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A NOTE ON NANO FUZZY CLOSURE AND BICLOSURE SPACES

Abstract: The aim of this paper is to present, clarify and analyze Nano fuzzy closure spaces and Nano fuzzy bi-closure spaces in relation to Nano fuzzy topological spaces. We have tried to analyze the basic properties of these new types of spaces.

Keywords: Nano Fuzzy Topological Spaces, Nano Fuzzy Closure Space, Nano Fuzzy Biclosure Space.

Mathematics Subject Classification (2010) No.: 03E72, 54A05, 54A40.

1. Introduction

Thivagar L. *at al*. [9, 6] introduced the concept of Nano topological spaces which were defined in terms of lower approximation, upper approximation and boundary region of a subset of a universe U using an equivalence relation on it and also defined Nano closed sets, Nano interior and Nano closure. Further, Bhuvneshwari K. *et al*. [2] introduced Nano generalized closed set in Nano topological spaces in 2014. B. A. Deole [5] has introduced Nano closure and Nano biclosure spaces in Nano topological spaces.

After the theory of fuzzy sets, given by L. Zadeh [11], fuzzfication of topological spaces was done. This work is done by C. L. Chang [4] and defined fuzzy topological spaces.

R. Navalakhe *et al*. [7] defined Nano fuzzy topological spaces with respect to a fuzzy subset λ of an universe which is defined in terms of lower and upper approximations of λ and studied Nano fuzzy closure and Nano fuzzy interior of a fuzzy subset. In this article we have presented the idea of Nano fuzzy closure spaces and Nano fuzzy bi-closure spaces and examined their characteristics.

2. Preliminaries

In this section we have narrated some of the important definition and results which are helpful in defining Nano fuzzy closure and Nano fuzzy bi-closure spaces in Nano fuzzy topological spaces.

Definition 2.1 [5]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U and $X \subseteq U$. Then Nano closure operator is a function: $Ncl_R: P(X) \to P(X)$ such that for all $A \subseteq X$

$$Ncl_R = \begin{cases} L_{iR}(X) \; if \; A \subseteq \quad L_{iR}(X) \\ B_{iR}(X) \; if \; A \subseteq \quad B_{iR}(X) \\ X; ot \;\; erwise \; and \; \phi \; if \; A = \phi \end{cases}$$

where L_i's are elements of $L_R(X)$ and B_i's are elements of $B_R(X)$. Which satisfies three conditions:

1. $Ncl_R(\phi) = \phi$

2. $A \subseteq Ncl_R(A)$

3. $Ncl_R(A \quad B) = Ncl_R(A) \quad Ncl_R(B)$

Hence, (X, Ncl_R) is called Nano closure space.

Definition 2.2 [7, 8]: Let U be the universe, R_1 and R_2 be equivalence relations on U. P_1 and P_2 are subsets of U. Then $\tau_{R1}(P_1)$ and $\tau_{R2}(P_2)$ satisfies the following axioms:

1. U and $\quad \in \tau_{R1}(P_1)$ and $\tau_{R2}(P_2)$.

2. The union of the elements of any sub collection of $\tau_{R1}(P_1)$ is in $\tau_{R1}(P_1)$ and $\tau_{R2}(P_2)$ is in $\tau_{R2}(P_2)$.

3. The intersection of the elements of any finite sub collection of $\tau_{R1}(P_1)$ is in $\tau_{R1}(P_1)$ and $\tau_{R2}(P_2)$ is in $\tau_{R2}(P_2)$.

Hence, $\tau_{R1}(P_1)$ and $\tau_{R2}(P_2)$ are called the Nano (τ_1, τ_2) bitopology on U with respect to $P_1$and P_2, $(U, \tau_{R_{1,2}}(X))$ is called Nano (τ_1, τ_2) bitopological space. Elements of the $\tau_{R_{1,2}}(X)$ are known as Nano $(1,2)$ open sets in U and elements of $[\tau_{R_{1,2}}(X)]^C$ are called Nano $(1,2)$ closed sets.

Definition 2.3 [1]: If $(U, \tau_{R_{1,2}}(X))$ is a Nano bitopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

1. The Nano $(1,2)$ closure of A is defined as the intersection of all Nano $(1,2)$ closed sets containing A and it is denoted by $N\tau_{R_{1,2}}cl(A)$. It is the smallest Nano $(1,2)$ closed set containing A.

