak{E})$*

Definition 2.2: *Let* $\mathfrak{V}$ *represent the universe,an equivalence relation on* $\mathfrak{V}$ *denote* $\mathcal{R}$ *and* $T_{\mathcal{R}}(\mathfrak{E}) = \{\mathfrak{E}, \Phi, \mathcal{L}_{\mathcal{R}}(\mathfrak{E}), \mathcal{U}_{\mathcal{R}}(\mathfrak{E}), \mathcal{B}_{\mathcal{R}}(\mathfrak{E})\}$, *where* $\mathfrak{E} \subseteq \mathfrak{V}$. *Under these conditions,* $\mathcal{R}(\mathcal{X})$ *Proceeding with the given postulates*:

1. Φ *and* $\mathfrak{V}$ *belong to* $\mathcal{T}_{\mathcal{R}}(\mathfrak{E})$.

2. *Any subset of the union of elements* $\mathcal{T}_{\mathcal{R}}(\mathfrak{E})$ *remains within* $\mathcal{T}_{\mathcal{R}}(\mathfrak{E})$.

3. *Any finite subset of the intersection of elements* $\mathcal{T}_{\mathcal{R}}(\mathfrak{E})$ *is contained in* $\mathcal{T}_{\mathcal{R}}(\mathfrak{E})$. *In other words,* $\mathcal{T}_{\mathcal{R}}(\mathfrak{E})$ *forms a topology on* $\mathfrak{V}$ *known as the nano topology on* $\mathfrak{V}$ *concerning* $\mathfrak{E}$. $(\mathfrak{V}, \mathcal{T}_{\mathcal{R}}(\mathfrak{E}))$ *constitutes the nano topological space. The sets within* $\mathcal{T}_{\mathcal{R}}(\mathfrak{E})$ *are denoted as nano open sets and the dual nano topology of* $[\mathcal{T}_{\mathcal{R}}(\mathfrak{E})]$ *is represented by* $[\mathcal{T}_{\mathcal{R}}(\mathfrak{E})]^c$.

In this context, $\mathfrak{T}_{\mathcal{R}}(\mathfrak{E})$ *is termed the Nano Topology* [5] *of the universal set* $\mathfrak{V}$ *with respect to the subset* $\mathfrak{E}$. *The pair* $(\mathfrak{V}, \mathfrak{T}_{\mathcal{R}}(\mathfrak{E}))$ *constitutes a nano topological space and its constituent elements are referred to as nano-open sets.*

Definition 2.3: $(\mathfrak{V}, \mathfrak{T}_{\mathcal{R}}(\mathfrak{E}))$ *creates a nanotopological space. In this case, the set* $\mathfrak{Y}\mathcal{R}(\mathfrak{E})$ *consists of two groups, namely* $\{\mathfrak{N} \cup (\mathfrak{N}' \cap \mathfrak{Y}) : \mathfrak{N}, \mathfrak{N}' \in \mathfrak{T}(\mathfrak{E})\}$. *The combination* $\mathfrak{T}_{\mathcal{R}}(\mathfrak{E})$ *is expressed as the microtopology* $\mathfrak{Y}$; *where* $\mathfrak{Y}$ *is not nanotopology elements of* $\mathfrak{T}_{\mathcal{R}}(\mathfrak{E})$.

Definition 2.4: *Micro Topology* $\mathfrak{Y}\mathcal{R}(\mathfrak{E})$ *adheres to the following postulates*:

1. *Both the universal set* $(\mathfrak{E})$ *and the empty set* Φ *are elements of* $\mu_{\mathcal{R}}(\mathfrak{E})$.

2. *Any subset of the union of elements of* $\mu_{\mathcal{R}}(\mathfrak{E})$ *remains within* $\mu_{\mathcal{R}}(\mathfrak{E})$.

3. *Any finite subset of the intersection of elements of* $\mu_{\mathcal{R}}(\mathfrak{E})$ *is contained within* $\mu_{\mathcal{R}}(\mathfrak{E})$. *Thus, the Micro topology* $\mu_{\mathcal{R}}(\mathfrak{E})$ *is defined as* $\mu_{\mathcal{R}}(\mathfrak{E}) = \{\mathfrak{N} \cup (\mathfrak{N}' \cap \mu)\}$ *for* $\mathfrak{N}$ *and* $\mathfrak{N}' \in \mu_{\mathcal{R}}(\mathfrak{E})$, *where* $\mu \notin \mathfrak{T}_{\mathcal{R}}(\mathfrak{E})$. *This constitutes the Micro topology on the set* $\mathfrak{V}$ *concerning* $\mathfrak{E}$.

The trio $(\mathfrak{V}, \mathfrak{T}_{\mathcal{R}}(\mathfrak{E}),\ \mu_{\mathcal{R}}(\mathfrak{E}))$ *is denoted as the Micro topological space and the elements of* $\mu_{\mathcal{R}}(\mathfrak{E})$ *are known as Mic-open sets. Moreover, the complement of a Mic-open set is defined as a Mic-closed set.*

Next, $\mathfrak{Y}_{\mathcal{R}}(\mathfrak{E})$ *is called the microtopology of* $\mathfrak{E}$ *and* $\mathfrak{V}$. *Triple* $(\mathfrak{E}, \mathfrak{T}_{\mathcal{R}}(\mathfrak{E}), \mathfrak{Y}_{\mathcal{R}}(\mathfrak{E}))$ *called micro-topological space. Elements in* $\mathfrak{Y}_{\mathcal{R}}(\mathfrak{E})$ *are slightly open and their complements are slightly off.*

Definition 2.5: *Consider the microtopological space* $(\mathfrak{V}, \mathfrak{Y}_{\mathcal{R}}(\mathfrak{E}))$ *and Selje topology be defined as* $SJ_{\mathcal{R}}(\mathfrak{E}) = \{(S - \boldsymbol{J}) \cup (S - J') : S \in \mathfrak{Y}_{\mathcal{R}}(\mathfrak{E})$ *and for fixed* $\boldsymbol{J}, J' \notin \mathfrak{Y}_{\mathcal{R}}(\mathfrak{E}), \boldsymbol{J} \cup J' = \mathfrak{V}\}$

Definition 2.6: *The Selje topology* $SJ_{\mathcal{R}}(\mathfrak{E})$ *satisfies the following axioms*

1. *Both the universal set* $\mathfrak{V}$ *and the empty set* Φ *are elements of* $\mathfrak{T}_{\mathcal{R}}(\mathfrak{E})$.

