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NEUTROSOPHIC SEMI δ-PRE IRRESOLUTE MAPPINGS

Abstract. This paper introduces a new class of mappings called neutrosophic semi *δ-pre irresolute* mappings in neutrosophic topological spaces and discusses some of its properties and characterizations.

Keywords: Neutrosophic Set, Neutrosophic Topology, Neutrosophic Semi *δ*-Pre Open Sets, Neutrosophic Semi *δ*-Pre Continuous and Neutrosophic Semi *δ-Pre Irresolute* mappings.

Mathematics Subject Classification No.: 54A.

1. Introduction

After the introduction of fuzzy sets [15] and intuitionistic fuzzy sets [4], Smarandache [10] created a neutrosophic set on a nonempty set by considering three components, namely membership, Indeterminacy,and non-membership whose sums lie between 0 and 3. In 2008, Lupiáñez [8] introduced the neutrosophic topology as an extension of intuitionistic fuzzy topology. Since, 2008 many authors such as Lupiáñez [8], Salama *et.al*. [10, 11], Acikgoz and his coworkers [1], Dhavaseelan *et.al*. [5], Al-Musaw [2], and others have contributed to neutrosophic topological spaces. Recently many weak and strong forms of neutrosophic open sets and neutrosophic continuity have been investigated by various authors [1, 2, 5, 6, 7, 12–14]. In this paper, we introduce a new class of mappings called neutrosophic fuzzy semi *δ*-pre irresolute mappings and obtain some of their characterizations and properties.

2. Preliminaries

This section contains some basic definitions and preliminary results which will be needed in the sequel.

Definition 2.1 [12]: A Neutrosophic set (**NS**) in X is a structure

$$A = \{< x, \mu_A(x), \varpi_A(x), \gamma_A(x) > : x \in X\}$$

where $\mu_A : X \to]^{-}0, 1^{+}[, \varpi A : X \to]^{-}0, 1^{+}[,$ and $\gamma_A : X \to]^{-}0, 1^{+}[$ denote the membership, indeterminacy, and non-membership of A, satisfying the condition that $^{-}0 \leq \mu_A(x) + \varpi_A(x) + \gamma_A(x) \leq 3^{+}, \forall x \in X$.

In real-life applications in scientific and engineering problems, using a neutrosophic set with values from a real standard or a non-standard subset of $\left]-0,\ 1+\right[$ is difficult. Hence, we consider the neutrosophic set which takes the value from the closed interval [0,1] and the sum of membership, indeterminacy, and non-membership degrees of each element of the universe of discourse lies between 0 and 3.

Definition 2.2 [10]: Let X be a nonempty set and let the neutrosophic sets A and neutrosophic set B be in the form $A = \{< x, \mu_A(x), \varpi_A(x), \gamma_A(x) > : x \in X\}$, $B = \{< x, \mu_B(x), \varpi_B(x), \gamma_B(x) > : x \in X\}$ and let $\{A_i : i \in J\}$ be an arbitrary family of neutrosophic sets in X. Then:

(a) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x), \varpi_A(x) \leq \varpi_B(x),$ and $\gamma_A(x) \geq \gamma_B(x)$.

(b) $A = B$ if $A \subseteq B$ and $B \subseteq A$.

(c) $A^c = \{< x, \gamma_A(x), \varpi_A(x), \mu_A(x) > : x \in X\}$.

(d) $\cap A_i = \{< x, \wedge \mu_{Ai}(x), \wedge \varpi_{Ai}(x), \vee \gamma_{Ai}(x) > : x \in X\}$.

(e) $\cup A_i = \{< x, \vee \mu_{Ai}(x), \vee \varpi_{Ai}(x), \wedge \gamma_{Ai}(x) > : x \in X\}$.

(f) $\tilde{\mathbf{0}} = \{< x, 0, 0, 1 > : x \in X\}$ and $\tilde{\mathbf{1}} = \{< x, 1, 1, 0 > : x \in X\}$

Definition 2.3 [8, 9]: A neutrosophic topology on a nonempty set X is a family τ of neutrosophic sets in X that satisfies the following axioms.