2. The Nano $(1,2)$ interior of A is defined as the union of all Nano $(1,2)$ open subsets of A contained in A and it is denoted by $N\tau_{R_{1,2}}Int(A)$. It is the largest Nano $(1,2)$ open subset of A.

Definition 2.4 [5]: Let U be a non-empty finite set of objects called the universe and R_1 and R_2 be two equivalence relations on U and $X \subseteq U$. Then Nano closure operator is a function: $Ncl_{R_i}: P(X) \to P(X)$ where $i = \{1,2\}$ such that for all $A \subseteq X$

$$Ncl_{R_i} = \begin{cases} L_{iR}(X) \; if \; A \subseteq \quad L_{iR}(X) \\ B_{iR}(X) \; if \; A \subseteq \quad B_{iR}(X) \\ X; ot \;\; erwise \; and \; \phi \; if \; A = \phi \end{cases}$$

where L_i's are elements of $L_R(X)$ and B_i's are elements of $B_R(X)$. Which satisfies three conditions:

1. $Ncl_{R_1}(\phi) = \phi$ and $Ncl_{R_2}(\phi) = \phi$

2. $A \subseteq Ncl_{R_1}(A)$ and $A \subseteq Ncl_{R_2}(A)$

3. $Ncl_{R_1}(A \quad B) = Ncl_{R_1}(A) \quad Ncl_{R_1}(B)$ and

$Ncl_{R_2}(A \quad B) = Ncl_{R_2}(A) \quad Ncl_{R_2}(B)$.

That is there are two closure spaces (X, Ncl_{R_1}) and (X, Ncl_{R_2}). Hence, $(X, Ncl_{R_1}, Ncl_{R_2})$ is called Nano biclosure space.

Definition 2.5 [5]: Let $(X, Ncl_{R_1}, Ncl_{R_2})$ be a Nano biclosure space. A Nano biclosure space $(Y, Ncl_{R_3}, Ncl_{R_4})$ is called a Nano biclosure subspace of $(X, Ncl_{R_1}, Ncl_{R_2})$ if $Y \subseteq X$ and $Ncl_{R_j} = Ncl_{R_i}\ \ Y$ for each $i = \{1,2\}$, $j = \{3,4\}$ and each subset $A \subseteq Y$.

Definition 2.6 Properties of Fuzzy Approximation Space [3, 10]: Let R be an arbitrary relation from X to Y. The lower and upper approximation operators of a fuzzy set $\underline{\mathrm{R}}$ and $\overline{\mathrm{R}}$ satisfies the following properties: for all $\alpha, \beta \in F(X)$,

(FL1) $\underline{R}(\alpha) = (\overline{R}(\alpha^C))^C$

(FU1) $\overline{R}(\alpha) = (\underline{R}(\alpha^C))^C$

(FL2) $\underline{R}(\alpha \ \ \beta) = \underline{R}(\alpha) \ \ \underline{R}(\beta)$

(FU2) $\overline{R}(\alpha \ \ \beta) = \overline{R}(\alpha) \ \ \overline{R}(\beta)$

(FL3) $\alpha \leq \beta \Rightarrow \underline{R}(\alpha) \leq \underline{R}(\beta)$

(FU3) $\alpha \leq \beta \Rightarrow \overline{R}(\alpha) \leq \overline{R}(\beta)$

(FL4) $\underline{R}(\alpha \ \ \beta) = \underline{R}(\alpha) \ \ \underline{R}(\beta)$

(FU4) $\overline{R}(\alpha \ \ \beta) = \overline{R}(\alpha) \ \ \overline{R}(\beta)$

Definition 2.7 [7]: Let X be a non-empty finite set, R be an equivalence relation on X, $\lambda \leq X$ be a fuzzy subset and $\tau_R(\lambda) = \{1_\lambda, 0_\lambda, \underline{R}(\lambda), \overline{R}(\lambda), Bd(\lambda)\}$. Then by property (2.6), $\tau_R(\lambda)$ atisfies the following axioms

i. $0_\lambda, 1_\lambda \in \tau_{(R)}(\lambda)$ where $0_\lambda : \lambda \to I$ denotes the null fuzzy sets and $1_\lambda : \lambda \to I$ denotes the whole fuzzy set.

ii. Arbitrary union of members of $\tau_{(R)}(\lambda)$ is a member of $\tau_{(R)}(\lambda)$.

iii. Finite intersection of members of $\tau_{(R)}(\lambda)$ is a member of $\tau_{(R)}(\lambda)$.