2. *Any subset of the union of elements from* $SJ_{\mathcal{R}}(\mathfrak{E})$ *remains within* $SJ_{\mathcal{R}}(\mathfrak{E})$.

3. *Any finite subset of the intersection of elements within* $SJ_{\mathcal{R}}(\mathfrak{E})$ *is contained within* $SJ_{\mathcal{R}}(\mathfrak{E})$.

The triplet $(\mathfrak{E}, \mathfrak{Y}_{\mathcal{R}}(\mathfrak{E}), SJ_{\mathcal{R}}(\mathfrak{E}))$ *is labeled as Selje topological space. Then, the components of Selje topology are Selje-Open* ($\boldsymbol{SJ}$*-Open*) *sets and*

their complements are Selje-closed (SJ *-closed*) *sets. The collection of Selje closed sets of Selje topology is denoted as* $SJCL(\mathfrak{E})$.

3. Theoretical Foundations and Comparative Analysis of Nano, Micro and Selje Topologies

The theorems compare nano, micro and Selje Topological Spaces, showing that Selje Topology offers finer approximations, better scalability for complex systems and generalizes the other two. They demonstrate why Selje Topological Space is superior for handling complex, multifactorial applications with improved precision and flexibility.

Theorem 3.1 establishes a hierarchical relationship between nano, micro and Selje Topological Spaces, showing that Selje topological space provides the most refined approximations, followed by micro and nano topologies. The inclusions between closures and interiors reflect the increasing precision of each space.

Theorem 3.1: Inclusion in Nano, Micro and Selje Topologies: *Let* $X \subseteq U$ *be a subset in the universe U. The relationships between the approximations in nano, micro and Selje topologies are given by*:

$$L_R(X) \subseteq Mic-cl(X) \subseteq SJ_R-cl(X)$$

and

$$SJ_R-\text{int}(X) \subseteq Mic-\text{int}(X) \subseteq U_R(X)$$

where $LR(X)$ *and* $UR(X)$ *are the lower and upper approximations in nano topology,* $Mic-cl(X)$ *and* $Mic-\text{int}(X)$ *are the micro closure and interior in micro topology and* $SJ_{\mathcal{R}}-cl(X)$ *and* $SJ_{\mathcal{R}}-\text{int}(X)$ *are the Selje closure and interior, respectively.*

Proof: In nano topology, $L_{\mathcal{R}}(X) \subseteq X \subseteq U_{\mathcal{R}}(X)$.

In micro topology, $L_{\mathcal{R}}(X) \subseteq Mic-cl(X)$ and $Mic-\text{int}(X) \subseteq UR(X)$.

In Selje topology, $Mic-cl(X) \subseteq SJ_{\mathcal{R}}-cl(X)$

and $SJ_{\mathcal{R}}-\text{int}(X) \subseteq Mic-\text{int}(X)$.

Thus, the theorem follows. □

Lemma 3.2 states that if a function is continuous in nano topology, it will also be continuous in both micro and Selje topologies. This is because micro and Selje topologies generalize the structures of nano topology, preserving the continuity of functions across these spaces.

Lemma 3.2 Preservation of Continuity in Micro and Selje Topologies: *If* $f: U \to V$ *is continuous in nano topology, then f is continuous in both micro and Selje topological spaces.*

Proof: In nano topology, $f^{-1}(V') \in \tau_R(U)$ for any nano-open set $V' \subseteq V$. In micro topology, since micro-open sets are unions or intersections of nano-open sets, $f^{-1}(W') \in \mu_R(U)$. Similarly, in Selje topology, $f^{-1}(S') \in SJ_R(U)$. Hence, f is continuous in both micro and Selje topologies. □

Theorem 3.3 demonstrates that Selje Topological Space scales better than nano and micro topologies. As system complexity increases, Selje retains higher precision in approximating sets, making it ideal for complex systems.

Scalability here refers to how well the different topologies handle an increase in system complexity. As the complexity of the dataset (e.g., the number of variables, the amount of data) increases, the precision of approximations made by each topology changes.

Theorem 3.3 Scalability of Approximations: *For any subset* $A \subseteq U$, *we have*:

$$\lim_{complexity(A)\to\infty} precision(SJ_R-cl(A)) > precision(Mic-cl(A)) > precision(L_R(A))$$

Proof: 1. In nano topology, $\text{precision}(L_R(A)) = \frac{|L_R(A)|}{|A|}$ tends to 0 as $|A| \to \infty$.

2. In micro topology, precision $(Mic-cl(A)) = \frac{|\, Mic-cl(A)\,|}{|\, A\,|}$, which is more precise than in nano topology.

3. In Selje topology, precision $(SJ-cl(A)) = \frac{|\, SJ_R-cl(A)\,|}{|\, A\,|}$, which remains precise even as complexity increases. □

Theorem 3.4 shows that the intersection of Selje-open sets provides a finer approximation than nano-open or micro-open sets. Selje Topology captures more intricate relationships, making it more powerful for handling complex data.

Theorem 3.4 Finer Set Operations in Selje Topology: *For any subsets* $A, B \subseteq U$ *, the intersection of Selje-open sets provides a finer approximation than the intersection of micro-open or nano-open sets*:

$$SJ_R - int(A \cap B) \subseteq Mic-int(A \cap B) \subseteq L_R(A \cap B)$$

Proof: 1. In nano topology, $L_R(A \cap B) = \{x \in U \mid x \in L_R(A) \cap L_R(B)\}$. 2. In micro topology, $Mic-int(A \cap B) = \{x \in U \mid x \in Mic-int(A) \cap Mic-int(B)\}$. 3. In Selje topology, $SJ_R-int(A \cap B) = \{x \in U \mid x \in SJ_R-int(A) \cap SJ_R-int(B)\}$, thus, providing a finer approximation. □

The below corollary states that Selje Topological Space generalizes both nano and micro topologies, but not all Selje-open sets are nano-open or micro-open, offering a broader and more flexible structure.