(NT_1) $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}} \in \tau$

(NT_2) Finite intersection of members of τ is a member of τ

(NT_3) Anyunion of members of τ is a member of τ

In this case, the pair (X, τ) is called a neutrosophic topological space and each neutrosophic set in τ is known as a neutrosophic open set in X. The complement A^c of a neutrosophic open set A is called a neutrosophic closed set in X.

Definition 2.4 [5]: Let $\alpha, \eta, \beta \in [0,1]$ and $0 \leq \alpha + \eta + \beta \leq 3$. A neutrosophic point $x_{(\alpha,\eta,\beta)}$ of X is a neutrosophic set in X defined by

$$x_{(\alpha\eta,\beta)}(y) = \begin{cases} (\alpha, \eta, \beta) & if \ y = x \\ (0, 0, 1) & if \ y \neq x \end{cases}$$

Definition 2.5 [1]: Let $x_{(\alpha,\eta,\beta)}$ be a neutrosophic point in X and $A = \{< x, \mu_A, \varpi_A, \gamma_A > : x \in X\}$ is a neutrosophic set in X. Then $x_{(\alpha,\eta,\beta)} \subseteq A$ if and only if $\alpha \subseteq \mu_A(x), \eta \subseteq \varpi_A$, and $\beta \supseteq \nu_A(x)$.

Definition 2.6 [1]: A neutrosophic point $x_{(\alpha,\eta,\beta)}$ is said to be quasi-coincident (q-coincident, for short) with A, denoted by $x_{(\alpha,\eta,\beta)} q A$ iff $x_{(\alpha,\eta,\beta)} \not\subseteq A^c$. If $x_{(\alpha,\eta,\beta)}$ is not quasi-coincident with A, we denote by $\neg(x_{(\alpha,\eta,\beta)} q A)$.

Definition 2.7 [1]: Two neutrosophic set A and B of X are said to be q-coincident (denoted by $A_q B$) if $A \not\subseteq B^c$.

Lemma 2.8 [1]: *For any two neutrosophic sets* A *and* B *of* X, $\neg(A_q B) \Leftrightarrow A \subset B^c$ *where* $\neg(A_q B)$ *is not q-coincident with* B.

Definition 2.9 [9]: Let (X, τ) be a *NTS* and $F \in N(X)$. Then the neutrosophic interior and neutrosophic closure of A are defined by:

$$cl(F) = \cap \{H : H \in NC(X) \text{ and } F \subseteq H\}$$

$$\text{int}(F) = \cup \{K : K \in \tau \text{ and } K \subseteq F\}$$

Definition 2.10 [3]: A neutrosophic set A of a *NTS* (X, τ) is called neutrosophic regular open (resp.neutrosophic regular closed) if $A = \text{int}(cl(A))$ (resp. $A = cl(\text{int}(A))$).

Definition 2.11 [1]: The δ-interior (denoted by $\delta\,\text{int}$) (resp.δ-closure (denoted by δcl)) of a neutrosophicset A of a $NTS(X, \tau)$ is the union of all neutrosophic regular open sets contained in (resp.intersection of all neutrosophic regular closed sets containing) A.

Definition 2.12 [3, 6, 13]: A neutrosophic set A of a *NTS* (X, τ) is called neutrosophic semi open (resp.neutrosophic pre open, neutrosophic α-open, neutrosophic semi preopen, neutrosophic δ-open, neutrosophic δ-preopen, neutrosophic δ-semi open, neutrosophic *b*-open) if

$$A \subseteq cl(\text{int}(A))(\text{resp.}A \subseteq \text{int}(cl(A)), A \subseteq \text{int}(cl(\text{int}(A))), A \subseteq cl(\text{int}(cl(A))),$$
$$A = \delta\,\text{int}(A),\ A \subseteq \text{int}(\delta cl(A)), A \subseteq cl(\delta\,\text{int}(A)), A \subseteq cl(\text{int}(A)) \cup \text{int}(cl(A)).$$

Definition 2.13 [11]: A neutrosophic set A of a neutrosophic topological space (X, τ) is called neutrosophic semi δ-preopen if there exists an eutrosophic δ-pre open set O in X such that $O \subseteq A \subseteq \delta cl(O)$.