That is, $\tau_{(R)}(\lambda)$ is a topology on X called the Nano fuzzy topology on X with respect to λ.

We call $(X, \tau_{(R)}(\lambda))$ as the Nano fuzzy topological space (NFTS). The elements of the Nano fuzzy topological space that is $\tau_{(R)}(\lambda)$, are called Nano fuzzy open sets and elements of $[\tau_{(R)}(\lambda)]^C$ are called Nano fuzzy closed sets.

Definition 2.8 [7]: Let $(X, \tau_{(R)}(\lambda))$ be a Nano fuzzy topological space with respect to λ where $\lambda \leq X$ and if $\mu \leq X$ then the Nano fuzzy interior of μ is defined as union of all Nano fuzzy open subsets of μ and it is denoted by $NfInt(\mu)$. That is, it is the largest Nano fuzzy open subset contained in μ.

Similarly, the Nano fuzzy closure of μ is defined as the intersection of all Nano fuzzy closed sets containing μ. It is denoted by $NfCl(\mu)$ and it is the smallest Nano fuzzy closed set containing μ.

Definition 2.9 [1]: Let X be a non-empty finite set, R_1 and R_2 be equivalence relations on X, $\lambda_1, \lambda_2 \leq X$ be fuzzy subsets and $\tau_{1,2_R}(\lambda) = \{\tau_{1_R}(\lambda_1), \tau_{2_R}(\lambda_2)\}$. Then $\tau_{1,2_R}(\lambda)$satisfies the following axioms:

1. $0_{\lambda_1}, 1_{\lambda_1} \in \tau_{1_R}(\lambda_1)$where $0_{\lambda_1}: \lambda_1 \to I$ denotes the null fuzzy sets and $1_{\lambda_1}: \lambda_1 \to I$ denotes the whole fuzzy set and $0_{\lambda_2}, 1_{\lambda_2} \in \tau_{2_R}(\lambda_2)$ where $0_{\lambda_2}: \lambda_2 \to I$ denotes the null fuzzy sets and $1_{\lambda_2}: \lambda_2 \to I$ denotes the whole fuzzy set.

2. Arbitrary union of members of $\tau_{1R}(\lambda_1)$ and $\tau_{2R}(\lambda_2)$ are in $\tau_{1R}(\lambda_1)$ and $\tau_{2R}(\lambda_2)$ respectively.

3. Finite intersection of members of $\tau_{1R}(\lambda_1)$ and $\tau_{2R}(\lambda_2)$ are in $\tau_{1R}(\lambda_1)$ and $\tau_{2R}(\lambda_2)$ respectively.

That is, $\tau_{1R}(\lambda_1)$ and $\tau_{2R}(\lambda_2)$ are called the Nano fuzzy bitopology $\tau_{1,2_R}(\lambda)$ on X with respect to $\lambda_1 \, and \, \lambda_2$. We call $\left(X, \tau_{1,2_R}(\lambda)\right)$ as the Nano fuzzy bitopological space (NFBTS). The elements of the Nano fuzzy bitopological space are called Nano fuzzy $(1,2)$ open sets and elements of $[\tau_{1,2_R}(\lambda)]^C$ are called Nano fuzzy $(1,2)$ closed sets.

3. Nano fuzzy Closure Spaces

Definition 3.1: Let X be a non-empty finite set of objects which called the universe and R be an equivalence relation defined on X and λ be an fuzzy subset of X. Then Nano fuzzy closure operator is a function $NfCl_R: F(X) \to F(X)$ where $F(X)$ is the set of all fuzzy subsets of X, such that for all $\mu \leq \lambda$

$$NfCl_R = \begin{cases} \overline{R}_i(\lambda)\ if\ \mu \leq \overline{R}_i(\lambda) \\ Bd_i(\lambda)\ if\ \mu \leq Bd_i(\lambda) \\ \lambda; ot\ \ erwise\ and\ 0_\lambda\ if\ \mu = 0_\lambda \end{cases}$$

Where R_i's are elements of $\overline{R}(\lambda)$ and Bd_i's are elements of $Bd(\lambda)$. Which satisfies three conditions:

1. $NfCl_R(0_\lambda) = 0_\lambda$

2. $\mu \leq NfCl_R(\mu)$

3. $NfCl_R(\mu\ \ \beta) = NfCl_R(\mu)\ \ NfCl_R(\beta)$

Hence, $(X, NfCl_R)$ is called Nano fuzzy closure space.