Corollary: (Generalization of Nano and Micro Topologies). *Selje Topological Space generalizes both nano and micro topologies. Every nano-open and micro-open set is a Selje-open set, but not every Selje-open set is nano-open or micro-open.*

Proof: By the definition of Selje Topology, $\tau_R(U) \subseteq \mu_R(U) \subseteq SJ_R(U)$,

meaning all nano-open and micro-open sets belong to the Selje Topology. However, Selje-open sets can contain additional elements that nano and micro topologies cannot capture. □

4. Topological Analysis of Climate Change Impact: A Comparative Study Using Nano, Micro and Selje Topologies

This application focuses on differentiating three topological spaces-nano topology, micro topology and Selje Topology-through the lens of climate change impact analysis. Climate change, a multifactorial process, affects various sectors like agriculture, health and the economy, with factors such as temperature rise, rainfall patterns and sea level rise influencing different regions in diverse ways.

By modeling these factors within each topological space, we aim to identify which regions and sectors are most affected. The process involves analyzing key climate-related variables, applying each topological method to assess their significance and comparing the results to determine how each topology captures critical factors. The comparison highlights the strengths of each topology, with special focus on how Selje Topology refines the relationships between variables, offering a more detailed and precise analysis compared to nano and micro topologies. In the end, the betterment of each topological space is analyzed, showing how they differ in precision, scalability and flexibility in identifying the most impactful factors of climate change on different regions.

4.1 Methodology for Topological Analysis of Climate Change Impact: The following structured steps outline the methodology used for applying nano, micro and Selje topologies to analyze climate change impacts:

- ***Data Preparation*:** Collected and standardized climate data, focusing on critical factors such as temperature rise, rainfall patterns, sea level rise, greenhouse gas emissions, deforestation and other socio-economic variables across various regions and sectors. This data was organized to ensure consistency and comparability across different regions.

- **Topological Space Application:** Applied nano, micro and Selje Topological Spaces to the climate data to assess the relationships between the key factors. The topologies were used to study how these factors interact and influence one another in various regions, allowing for the identification of underlying patterns in the data. Special attention was paid to how the different topological spaces handle these relationships, particularly their set approximations and scalability.

· ***Critical Factor Identification*:** Determined the most significant climate-related factors for each region by analyzing the topological spaces. The analysis focused on identifying which variables-such as temperature rise, rainfall variability, or deforestation-had the greatest impact on environmental, economic and health outcomes in specific regions.

· ***Visualization and Analysis*:** Generated diagrams, tables and comparative metrics to visualize the relationships between climate factors and the regions they affect. These visualizations highlight the differences in performance between nano, micro and Selje topologies. (Add visual aids such as graphs comparing factor influence across regions for each topology to show how Selje provides deeper insights.)

· ***Comparison of Topologies*:** Compared the efficiency and flexibility of nano, micro and Selje Topologies in analyzing the climate change impact. This comparison focused on determining which topology provided the most accurate and scalable analysis for multi- factorial climate systems. Results showed that while all three topologies identified key variables, Selje topology allowed for more detailed insights into variable interactions, offering superior scalability and precision in the analysis of complex datasets.

4.2 Topological Analysis of Climate Change Impact: The table below presents the data collected for climate change impact analysis. This data is then processed to compare the performance of nano, micro and Selje Topological Spaces.

Region	Temp-rature (Te)	Rainfall (Ra)	Sea Level (Sl)	GHG Emis-sions (GHG)	Defores-tation(D)	Agri. Prod. (Ap)	Health Impacts (Hi)	Eco-nomic Costs (Ec)	Impact Rate(Ir)
Coastal Regions (Cr)									Medium
Agri cultural Lands (Al)									Medium
Forested Areas (Fa)									High
Urban Areas (Ua)									High

Region	Temp-rature (Te)	Rainfall (Ra)	Sea Level (Sl)	GHG Emis-sions	Defores-tation(D)	Agri. Prod. (Ap)	Health Impacts (Hi)	Eco-nomic Costs	Impact Rate(Ir)
Island Nations (In)									Medium
River Basins (Rb)									High
Energy Sector (Es)									High
Fisheries (Fs)									High
Tourism Indus-try (Ti)									High
Health care Sys-tems (Hs)									Medium

Table 1: Impact of Climate Change on Various Regions and Sectors

Let the set of region be $\mathfrak{E} = \{Cr,Al,F,Ua,In,Rb,Es,Fs,Ti,Hs\}$

and

$$\mathfrak{G} = \{Te,Ra,Sl,GHG,De,Ap,Hi,Ec,Ir\}.$$

It splits into two cases where

$$\mathfrak{H} = \{Te,Ra,Sl,GHG,De,Ap,Hi,Ec\} \text{ and } \mathfrak{I} = \{Ir\}$$

The group of Equivalence types $\mathfrak{V}/\mathfrak{H}$ corresponding to $\mathfrak{H}$ is given by

$$\mathfrak{V}/\mathfrak{H} = \{\{Cr\},\{Al,In\},\{Ua\},\{Fa,Rb,Es\},\{Ti\},\{Fs,Hs\}\},$$

$$\mathfrak{E} = \{Fa,Ua,Rb,Es,Fs,Ti\}$$

Case 1: When Impact level is High

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Fa,Ua,Rb,Es,Ti},{Fs,Hs},{Fa,Ua,Rb,Es,Fs,Ti,Hs}}

$\mu_{\mathcal{R}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Al},{Fa,Ua,Rb,Es,Ti},{Al,Fa,Ua,Rb,Es,Ti},{Fs,Hs},{Al,Fs,Hs},
{Fa,Ua,Rb,Es,Fs,Ti,Hs},{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs}}

$SJ_{\mathfrak{H}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Cr,Fa,In,Es,Ti},{Fa,Es,Ti},{Al,Ua,Rb,Fs,Hs},
{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs},{Al},{Al,Fa,Ti},
{Cr,Al,Fa,In,Es,Ti},{Ua,Rb},{Cr,Fa,Ua,In,Rb,Es,Ti},{Fa,Ua,Rb,Es,Ti},
{Al,Ua,In},{Cr,Al,Fa,Ua,In,Rb,Es,Ti},{Al,Fa,Ua,Rb,Es,Ti},{Fs,Hs},
{Cr,Fa,In,Es,Fs,Ti,Hs},{Fa,Es,Fs,Ti,Hs},
{Al,Fs,Hs},{Cr,Al,Fa,In,Es,Fs,Hs},{Al,Fa,Es,Fs,Ti,Hs},
{Ua,Rb,Fs,Hs},{Cr,Fa,Ua,In,Rb,Es,Ti,Hs},{Fa,Ua,Rb,Es,Fs,Ti,Hs}}