The family of all neutrosophic semi δ-pre open set so fan neutrosophic topological space (X, τ) is denoted by $NS\delta PO(X)$.

Definition 2.14 [11]: A neutrosophic set A in a neutrosophic topological space (X, τ) is called neutrosophic semi δ-preclosed) if $A^c \in NS\delta PO(X)$. The

family of all neutrosophic semi δ-preclosed) sets of an neutrosophictopological space (X, τ) is denoted by $NS\delta PC(X)$.

Remark 2.15 [11]: Every neutrosophic semi preopen (resp.neutrosophic δ-preopen) set is neutrosophic semi δ-preopen. But the separate converse may not be true.

Definition 2.16 [11]: Let (X, τ) be an neutrosophic topological space and A be an neutrosophic set of X. Then the neutrosophic semi δ-preinterior (denoted by $s\delta p\,\mathrm{int}$) and neutrosophic semi δ-preclosure (denoted by $s\delta pcl$) of A respectively defined as follows:

$$s\delta p\,\mathrm{int}(A) = \cup \{O : O \subseteq A; O \in NS\delta PO(X)\},$$

$$s\delta pcl(A) = \cap \{O : O \supseteq A; O \in NS\delta PC(X)\}.$$

Definition 2.17 [11]: Let A be an neutrosophic set A of an neutrosophic topological space (X, τ) and $x_{(\alpha,\eta,\beta)}$ be an neutrosophic point of X. A is called:

(a) Neutrosophic semi δ-pre neighborhood of $x_{(\alpha,\eta,\beta)}$ if there exists an neutrosophic set $O \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)} \in O \subseteq A$.

(b) Neutrosophic semi δ-pre Q-neighborhood of $x_{(\alpha,\eta,\beta)}$ if there exists an neutrosophic set $O \in NS\delta PO(X)$ such that $x(\alpha, \eta, \beta) \in O \subseteq A$.

Definition 2.19 [9,11]: A mapping $f : (X, \tau) \to (Y, \sigma)$ is called:

(a) Neutrosophic continuous if $f^{-1}(A)$ is a neutrosophic open set in X for each neutrosophic open set A of Y.

(b) Neutrosophic semi δ-pre continuous if $f^{-1}(A) \in NS\delta PO(X)$ for every neutrosophic open set A of Y.

3. Neutrosophic Semi δ-preir Resolute Mappings

In this section, we introduce the concept of neutrosophic semi δ-pre irresolute mappings and study some of their properties in neutrosophic topological spaces.

Definition 3.1: A mapping f from aneutrosophic topological space (X, τ) to another neutrosophic topological space (Y, σ) is said to be neutrosophic semi δ-pre irresolute if $f^{-1}(A) \in NS\delta PO(X)$ for every neutrosophic set $\delta \in NS\delta PO(Y)$.

Remark 3.2: Every neutrosophic semi δ-pre irresolute mapping is neutrosophic semi δ-pre continuous but the converse may not be true.

Example 3.3: Let $X = \{a, b\}, Y = \{p, q\}$, and neutrosophic sets U defined as follows:

$$U = \{< a, 0.5, 0.4, 0.5 >, < b, 0.4, 0.4, 0.6 >\}$$

let $\tau = \{\tilde{0}, U, \tilde{1}\}$ and $\sigma = \{\tilde{0}, \tilde{1}\}$ be neutrosophic topologies on X and Y respectively. Then the mapping $f : (X, \tau) \to (Y, \sigma)$ defined by $f(a) = p$ and $f(b) = q$ is neutrosophic semi δ-pre continuous and hence neutrosophic continuous but not neutrosophic semi δ-pre irresolute.