Definition 3.2: The elements of Nano fuzzy closure space are called Nano fuzzy open sets in Nano fuzzy closure spaces. The complement of Nano fuzzy open sets is called Nano fuzzy closed sets with respect to the Nano fuzzy closure space.

Definition 3.3: A fuzzy subset α of a Nano fuzzy closure space $(X, NfCl_R)$ is called Nano fuzzy closed if $\left(NfCl_R(\alpha)\right) = \alpha$.

The complement of Nano fuzzy closed set is called Nano fuzzy open.

4. Nano fuzzy Bi-closure Spaces

Definition 4.1: Let X be a non-empty finite set of objects which is called the universe and R_1 and R_2 be two equivalence relations on X and λ be any fuzzy subset of X. Then Nano fuzzy closure operator is a function: $NfCl_{R_i}: F(X) \to F(X)$, where $i = \{1,2\}$, and $F(X)$ is the set of all fuzzy subsets of X, such that for all $\mu \leq \lambda$

$$NfCl_R = \begin{cases} \overline{R}_i(\lambda)\ if\ \mu \leq \overline{R}_i(\lambda) \\ Bd_i(\lambda)\ if\ \mu \leq Bd_i(\lambda) \\ \lambda; ot\ \ erwise\ and\ 0_\lambda\ if\ \mu = 0_\lambda \end{cases}$$

Where R_i's are elements of $\overline{R}(\lambda)$ and Bd_i's are elements of $Bd(\lambda)$. Which satisfies three conditions:

1. $NfCl_{R_1}(0_\lambda) = 0_\lambda$ and $NfCl_{R_2}(0_\lambda) = 0_\lambda$

2. $\mu \leq NfCl_{R_1}(\mu)$ and $\mu \leq NfCl_{R_2}(\mu)$

3. $NfCl_{R_1}(\mu \ \ \beta) = NfCl_{R_1}(\mu) \ \ NfCl_{R_1}(\beta)$

and

$NfCl_{R_2}(\mu \ \ \beta) = \ NfCl_{R_2}(\mu) \ \ NfCl_{R_2}(\beta)$

That is there are two fuzzy closure spaces $(X, NfCl_{R_1})$ and $(X, NfCl_{R_2})$.

Hence, $(X, NfCl_{R_1}, NfCl_{R_2})$ is called Nano fuzzy biclosure space.

Definition 4.2: The elements of Nano fuzzy biclosure space are called Nano fuzzy open sets in Nano fuzzy bi-closure spaces. The complement of Nano fuzzy open sets is called Nano fuzzy closed sets with respect to the Nano fuzzy biclosure space.

Definition 4.3: Let $(X, NfCl_{R_1}, NfCl_{R_2})$ be a Nano fuzzy biclosure space. A Nano fuzzy biclosure space $(Y, NfCl_{R_3}, NfCl_{R_4})$ is called a Nano fuzzy biclosure subspace of $(X, NfCl_{R_1}, NfCl_{R_2})$ if $Y \subseteq X$ and $NfCl_{R_j} = NfCl_{R_i} \ \ Y$ for each $i = \{1,2\}, j = \{3,4\}$ and each fuzzy subset $\lambda \leq Y$.

Remark 4.4: 1. Nano fuzzy open sets of Nano fuzzy bi-closure space are open in both Nano fuzzy closure spaces.

2. A fuzzy subset α of a Nano fuzzy bi-closure space $(X, NfCl_{R_1}, NfCl_{R_2})$ is called Nano fuzzy closed if $NfCl_{R_1}\left(NfCl_{R_2}(\alpha)\right) = \alpha$.

The complement of Nano fuzzy closed set is called Nano fuzzy open.

3. α is a Nano fuzzy closed subset of Nano fuzzy biclosure space $(X, NfCl_{R_1}, NfCl_{R_2})$ if and only if α is Nano fuzzy closed subset of both $(X, NfCl_{R_1})$ and $(X, NfCl_{R_2})$.