Phase I: Te is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Fa,Ua,Rb,Es},{Al,Fa,Ua,In,Rb,Es,Fs,Ti,Hs},{Al,In,Fs,Ti,Hs}}

$\mu_{\mathcal{R}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Al},{Fa,Ua,Rb,Es},{Al,Fa,Ua,Rb,Es},
{Al,Fa,Ua,In,Rb,Es,Fs,Ti,Hs}}

$SJ_{\mathfrak{H}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Al},{Al,Fa,Es},{Al,Fa,Es,Ti},{Cr,Al,Fa,In,Es,Ti},{Ua,Rb},
{Fa,Ua,Rb,Es},{Fa,Ua,Rb,Es,Ti},{Cr,Fa,Ua,In,Rb,Es,Ti},{Al,Ua,Rb},
{Al,Fa,Ua,Rb,Es},{Fa,Es},{Fa,Es,Ti},{Cr,Fa,In,Es,Ti},
{Al,Ua,Rb,Fs,Hs},{Al,Fa,Ua,Rb,Es,Fs,Hs},{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs},
{Fa,Es},{Fa,Es,Ti},{Al,Fa,Ua,Rb,Es,Ti},{Cr,Al,Fa,Ua,In,Rb,Es,Ti}}

Phase II: Ra is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$, {Fa,Ua,Rb,Es,Ti},{Fa,Ua,Rb,Es,Fs,Ti,Hs},{Fs,Hs}}

$\mu_{\mathcal{R}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Al},}Fa,Ua,Rb,Es,Ti},{Al,Fa,Ua,Rb,Es,Ti},
{Fa,Ua,Rb,Es,Fs,Ti,Hs},{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs},{Fs,Hs},{Al,Fs,Hs}}

$SJ_{\mathfrak{H}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Cr,Fa,In,Es,Ti},{Fa,Es,Ti},{Al,Ua,Rb,Fs,Hs},
{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs},{Al},{Al,Fa,Es,Ti},{Cr,Al,Fa,In,Es,Ti},
{Ua,Rb},{Cr,Fa,Ua,In,Rb,Es,Ti},{Fa,Ua,Rb,Es,Ti},{Al,Ua,Rb},
{Cr,Al,Fa,Ua,In,Rb,Es,Ti},{Al,Fa,Ua,Rb,Es,Ti},{Fs,Hs},
{Cr,Fa,In,Es,Fs,Ti,Hs},{Fa,Es,Fs,Ti,Hs},{Al,Fs,Hs},
{Cr,Al,Fa,In,Es,Fs,Ti,Hs}

Phase III: Sl is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E}) =$ {Φ,𝔙, {Fa,Rb,Es,Ti},{Cr,Fa,Ua,Rb,Es,Fs,Ti,Hs},{Cr,Ua,Fs,Hs}}

$\mu_{\mathcal{R}}(\mathfrak{E}) =$ {Φ,𝔙, {Al},{Fa,Rb,Es,Ti},{Al,Fa,Rb,Es,Ti},{Cr,Fa,Ua,Rb,Es,Fs,Ti,Hs},
{Cr,Al,Fa,Ua,Rb,Es,Fs,Ti,Hs},{Cr,Ua,Fs,Hs},{Cr,Al,Ua,Fs,Hs}}

$\mathcal{SJ}_{\mathfrak{H}}(\mathfrak{E}) =$ {Φ,𝔙,{Al},{Al,Fa},{Al,Fa,Es,Ti},{Cr,Al,Fa,Es,Ti},{Cr,Al},
{Cr,Al,Fa,In,Es,Ti},{Rb},{Fa,Rb},{Fa,Rb,Es,Ti},{Cr,Fa,Rb,Es,Ti},
{Al,Rb},{Al,Fa,Rb},{Al,Fa,Rb,Es,Ti},{Cr,Al,Fa,Rb,Es,Ti},
{Cr,Al,Rb},{Cr,Al,Fa,In,Rb,Es,Ti},{Al,Ua,Rb,Fs,Hs},
{Al,Fa,Ua,Rb,Fs,Hs},{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs},
{Cr,Al,Fa,Ua,Rb,Es,Fs,Ti,Hs},{Ua,Fs,Hs},{Fa,Ua,Fs,Hs},
{Fa,Ua,Es,Fs,Hs},{Cr,Fa,Ua,Es,Fs,Ti,Hs},{Cr,Ua,Fs,Hs},
{Cr,Fa,Ua,In,Es,Fs,Ti,Hs},{Al,Ua,Fs,Hs}{Al,Fa,Ua,Fs,Hs}
{Al,Fa,Ua,Es,Fs,Ti,Hs},{Cr,Al,Fa,Ua,Es,Fs,Ti,Hs},{Cr,Al,Ua,Fs,Hs},
{Cr,Al,Fa,Ua,In,Es,Fs,Ti,Hs},{Fa},{Fa,Es,Ti},{Cr,Fa,Es,Ti},{Cr},
{Cr,Fa,In,Es,Ti},{Cr,Al,Ua,Rb,Fs,Hs},{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs}}

Phase IV: GHG is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E}) =$ {Φ,𝔙,{Fa,Rb,Es,Ti},{Fa,Ua,In,Rb,Es,Fs,Ti,Hs},{Fa,Ua,Rb,Fs,Hs}}

$\mu_{\mathcal{R}}(\mathfrak{E}) =$ {Φ,𝔙,{Al},{Fa,Rb,Es,Ti},{Al,Fa,Rb,Es,Ti},{Fa,Ua,Rb,Es,Fs,Ti,Hs},
{Al,Fa,Rb,Es,Fs,Ti,Hs},{Fa,Ua,Rb,Es,Fs,Ti,Hs},{Al,Fa,Ua,Rb,Es,Fs,Hs}}