Consider the following example:

Example 3.4: Example 3.4. Let $X = \{a, b\}$, $Y = \{p, q\}$, and neutrosophic sets V defined as follows:

$$V = \{< a, 0.4, 0.3, 0.6 >, < b, 0.5, 0.3, 0.5 >\}$$

let $\tau = \{\tilde{0}, \tilde{1}\}$ and $\sigma = \{\tilde{0}, V, \tilde{1}\}$ be neutrosophic topologies on X and Y respectively. Then the mapping $g : (X, \tau) \to (Y, \sigma)$ defined by $g(a) = p$ and $g(b) = q$ is neutrosophic semi δ-*pre irresolute* but not neutrosophic continuous.

Remark 3.5: Example (3.3) and Example (3.4) show that the concepts of neutrosophic semi δ-*pre irresolute* and neutrosophic continuous mappings are independent.

Theorem 3.6: *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping then the following statements are equivalent*:

(a) *F is neutrosophic semi δ-pre irresolute*

(b) *If $f^{-1}(A) \in NS\delta PC(X)$ for every neutrosophic set $A \in NS\delta PC(Y)$.*

(c) *For every neutrosophic point $x_{(\alpha,\eta,\beta)}$ in X and every neutrosophic set $A \in NS\delta PO(Y)$ such that $f(x_{(\alpha,\eta,\beta)}) \in A$ there is an neutrosophic set $O \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)} \in O$ and $f(O) \subseteq A$.*

(d) *For every neutrosophic point $x_{(\alpha,\eta,\beta)}$ of X and every neutrosophic semi δ-pre neighborhood A of $f(x_{(\alpha,\eta,\beta)})$, $f^{-1}(A)$ is an neutrosophic semi δ-pre neighborhood of $x_{(\alpha,\eta,\beta)}$.*

(e) *For every neutrosophic point $x_{(\alpha,\eta,\beta)}$ of X and every neutrosophic semi δ-pre neighbor hood A of $f(x_{(\alpha,\eta,\beta)})$, there is an neutrosophic semi δ-pre neighborhood U of $x_{(\alpha,\eta,\beta)}$ such that $f(U) \subseteq A$.*

(f) *For every neutrosophic point $x_{(\alpha,\eta,\beta)}$ of X and every neutrosophic set $A \in NS\delta PO(Y)$ such that $f(x_{(\alpha,\eta,\beta)})_q A$, there is an neutrosophic set $O \in NS\delta PO(X)$ such that $x(\alpha, \beta)qO$ and $f(O) \subseteq A$.*

(g) *for every neutrosophic point $x_{(\alpha,\eta,\beta)}$ of X and every neutrosophic semi δ-pre Q-neighborhood A of $f(x(\alpha,\eta,\beta))$, $f^1(A)$ is an neutrosophic semi δ-pre Q-neighborhood of $x_{(\alpha,\eta,\beta)}$.*

(h) *for every neutrosophic point $x_{(\alpha,\eta,\beta)}$ of X and every neutrosophic semi δ-pre Q-neighborhood A of $f(x_{(\alpha,\eta,\beta)})$, there is an neutrosophic semi pre Q-neighborhood U of $x_{(\alpha,\eta,\beta)}$ such that $f(U) \subseteq A$.*

(j) $f(s\delta pcl(A)) \subseteq s\delta pcl(f(A))$, *for every neutrosophicset A of X.*

(j) $s\delta pcl(f^{-1}(O)) \subseteq f^{-1}(s\delta pcl(O))$, *for every neutrosophic set O of Y.*

(k) $f^{-1}(s\delta p\,\mathrm{int}(O)) \subseteq s\delta p\,\mathrm{int}(f^{-1}(O))$, *for every neutrosophic set* O *of* Y.

Proof: (a) **(b)** Obvious.

(a) ⇒ (c) Let $x_{(\alpha,\eta,\beta)}$ be an eutrosophic point of X and $A \in NS\delta PO(Y)$ such that $f(x_{(\alpha,\eta,\beta)}) \in A$. Put $O = f^{-1}(A)$, then by (a), $O \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)} \in O$ and $f(O) \subseteq A$.