4. Let α be a Nano fuzzy closed subset of a Nano fuzzy biclosure space $(X, NfCl_{R_1}, NfCl_{R_2})$

The following conditions are equivalent.

1. $NfCl_{R_2}\left(NfCl_{R_1}(\alpha)\right) = \alpha$

2. $NfCl_{R_1}(\alpha) = \alpha, NfCl_{R_2}(\alpha) = \alpha$

Remark 4.5: Let α be a fuzzy subset of a Nano fuzzy biclosure space $(X, NfCl_{R_1}, NfCl_{R_2})$. If α is a Nano fuzzy open set in $(X, NfCl_{R_1}, NfCl_{R_2})$, then

$$NfCl_{R_1}\left(NfCl_{R_2}(1_\lambda \quad \alpha)\right) = NfCl_{R_2}\left(NfCl_{R_1}(1_\lambda \quad \alpha)\right)$$

Proposition 4.6: Let $(X, NfCl_{R_1}, NfCl_{R_2})$ be a Nano fuzzy biclosure space and let $\alpha \leq X$. Then

1. α is Nano fuzzy open if and only if $\alpha = 1_\lambda \quad \left(NfCl_{R_1}\left(NfCl_{R_2}(1_\lambda \quad \alpha)\right)\right)$

2. If α is Nano fuzzy open and $\alpha \leq \mu$, then $\alpha \leq 1_\lambda \quad \left(NfCl_{R_1}\left(NfCl_{R_2}(1_\lambda \quad \alpha)\right)\right)$.

Proof: 1. Let $(X, NfCl_{R_1}, NfCl_{R_2})$ be a Nano fuzzy biclosure space and let $\mu \leq X$ and μ is Nano fuzzy open then $1_\lambda \quad \alpha$ is Nano fuzzy closed in Nano fuzzy biclosure space. So, by definition, $NfCl_{R_1}\left(NfCl_{R_2}(1_\lambda \quad \alpha)\right) = 1_\lambda \quad \alpha$. This implies that $\alpha = 1_\lambda \quad \left(NfCl_{R_1}\left(NfCl_{R_2}(1_\lambda \quad \alpha)\right)\right)$.

2. By part (1) obvious.

Proposition 4.7: Let $(X, NfCl_{R_1}, NfCl_{R_2})$ is a Nano fuzzy biclosure space. If α and β are two Nano fuzzy closed subsets of $(X, NfCl_{R_1}, NfCl_{R_2})$. Then $\alpha \quad \beta$ is also Nano fuzzy closed in $(X, NfCl_{R_1}, NfCl_{R_2})$.

Proposition 4.8: Let $(X, NfCl_{R_1}, NfCl_{R_2})$ is a Nano fuzzy biclosure space. If α and β are two Nano fuzzy closed subsets of $(X, NfCl_{R_1}, NfCl_{R_2})$ then $\alpha \quad \beta$ is Nano fuzzy closed if $NfCl_{R_1}$ and $NfCl_{R_2}$ are disjoint.

Proof: Let α and β are two Nano fuzzy closed subsets of $(X, NfCl_{R_1}, NfCl_{R_2})$.

Then $NfCl_{R_1}\left(NfCl_{R_2}(\alpha)\right) = \alpha$ and $NfCl_{R_1}\left(NfCl_{R_2}(\beta)\right) = \beta$

Now,

$$NfCl_{R_1}\left(NfCl_{R_2}(\alpha \quad \beta)\right) = NfCl_{R_1}\left(NfCl_{R_2}(\alpha) \quad NfCl_{R_2}(\beta)\right)$$
$$= NfCl_{R_1}\left(NfCl_{R_2}(\alpha)\right) \quad NfCl_{R_1}\left(NfCl_{R_2}(\beta)\right) = \alpha \quad \beta$$

Therefore $\alpha \ \ \beta$ is Nano closed if $NfCl_{R_1}$ and $NfCl_{R_2}$ are disjoint.

Proposition 4.9: If $(Y, NfCl_{R_3}, NfCl_{R_4})$ is a Nano fuzzy biclosure subspace of $(X, NfCl_{R_1}, NfCl_{R_2})$, then for every Nano fuzzy open subset ϑ of $(X, NfCl_{R_1}, NfCl_{R_2})$, $\vartheta \ \ Y$ is an Nano fuzzy open set in $(Y, NfCl_{R_3}, NfCl_{R_4})$.