$\mathcal{SJ}_{\mathfrak{H}}(\mathfrak{E}) =$ {Φ,𝔙, {Cr,Fa,In,Es,Ti},{Fa,Es,Ti},{Fa,Es},{Al,Ua,Rb,Fs,Hs},
{Al,Fa,Ua,Rb,Es,Fs,Hs},{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs},{Al},
{Cr,Al,Fa,In,Es,Ti},{Al,Fa,Es,Ti},{Al,Fa,Es},{Rb},{Cr,Fa,In,Rb,Es,Ti},
{Fa,Rb,Es,Ti},{Fa,Rb,Es},{Al,Rb},{Cr,Al,Fa,In,Rb,Es,Ti},
{Al,Fa,Rb,Es,Ti},{Al,Fa,Rb,Es},{Ua,Rb,Fs,Hs},
{Cr,Al,Fa,In,Rb,Es,Fs,Ti,Hs},{Al,Fa,Rb,Es,Fs,Ti,Hs},
{Al,Fa,Rb,Es,Fs,Hs}}

Phase V: De is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E}) =$ {Φ,𝔙,{Fa,Ua,Rb,Es,Ti},{Fa,Ua,Rb,Es,Fs,Ti,Hs},{Fs,Hs}}

$\mu_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Al\},\{Fa,Ua,Rb,Es,Ti\},\{Al,Fa,Ua,Rb,Es,Ti\},$
$\{Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Fs,Hs\},\{Al,Fs,Hs\}\}$

$\mathcal{SJ}_{\mathfrak{H}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Cr,Fa,In,Es,Ti\},\{Fa,Es,Ti\},\{Al,Ua,Rb,Fs,Hs\},$
$\{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Al\},\{Al,Fa,Es,Ti\},\{Cr,Al,Fa,In,Es,Ti\},$
$\{Ua,Rb\},\{Cr,Fa,Ua,In,Rb,Es,Ti\},\{Fa,Ua,Rb,Es,Ti\},\{Al,Ua,Rb\},$
$\{Cr,Al,Fa,Ua,In,Rb,Es,Ti\},\{Al,Fa,Ua,Rb,Es,Ti\},\{Fs,Hs\},$
$\{Cr,Fa,In,Es,Fs,Ti,Hs\},\{Fa,Es,Fs,Ti,Hs\},\{Al,Fs,Hs\},$
$\{Cr,Al,Fa,In,Es,Fs,Ti,Hs\}\}$

Phase VI: Ap is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Fa,Ua,Rb,Es,Ti\},\{Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Fs,Hs\}\}$

$\mu_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Al\},\{Fa,Ua,Rb,Es,Ti\},\{Al,Fa,Ua,Rb,Es,Ti\},$
$\{Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Fs,Hs\},\{Al,Fs,Hs\}\}$

$\mathcal{SJ}_{\mathfrak{H}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\ \{Cr,Fa,In,Es,Ti\},\{Fa,Es,Ti\},\{Al,Ua,Rb,Fs,Hs\},$
$\{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Al\},\{Al,Fa,Es,Ti\},\{Cr,Al,Fa,In,Es,Ti\},$
$\{Ua,Rb\},\{Cr,Fa,Ua,In,Rb,Es,Ti\},\{Fa,Ua,Rb,Es,Ti\},\{Al,Ua,Rb\},$
$\{Cr,Al,Fa,Ua,In,Rb,Es,Ti\},\{Al,Fa,Ua,Rb,Es,Ti\},\{Fs,Hs\},$
$\{Cr,Fa,In,Es,Fs,Ti,Hs\},\{Fa,Es,Fs,Ti,Hs\},\{Al,Fs,Hs\},$
$\{Cr,Al,Fa,In,Es,Fs,Ti,Hs\}\}$

Phase VII: Hi is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Fa,Rb,Es,Ti\},\{Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Ua,Fs,Hs\}\}$

$\mu_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Al\},\{Fa,Rb,Es,Ti\},\{Al,Fa,Rb,Es,Ti\},\{Fa,Ua,Rb,Es,Fs,Ti,Hs\},$
$\{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Ua,Fs,Hs\},\{Al,Ua,Fs,Hs\}\}$

$\mathcal{SJ}_{\mathfrak{H}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Al\},\{Al,Fa,Es,Ti\},\{Cr,Al,Fa,In,Es,Ti\},\{Rb\},\{Fa,Rb,Es,Ti\},$
$\{Cr,Fa,In,Rb,Es,Ti\},\{Al,Rb\},\{Al,Fa,Rb,Es,Ti\},Cr,Al,Fa,Ua,In,Rb,Es,Ti\},$
$\{Ua,Rb,Fs,Hs\},\{Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Cr,Fa,Ua,In,Rb,Es,Fs,Ti,Hs\},$
$\{Al,Ua,Rb,Fs,Hs\},\{Al,Fa,Ua,Es,Fs,Ti,Hs\},\{Cr,Al,Fa,Ua,In,Es,Fs,Ti,Hs\},$
$\{Fa,Es,Ti\},\{Cr,Fa,In,Es,Ti\}\}$

Phase VIII: Ec is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Fa,Ua,Rb,Es,Ti\},\{Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Fs,Hs\}\}$

$\mu_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Al\},\{Fa,Ua,Rb,Es,Ti\},\{Al,Fa,Ua,Rb,Es,Ti\},$
$\{Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Fs,Hs\},\{Al,Fs,Hs\}\}$

$\mathcal{SJ}_{\mathfrak{H}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Cr,Fa,In,Es,Ti\},\{Fa,Es,Ti\},\{Al,Ua,Rb,Fs,Hs\},$
$\{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Al\},\{Al,Fa,Es,Ti\},\{Cr,Al,Fa,In,Es,Ti\},$
$\{Ua,Rb\},\{Cr,Fa,Ua,In,Rb,Es,Ti\},\{Fa,Ua,Rb,Es,Ti\},\{Al,Ua,Rb\},$
$\{Cr,Al,Fa,Ua,In,Rb,Es,Ti\},\{Al,Fa,Ua,Rb,Es,Ti\},\{Fs,Hs\},$
$\{Cr,Fa,In,Es,Fs,Ti,Hs\},\{Fa,Es,Fs,Ti,Hs\},\{Al,Fs,Hs\},$
$\{Cr,Al,Fa,In,Es,Fs,Ti,Hs\}\}$

Following the aforementioned analysis of CrRase Cr, it has been determined that the principal factors affecting climate change impact are Rainfall, Deforestation, Algricultural Productivity and Economic Crosts.