(c) ⇒ (a) Let $A \in NS\delta PO(Y)$ and $x_{(\alpha,\eta,\beta)} \in f^{-1}(A)$. Then $f(x_{(\alpha,\eta,\beta)}) \in A$. Now by (c) there is an eutrosophic set $O \in NS\delta PO(X)$ such that $x(\alpha, \eta, \beta) \in O$ and $f(O) \subseteq A$. Then $x_{(\alpha,\eta,\beta)} \in O \subseteq f^{-1}(A)$. Hence, $f^{-1}(A) \in NS\delta PO(X)$.

(a) ⇒ (d) Let $x_{(\alpha,\eta,\beta)}$ be a neutrosophic point of X, and let A be a semi δ-pre neighborhood of $f(x_{(\alpha,\eta,\beta)})$.Then there is an eutrosophic set $U \in NS\delta PO(X)$ such that $f(x_{(\alpha,\eta,\beta)}) \in U \subseteq A$. Now $f^{-1}(U) \in NS\delta PO(X)$ and $f^{-1}(U) \subseteq f^{-1}(A)$. Thus, $f^{-1}(A)$ is an eutrosophic semi δ-pre neighborhood of $x_{(\alpha,\eta,\beta)}$ in X.

(d) ⇒ (e) Let $x_{(\alpha,\eta,\beta)}$ be a neutrosophic point of X, and let A be a semi δ-pre neighborhood of $f(x_{(\alpha,\eta,\beta)})$. Then $U = f^{-1}(A)$ is an eutrosophic semi δ-pre neighborhood of $x_{(\alpha,\eta,\beta)}$ and $f(U) = f(f^{-1}(A)) \subseteq A$.

(e) ⇒ (c) Let $x_{(\alpha,\eta,\beta)}$ be an neutrosophic point of X and $A \in NS\delta PO(Y)$ such that $f(x_{(\alpha,\eta,\beta)}) \in A$. So there is neutrosophic semi δ-pre neighborhood U of $x_{(\alpha,\eta,\beta)}$ in X such that $x_{(\alpha,\eta,\beta)} \in U$ and $f(U) \subseteq A$. Hence there is an eutrosophic set $O \in NS\delta PO(X)$ such that $x(\alpha, \eta, \beta) \in O \subseteq U$ and so $f(O) \subseteq f(U) \subseteq A$.

(a) ⇒ (f) Let $x_{(\alpha,\eta,\beta)}$ be an neutrosophic point of X and $A \in NS\delta PO(Y)$ such that $f(x_{(\alpha,\eta,\beta)})_q \in A$. Let $O = f^{-1}(A)$. Then $O \in NS\delta PO(X)$, $x_{(\alpha,\eta,\beta)q}O$ and $f(O) = f(f^{-1}(A)) \subseteq A$.

(f) ⇒ (a) Let $A \in NS\delta PO(Y)$ and $x_{(\alpha,\eta,\beta)} \in f^{-1}(A)$ clearly $f(x_{(\alpha,\eta,\beta)}) \in A$ choose the Neutrosophic point x^c defined as

$$x^c{}_{(\alpha,\eta,\beta)}(z) = \begin{cases} (\beta, \eta, \alpha) & \text{if } z = x \\ (1, 1, 0) & \text{if } z \neq x \end{cases}$$

Then $f(x^c{}_{(\alpha,\eta,\beta)})_q \quad A$ and so by (f) there exists an neutrosophic set $O \in NS\delta PO(X)$ such that $x^c{}_{(\alpha,\eta,\beta)q} \quad O$ and $f(O) \subseteq A$. Now $x^c{}_{(\alpha,\eta,\beta)q}\, O$ implies $x_{(\alpha,\eta,\beta)} \in O$. Thus, $x_{(\alpha,\eta,\beta)} \subseteq f^{-1}A$. Hence, $f^{-1}(A) \in NS\delta PO(X)$.