Proof: Let ϑ be a Nano fuzzy open set in $(X, NfCl_{R_1}, NfCl_{R_2})$, then by property we can say that ϑ is Nano fuzzy open in both $NfCl_{R_1}$ and $NfCl_{R_2}$.

Thus,

$NfCl_{R_j}(Y \quad \vartheta \ \ Y) = NfCl_{R_i}(Y \quad \vartheta \ \ Y) \ \ Y \leq NfCl_{R_i}(X \quad \vartheta) \ \ Y = (X \quad \vartheta) \ \ Y = Y \quad (\vartheta \ \ Y)$ for each $i = \{1,2\}, j = \{3,4\}$. Consequently, $\vartheta \ \ Y$ is Nano fuzzy open in both $(Y, NfCl_{R_3})$ and $(Y, NfCl_{R_4})$. Therefore, $\vartheta \ \ Y$ is Nano fuzzy open in $(Y, NfCl_{R_3}, NfCl_{R_4})$.

Proposition 4.10: Let $(X, NfCl_{R_1}, NfCl_{R_2})$ be a Nano fuzzy biclosure space and let $(Y, NfCl_{R_3}, NfCl_{R_4})$ be a Nano fuzzy biclosure subspace of $(X, NfCl_{R_1}, NfCl_{R_2})$. If α is a Nano fuzzy closed subset of $(Y, NfCl_{R_3}, NfCl_{R_4})$, then α is also a Nano fuzzy closed subset of $(X, NfCl_{R_1}, NfCl_{R_2})$.

Proof: Let α be a Nano fuzzy closed subset of $(Y, NfCl_{R_3}, NfCl_{R_4})$. Then $NfCl_{R_3}(\alpha) = \alpha$ and $NfCl_{R_4}(\alpha) = \alpha$. Since α is Nano fuzzy closed subset of both $(X, NfCl_{R_1})$ and $(X, NfCl_{R_2})$.

Consequently, α is a Nano fuzzy closed subset of both $(X, NfCl_{R_1})$ and $(X, NfCl_{R_2})$. Therefore, α is a Nano fuzzy closed subset of $(X, NfCl_{R_1}, NfCl_{R_2})$.

REFERENCES

[1] Arul Selvaraj, X. and Balakrishna, U. Z. (2021): Open Sets and Maps in Nano Bitopological Spaces, *Journal of Physics*: Conference Series, 2070012033, doi:10.1088/1742-6596/2070/1/012033.

[2] Bhuvaneswari, K. and GnanapriyaMythili, K. (2014): Nano generalized closed sets in Nano topological spaces, *International Journal of Scientific and Research Publications*, Vol. 4(5), pp. 1-3.

[3] Bin Qin (2014): Fuzzy approximation spaces, *Journal of Applied Mathematics*, Article ID 405802, pp. 1-10.

[4] Chang, C. L. (1968): Fuzzy topological spaces, *Journal of Mathematical Analysis and Applications*, Vol. 24, pp. 182-190.

[5] Deole, B. A. **(**2020): On Nano Biclosure Space, *JuniKhyat*, Vol. 10(6) (13), pp. 223-234.

[6] Dubois, D. and Prade, H. (1990): Rough fuzzy sets and fuzzy rough sets, *International Journal of General Systems*, Vol. 17, pp. 191-208.

[7] Navalakhe, R. and Rajwade, P. (2019): On Nano Fuzzy Topological Spaces, *International Review of Fuzzy Mathematics*, Vol. 14(2), pp. 127-136.

[8] Rajwade, P. and Navalakhe, R. (2024): Nano fuzzy bitopological spaces, *Ratio Mathematica*, Vol. 51, pp. 103-111. DOI: 10.23755/rm.v51i0.1297.

[9] Thivagar, L. and Richard, C. (2013): On Nano forms of weakly open sets, *International Journal of Mathematics and Statistics Invention*, Vol. 1(1), pp. 31-37.

[10] Tripathy, B. K. and Panda, G. K. (2012): Approximation equalities on rough intuitionistic fuzzy sets and an analysis of approximate equalities, *International Journal of Computer Science Issues*, Vol. 9(2), pp 1-10.

[11] Zadeh, L. A. (1965): Fuzzy sets, *Information and Control*, Vol. 8, pp. 338-353.

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