Case 2: When Impact level is Normal

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Cr,Al,In\},\{Cr,Al,In,Fs,Hs\},\{Fs,Hs\}\}$

$\mu_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Al\},\{Cr,Al,In\},\{Cr,Al,In,Fs,Hs\},\{Fs,Hs\},\{Al,Fs,Hs\}\}$

$\mathcal{SJ}_{\mathfrak{H}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Al\},\{Cr,Al\},\{Cr,Al,In,Ti\},\{Cr,Al,Fa,In,Es,Ti\},\{Al,Fs,Hs\},$
$\{Cr,Al,Fs,Hs\},\{Cr,Al,In,Fs,Ti,Hs\},\{Cr,Al,Fa,In,Es,Fs,Ti,Hs\},$
$\{Fs,Hs\},\{Cr,Fs,Hs\},\{Cr,In,Fs,Ti,Hs\},\{Cr,Fa,In,Es,Fs,Ti,Hs\},$
$\{Cr\},\{Cr,In,Ti\},\{Cr,Fa,In,Es,Ti\},\{Al,Ua,Rb,Fs,Hs\},$
$\{Cr,Al,Ua,Rb,Fs,Hs\},\{Cr,Al,Ua,In,Rb,Fs,Ti,Hs\}\}$

Phase I: Te is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Cr\},\{Cr,Al,In,Fs,Ti,Hs\},\{Al,In,Fs,Ti,Hs\}$

$\mu_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Al\},\{Cr\},\{Cr,Al\},\{Cr,Al,In,Fs,Ti,Hs\},\{Al,In,Fs,Ti,Hs\}$

$\mathcal{SJ}_{\mathfrak{H}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Cr,Fa,In,Es,Ti\},\{Cr\},\{Cr,In,Ti\},\{In,Ti\},\{Al,Ua,Rb,Fs,Hs\},$

{Cr,Al,Ua,Rb,Fs,Hs},{Cr,Al,Ua,In,Rb,Fs,Ti,Hs},
{Al,Ua,In,Rb,Fs,Ti,Hs},{Al},{Cr,Al,Fa,In,Es,Ti},{Cr,Al},{Cr,Al,In,Ti},
{Al,In,Ti},{Al,Fs,Hs} {Cr,Al,Fa,In,Es,Fs,Ti,Hs},{Cr,Al,In,Fs,Ti,Hs},
{Al,In,Fs,Ti,Hs},{Cr,Al,Fs,Hs}}

Phase II: Ra is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Fa,Ua,Rb,Es,Ti},{Fa,Ua,Rb,Es,Fs,Ti,Hs},{Fs,Hs}

$\mu_{\mathcal{R}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Al},{Fa,Ua,Rb,Es,Ti},{Al,Fa,Ua,Rb,Es,Ti},
{Fa,Ua,Rb,Es,Fs,Ti,Hs},{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs},{Fs,Hs},{Al,Fs,Hs}}

$\mathcal{SJ}_{\mathfrak{H}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Cr,Fa,In,Es,Ti},{Fa,Es,Ti},{Al,Ua,Rb,Fs,Hs},
{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs},{Al},{Al,Fa,Es,Ti},{Cr,Al,Fa,In,Es,Ti},
{Ua,Rb},{Cr,Fa,Ua,In,Rb,Es,Ti},{Fa,Ua,Rb,Es,Ti},
{Al,Ua,Rb},{Cr,Al,Fa,Ua,In,Rb,Es,Ti},{Al,Fa,Ua,Rb,Es,Ti},
{Fs,Hs},{Cr,Fa,In,Es,Fs,Ti,Hs},{Fa,Es,Fs,Ti,Hs},{Al,Fs,Hs},
{Cr,Al,Fa,In,Es,Fs,Ti,Hs}}

Phase III: Sl is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Al,In}{Cr,Ua,Fs,Hs},{Cr,Al,Ua,In,Fs,Hs}}

$\mu_{\mathcal{R}}(\mathfrak{E})$ ={Φ,$\mathfrak{V}$,{Al},{Al,In},{Cr,UaFs,Hs},{Cr,Al,Ua,Fs,Hs},{Cr,Al,Ua,In,Fs,Hs}}

$\mathcal{SJ}_{\mathfrak{H}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Cr,Fa,In,Es,Ti},{Cr},{In},{Cr,In},{Al,Ua,Rb,Fs,Hs},
{Al,Ua,In,Rb,Fs,Hs},{Cr,Al,Ua,Rb,Fs,Hs},{Cr,Al,Ua,In,Rb,Fs,Hs},
{Al},{Cr,Al,Fa,In,Es,Ti},{Al,In},{Cr,Al},{Cr,Al,In},{Ua,FsHs},
{Cr,Fa,Ua,In,Es,Fs,Ti,Hs},{Ua,In,Fs,Hs},{Cr,Ua,Fs,Hs},
{Cr,Ua,In,Fs,Hs},{Al,Ua,Fs,Hs},{Cr,Al,Fa,Ua,In,Es,Fs,Ti,Hs},
{Al,Ua,In,Fs,Hs},{Cr,Al,Ua,Fs,Hs},{Cr,Al,Ua,In,Fs,Hs}}

Phase IV: GHG is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Cr,Al,In},{Fa,Rb,Es,Fs,Hs},{Cr,Al,Fa,In,Rb,Es,Fs,Ti,Hs}}

$\mu_{\mathcal{R}}(\mathfrak{E})$ = {Φ,$\mathfrak{V}$,{Al},{Cr,Al,In},{Fa,Rb,Es,Fs,Hs},{Al,Fa,Rb,Es,Fs,Hs},
{Cr,Al,Fa,In,Rb,Es,Fs,Hs}}