(f) ⇒ (g) Let $x_{(\alpha,\eta,\beta)}$ be an neutrosophic point of X and A be semi δ-Q-neighborhood of $f(x(\alpha, \eta, \beta))$. Then there is a neutrosophic open set $A_1 \in NS\delta PO(Y)$ such that $f(x(\alpha, \eta, \beta))_q \subseteq A_1 \subseteq A$. By hypothesis, there is a neutrosophic set $O \in NS\delta PO(X)$ such that $x(\alpha, \eta, \beta)qO$ and $f(O) \subseteq A_1$. Thus, $x_{(\alpha,\eta,\beta)q}O \subseteq f^{-1}(A_1) \subseteq f^{-1}(A)$. Hence, $f^{-1}(A)$ is an neutrosophic semi δ-pre Q-neighborhood of $x_{(\alpha,\eta,\beta)}$.

(f) ⇒ (h) Let $x_{(\alpha,\eta,\beta)}$ be an eutrosophic point of X and A be a semi δ-pre-Q-neighborhood of $f(x(\alpha, \eta, \beta))$. Then $U = f^{-1}(A)$ is aneutrosophic semi δ-pre-Q-neighborhood of $x_{(\alpha,\eta,\beta)}$ and $f(U) = f(f^{-1}(A)) \subseteq A$.

(h) ⇒ (f) Let $x_{(\alpha,\eta,\beta)}$ be an eutrosophic point of X and $A \in NS\delta PO(Y)$ such that $f(x_{(\alpha,\eta,\beta)})_q A$. Then A is neutrosophic semi δ-pre-Q-neighborhood of $f(x(\alpha, \eta, \beta))$. So there is an eutrosophic semi δ-pre Q-neighborhood U of $x_{(\alpha,\eta,\beta)}$ such that $f(U) \subseteq A$. Now U being an eutrosophic semi δ-pre Q-neighborhood of $x_{(\alpha,\eta,\beta)}$. Then there exists an eutrosophicset $O \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)q}O \subseteq U$. Hence, $x_{(\alpha,\beta)q}O$ and $f(O) \subseteq f(U) \subseteq A$.

(b) ⇒ (i) Let A be an eutrosophic set of X . Since, $A = f^{-1}(f(A))$, we have

$A \subseteq f^{-1}(s\delta pcl(f^{-1}(A)))$. Now $s\delta pcl(f(A)) \in NS\delta PC(Y)$ and hence, $f^{-1}(s\delta pcl(f(A))) \in NS\delta PC(X)$. There for $es\delta pcl(A) \subseteq f^{-1}(s\delta pcl(f(A)))$ and $f(s\delta pcl(A)) \subseteq f(f^{-1}(s\delta pcl(A))) \subseteq s\delta pcl(f(A))$.

(i) ⇒ (b) Let $A \in NS\delta PC(Y)$ then $f(s\delta pcl(f^{-1}(A))) \subseteq s\delta pcl(f(f^{-1}(A)))$ $\subseteq s\delta pcl(A) = A$. Hence, $s\delta pcl(f^{-1}(A)) \subseteq f^{-1}(A)$ and $sof^{-1}(A) \in NS\delta PC(X)$.

(i) ⇒ (j) Let O be any neutrosophic set of Y, then $f^{-1}(O)$ is an neutrosophic set of X. Therefore by hypothesis (i), $f(s\delta pcl(f^{-1}(O)))$ $\subseteq s\delta pcl(f(f^{-1}(O))) \subseteq s\delta pcl(O)$. Hence, $s\delta pcl(f^{-1}(O)) \subseteq f^{-1}(s\delta pcl(O))$.

(j) ⇒ (i) Let A be any neutrosophic set of X, then $f^{-1}(A)$ is an neutrosophic set of Y, and by (j), $s\delta pcl(f^{-1}(f(A))) \subseteq f^{-1}(s\delta pcl(f(A)))$. Hence, $f(s\delta pcl(A)) \subseteq s\delta pcl(f(A))$.