$SJ_{\mathfrak{H}}(\mathfrak{E}) =$ {Φ,𝔙,{Al},{Cr,Al,In},{Al,Fa,Es},{Al,Fa,In,Es},{Cr,Al,Fa,In,Es,Ti},
{Rb,Fs,Hs},{Cr,In,Rb,Fs,Hs},{Fa,Rb,Es,Fs,Hs},{Fa,In,Rb,Fs,Hs},
{Cr,Fa,In,Rb,Es,Fs,Ti,Hs},{Al,Rb,Fs,Hs},{Cr,Al,In,Rb,Fs,Ti,Hs},
{Al,Fa,Rb,Es,Fs,Hs},{Al,Fa,In,Rb,Es,Fs,Hs},
{Cr,Al,Fa,In,Rb,Es,Fs,Ti,Hs},{Cr,In},{Fa,In},{Fa,In,Ti},
{Cr,Fa,In,Es,Ti},{Al,Ua,Rb,Fs,Hs},{Cr,Al,Ua,Fs,Rb,Fs,Hs},
{Al,Fa,Rb,Es,Fs,Hs},{Al,Fa,In,Rb,Es,Fs,Hs}}

Phase V: De is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E}) =$ {Φ,𝔙,{Fa,Ua,Rb,Es,Ti},{Fa,Ua,Rb,Es,Fs,Ti,Hs},{Fs,Hs}

$\mu_{\mathcal{R}}(\mathfrak{E}) =$ {Φ,𝔙,{Al},{Fa,Ua,Rb,Es,Ti},{Al,Fa,Ua,Rb,Es,Ti},{Fa,Ua,Rb,Es,Fs,Ti,Hs},
{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs},{Fs,Hs},{Al,Fs,Hs}}

$SJ_{\mathfrak{H}}(\mathfrak{E}) =$ {Φ,𝔙,{Cr,Fa,In,Es,Ti},{Fa,Es,Ti},{Al,Ua,Rb,Fs,Hs},
{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs},{Al},{Al,Fa,Es,Ti},{Cr,Al,Fa,In,Es,Ti},
{Ua,Rb},{Cr,Fa,Ua,In,Rb,Es,Ti},{Fa,Ua,Rb,Es,Ti},{Al,Ua,Rb},
{Cr,Al,Fa,Ua,In,Rb,Es,Ti},{Al,Fa,Ua,Rb,Es,Ti},{Fs,Hs},
{Cr,Fa,In,Es,Fs,Ti,Hs},{Fa,Es,Fs,Ti,Hs},{Al,Fs,Hs},
{Cr,Al,Fa,In,Es,Fs,Ti,Hs}}

Phase VI: Ap is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E}) =$ {Φ,𝔙,{Fa,Ua,Rb,Es,Ti},{Fa,Ua,Rb,Es,Fs,Ti,Hs},{Fs,Hs}}

$\mu_{\mathcal{R}}(\mathfrak{E}) =$ {Φ,𝔙,{Al},{Fa,Ua,Rb,Es,Ti},{Al,Fa,Ua,Rb,Es,Ti},
{Fa,Ua,Rb,Es,Fs,Ti,Hs},{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs},{Fs,Hs},{Al,Fs,Hs}}

$SJ_{\mathfrak{H}}(\mathfrak{E}) =$ {Φ,𝔙,{Cr,Fa,In,Es,Ti},{Fa,Es,Ti},{Al,Ua,Rb,Fs,Hs},
{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs},{Al},{Al,Fa,Es,Ti},{Cr,Al,Fa,In,Es,Ti},
{Ua,Rb},{Cr,Fa,Ua,In,Rb,Es,Ti},{Fa,Ua,Rb,Es,Ti},
{Al,Ua,Rb},{Cr,Al,Fa,Ua,In,Rb,Es,Ti},{Al,Fa,Ua,Rb,Es,Ti},{Fs,Hs},
{Cr,Fa,In,Es,Fs,Ti,Hs},{Fa,Es,Fs,Ti,Hs},{Al,Fs,Hs},
{Cr,Al,Fa,In,Es,Fs,Ti,Hs}}

Phase VII: Hi is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Cr,Al,In\},\{Cr,Al,Ua,In,Fs,Hs\},\{Ua,Fs,Hs\}\}$

$\mu_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Al\},\{Cr,Al,In\},\{Cr,Al,Ua,In,Fs,Hs\},\{Ua,Fs,Hs\},\{Al,Ua,Fs,Hs\}\}$

$\mathcal{SJ}_{\mathfrak{H}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Cr,Fa,In,Es,Ti\},\{Cr\},\{Cr,In\},\{Al,Ua,Rb,Fs,Hs\},$
$\{Cr,Al,Ua,Rb,Fs,Hs\},\{Cr,Al,Ua,In,Rb,Fs,Hs\},\{Al\},\{Cr,Al,Fa,In,Es,Ti\},$
$\{Cr,Al\},\{Cr,Al,In\},\{Al,Ua,Fs,Hs\},\{Cr,Al,Fa,Ua,In,Es,Fs,Ti,Hs\},$
$\{Cr,Al,Ua,Fs,Hs\},\{Cr,Al,Ua,In,Fs,Hs\},$
$\{Ua,Fs,Hs\},\{Cr,Fa,Ua,In,Es,Fs,Ti,Hs\},\{Cr,Ua,Fs,Hs\},$
$\{Cr,Ua,In,Fs,Hs\}\}$

Phase VIII: Ec is removed

$\mathfrak{T}_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Fa,Ua,Rb,Es,Ti\},\{Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Fs,Hs\}\}$

$\mu_{\mathcal{R}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Al\},\{Fa,Ua,Rb,Es,Ti\},\{Al,Fa,Ua,Rb,Es,Ti\},$
$\{Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Fs,Hs\},\{Al,Fs,Hs\}\}$

$\mathcal{SJ}_{\mathfrak{H}}(\mathfrak{E}) = \{\Phi,\mathfrak{V},\{Cr,Fa,In,Es,Ti\},\{Fa,Es,Ti\},\{Al,Ua,Rb,Fs,Hs\},$
$\{Al,Fa,Ua,Rb,Es,Fs,Ti,Hs\},\{Al\},\{Al,Fa,Es,Ti\},\{Cr,Al,Fa,In,Es,Ti\},$
$\{Ua,Rb\},\{Cr,Fa,Ua,In,Rb,Es,Ti\},\{Fa,Ua,Rb,Es,Ti\},\{Al,Ua,Rb\},$
$\{Cr,Al,Fa,Ua,In,Rb,Es,Ti\},\{Al,Fa,Ua,Rb,Es,Ti\},\{Fs,Hs\},$
$\{Cr,Fa,In,Es,Fs,Ti,Hs\},\{Fa,Es,Fs,Ti,Hs\},\{Al,Fs,Hs\},$
$\{Cr,Al,Fa,In,Es,Fs,Ti,Hs\}\}$

From both **Case 1** and **Case 2**, it is clear that Rainfall, Deforestation, Agricultural Productivity and Economic Costs play a crucial role in driving climate change outcomes.

Visualization and Analysis

To provide a clear comparison of the performance of nano, micro and Selje topologies in identifying critical climate factors, a heat map was generated (see Figure 1). This visual repre- sentation compares the ability of each topology to detect key factors, such as temperature rise, rainfall variability and deforestation, across various regions including Coastal, Agricultural, Urban, Forested and Island regions.

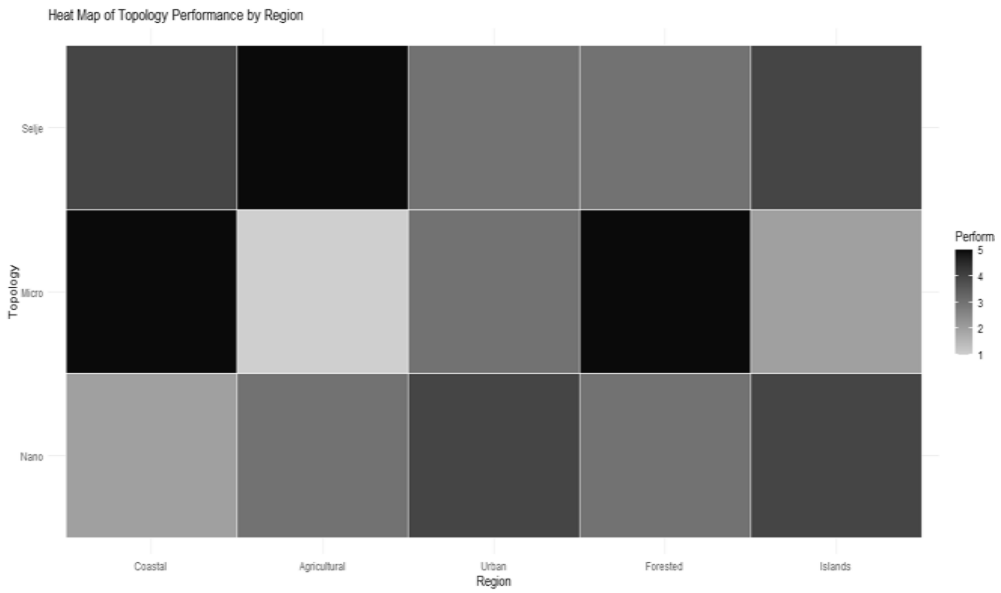


Figure 1: Heat Map of Topology Performance by Region

The heat map shows the performance score of each topology, with darker shades representing better performance in terms of accurately identifying impactful factors. As seen in the heat map, the Selje topology consistently demonstrates superior performance across all regions, particularly in complex environments like urban and forested areas, where multifactorial dependencies are prevalent.

5. Results and Discussion

Comparison of Nano, Micro and Selje Topological Spaces

In this analysis, nano topology, micro topology and Selje topology were applied to climate change impact factors to assess their efficiency in identifying critical variables. While all three topologies consistently identified Rainfall, Deforestation, Agricultural Productivity and Economic Costs as major factors, the depth of analysis, precision and flexibility differed significantly across the topologies.

Nano Topology

Strengths: Nano topology provides a simple binary classification of critical climate factors, making it effective for identifying whether a factor is part of a critical set.

Weaknesses: Its binary approach cannot capture the complexities of dynamic systems, leading to limitations in handling multifactorial relationships, scalability and interdependencies.

Micro Topology

Strengths: Micro topology refines nano topology by introducing micro-open and micro closed sets, allowing for more nuanced classifications and adaptable relationships between factors.

Weaknesses: While an improvement, micro topology still struggles with highly multifactorial systems, lacking the precision needed to fully address the complex, interconnected nature of climate factors.

Selje Topology

Strengths: Selje topology generalizes both nano and micro topologies, providing superior flexibility and precision. It uses Selje-open and Selje-closed sets to capture intricate relationships between climate factors, even in dynamic and multifactorial systems.

Theorem 1: Demonstrates finer approximations through better handling of closures and interiors.

Theorem 2: Highlights Selje's superior scalability, enabling it to handle complex systems more effectively.

Theorem 3: Proves Selje topology's ability to capture interdependencies through finer approximations of set intersections.

Better Performance: Selje topology offers deeper insights into the variability of climate impacts across regions. Unlike nano and micro, which treat factors as static, Selje allows for a dynamic understanding of how these factors fluctuate under different conditions and regions.

Weaknesses: The complexity of Selje topology may be unnecessary for simpler systems where its precision is not required.

Selje Topology's Superiority

While all three topologies identified the same major factors, Selje topology stands out due to its enhanced precision, scalability and ability to capture complex relationships.

Precision in Complex Systems: It handles intricate, multifactorial environments like climate change, providing a finer analysis of the interactions between key factors.

Scalability: As demonstrated in Theorem 3.3, Selje topology scales well with system complexity, retaining accuracy even as more variables are introduced.

Handling Nuanced Relationships: Theorem 3.4 shows that Selje topology excels in analyzing overlapping and interdependent factors, offering a more detailed understanding of cumulative impacts.

Flexibility: Unlike nano's rigid binary classification, Selje topology adapts to uncertainties and changing conditions, making it more versatile for dynamic systems.

6. Conclusion

While nano, micro and Selje topologies all identified the same key climate factors, Selje topology offers greater analytical power due to its flexibility, precision and scalability. These qualities make it the optimal choice for analyzing complex, multifactorial systems like climate change, where relationships between factors are dynamic and interdependent. Future research could explore Selje topology's application in other fields, such as optimizing smart grids or analyzing healthcare systems, where multifactorial interactions are critical. Its adaptability and precision make it well-suited for real-world applications in dynamic environments, providing deeper insights and better handling of complex systems.

Conflict of Interest

The authors affirm that there is no conflicts of interest pertaining to the research presented in this paper. No financial or personal relationships with any organizations or individuals that could potentially bias the findings or interpretations are reported.

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