(a) ⇒ (k) Let O be any neutrosophic set of Y, then $s\delta p\,\mathrm{int}(O) \in NS\delta PO(Y)$ and $f^{-1}(s\delta p\,\mathrm{int}(O)) \in NS\delta PO(X)$. Since, $f^{-1}(s\delta p\,\mathrm{int}(O)) \subseteq f^{-1}(O)$, then $f^{-1}(s\delta p\,\mathrm{int}(O)) \subseteq s\delta p\,\mathrm{int}(f^{-1}(O))$.

(i) ⇒ (a) Let $O \in NS\delta PO(Y)$, then $s\delta p\,\mathrm{int}(O) = O$ and $f^{-1}(O) \subseteq s\delta p\,\mathrm{int}(f^{-1}(O))$. Thus, $f^{-1}(O) = s\delta p\,\mathrm{int}(f^{-1}(O))$ and $f^{-1}(O) \in NS\delta PO(X)$. Hence, f is neutrosophic semi δ-*pre irresolute*

Definition 3.7: A mapping $f : (X, \tau) \to (Y, \sigma)$ is called neutrosophic *R*-open if the image of every neutrosophic open set of X is neutrosophic δ-open in Y.

Theorem 3.8: *Iff* $: (X, \tau) \to (Y, \sigma)$ *is neutrosophic* δ*-almost continuous and neutrosophic R-open mapping, then* f *is neutrosophic semi* δ*-pre irresolute.*

Proof: Let $A \in NS\delta PO(Y)$. Then there exist aneutrosophic set $O \in IF\delta PO(X)$ such that $O \subseteq A \subseteq \delta cl(O)$, therefore $f^{-1}(O) \subseteq f^{-1}(A) \subseteq f^{-1}(\delta cl(O)) \subseteq cl(f^{-1}(O))$ because f is neutrosophic *R*-open. Since f is neutrosophic δ-almost continuous and neutrosophic *R*-open, $f^{-1}(O) \in IF\delta PO(X)$. Hence, $f^{-1}(A) \in NS\delta PO(X)$.

Theorem 3.9: *Let* $f : (X, \tau) \to (Y, \sigma)$ *and* $g : (Y, \sigma) \to (Z, \eta)$ *be neutrosophic semi* δ*-pre irresolute mappings then* gof *is neutrosophic semi* δ*-pre irresolute.*

Proof: Let $A \in NS\delta PO(Z)$. Since g is neutrosophic semi δ-preirresolute, $g^{-1}(A) \in NS\delta PO(Y)$. Therefore, $(gof)^{-1}(A) = f^{-1}(g^{-1}(A)) \in NS\delta PO(X)$, because f is neutrosophic semi δ-*pre irresolute*. Hence, gof is neutrosophic semi δ-*pre irresolute*.

Theorem 3.10: *Let* $f : (X, \tau) \to (Y, \sigma)$ *is neutrosophic semi* δ*-pre irresolute and* $g : (Y, \sigma) \to (Z, \eta)$ *is neutrosophic semi* δ*-pre continuous mapping, then gof is neutrosophic semi* δ*-pre continuous.*

Proof: Let O be any neutrosophic open set of Z. Since g is neutrosophic semi δ-precontinuous $g^{-1}(O) \in NS\delta PO(Y)$. Therefore, $(gof)^{-1}(O) = f^{-1}(g^{-1}(O)) \in NS\delta PO(X)$ because f is neutrosophic semi δ-*pre irresolute*. Hence, gof is neutrosophic semi δ-precontinuous.

4. Conclusion

In this paper, a new class of mappings called neutrosophic fuzzy semi δ-pre irresolute mappings have been introduced, it is shown by examples that the concepts of neutrosophic fuzzy semi δ-pre irresolute mappings are stronger than the neutrosophic fuzzy semi δ-pre continuous mappings and independent of the neutrosophic fuzzy continuous mappings. Several characterizations and properties of this class of neutrosophic fuzzy mappings have been studied. In the future, we study the images and inverse images of neutrosophic compact, and neutrosophic connected spaces under these classes of mappings.

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