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TOPOLOGICAL SIMPLE ROUGH GROUPS

Abstract: In this paper, we introduce the concept of topological simple rough group using simple rough group. Also, we explore the filter of identity neighborhood in topological simple rough groups and we discuss some results related to these concepts.

Keywords: Topological Rough Group, Topological Rough Group Homomorphism, Topological Rough Normal Subgroup, Simple Rough Group, Topological Simple Rough Group, Filter of Identity Neighbourhood.

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1. Introduction

The rough set theory, initially proposed by Pawlak (1982) [19], has been utilized as an effective mathematical tool for modeling and processing incomplete information. In recent years, rough sets have been integrated with various mathematical theories such as algebra and topology.

Algebraic structures of rough sets have been studied by several authors, including Bonikowaski. Z, Kuroki. N, Wang. PP and Li. Z *et al*. [5, 14, 15, 16]. In 1994, Biswas and Nanda [4] introduced the concept of rough group and rough subgroups, which are based on upper approximation and are independent of lower approximation. Miao *et al*. [7] have enhanced the definitions of rough group and

rough subgroup, and have demonstrated their new properties. Conversely, Kuroki and Wang [15] outlined certain properties of lower and upper approximations in relation to the normal subgroups in 1996. Bagirmaz *et al*. [17] proposed the concept of topological rough groups, expanding the idea of a topological group to encompass the algebraic structures of rough groups.

In group theory, A group is simple if its only normal subgroups are the identity subgroup and the group itself [12]. The notion of a simple group was introduced by Galois about 180 years ago. Simple groups are the building blocks of all groups. In the concept of topological group, filters provide a powerful tool for understanding the topological properties like convergence, continuity, compactness, etc..

In this paper, we investigate the key principles of topological simple rough groups, which merge the structures of simple group and topological rough group. We give some examples to illustrate this concept and discuss the basis of topological simple rough group, which forms the foundation for studying their local properties, Also we explore the filter of identity neighborhoods, underscoring their role in analyzing the structure of topological simple rough groups.

2. Preliminaries

Definition 2.1 ([7]): Let U be a universe, C be a family of subsets of U, $C = \{X_1, X_2, \ldots, X_n\}$. C is called a classification of U if the following properties are satisfied:

1. $X_1 \cup X_2 \cup \ldots \cup X_n = U$;

2. $X_i \cap X_j = \phi, (i \neq j)$.

Definition 2.2 ([7]): Let $K = (U, R)$ be an approximation space and X be a subset of U. The sets

1. $\overline{X} = \{x \mid [x]_R \cap X \neq \phi\}$;

2. $\underline{X} = \{x \mid [x]_R \subseteq X\}$;

3. $BN(X) = \overline{X} - \underline{X}$

are called upper approximation, lower approximation and boundary region of X in K, respectively.

Definition 2.3 ([7]): Let $K = (U, R)$ be an approximation space and be a binary operation defined on U. A subset G of universe U is called a rough group if the following properties are satisfied:

1. $\forall x,\ y \in G, x * y \in \overline{G}$;

2. Association property holds in $\overline{G}$;

3. $\exists e \in \overline{G}$ such that $\forall x \in G, x * e = e * x = x$; e is called the rough identity element of rough group G;

4. $\forall x \in G, \exists y \in G$ such that $x * y = y * x = e$; y is called the rough inverse element of x in G;

Definition 2.4 ([7]): A non-empty subset H of rough group G is called its rough subgroup, if it is a rough group itself with respect to operation .

There is only one guaranteed trivial rough subgroup of rough group G, i.e., G itself. A necessary and sufficient condition for $\{e\}$ to be a trivial rough subgroup of rough group G is $e \in G$.

Definition 2.5 ([7]): A rough group is called a commutative rough group if for every $x, y \in G$, we have $x * y = y * x$.

Definition 2.6 ([7]): A rough subgroup N of rough group G is called a rough invariant subgroup, if $\forall a \in G, a * N = N * a$.

Definition 2.7 ([9]): Let G be a rough group and $A \subseteq G$. We say that A is symmetric if $A = A^{-1}$.

Definition 2.8 ([9]): Let $G_1 \subset U_1$ and $G_2 \subset U_2$ be rough groups. We say that G_1 and G_2 be rough homomorphism if there exists a surjection mapping $\varphi : \overline{G}_1 \to \overline{G}_2$ such that the following conditions (1)-(3) hold:

1. $\varphi|_{G_1}$ is a surjection mapping from G_1 to G_2;

2. For any $x, y \in G_1 \cup \{e\}$, we have $\varphi(x *_1 y) = \varphi(x) *_2 \varphi(y)$;

3. For any subset H of $G_1, \overline{H} = \varphi^{-1}\overline{(\varphi(H))}$.

If a rough homomorphism is a bijection, then we say that G_1 and G_2 are rough isomorphism.

Definition 2.9 ([3]): Let G be a topological group. A filter on G is a family η of non-empty subsets of G satisfying the two conditions:

1. If U and V are in η then $U \cap V$ is also in η;

2. If $U \in \eta$ and $U \subset W \subset G$, then $W \in \eta$.

Definition 2.10 ([3]): Let G be a topological group. A family ξ is called an open filter on G if there exists a filter η in G such that ξ is the intersection of η with the family of all open subsets of G.

Of course, this definition is equivalent to the following one: ξ is an open filter on G if ξ is a family of non-empty open subsets of G such that the intersection of any finite number of elements of ξ is also in ξ, and for each $U \in \xi$ and for every open subset W of G such that $U \subset W$, W also belongs to ξ.

Definition 2.11 ([17]): A topological rough group is a rough group $(G, *)$ together with a topology T on $\overline{G}$ satisfying the following two properties:

1. the mapping $f : G \times G \to \overline{G}$ defined by $f(x, y) = xy$ is continuous with respect to product topology on $G \times G$ and the topology T_G on $\overline{G}$ induced by T,

2. the inverse mapping $g : G \to G$ defined by $g(x) = x^{-1}$ is continuous with respect to the topology T_G on G induced by T.

Definition 2.12 ([17]): let G be a topological rough group and let H be a subgroup of G. Then, H is called a topological rough subgroup of G if

1. the mapping $f_H : H \times H \to \overline{H}$ defined by $f_H(x, y) = xy$ is continuous where $\overline{H}$ carries the topology induced by $\overline{G}$,

2. the inverse mapping $g_H : H \to H$ defined by $g_H(x) = x^{-1}$ is continuous.

Definition 2.13 ([1]): A mapping $\varphi : \overline{G}_1 \to \overline{G}_2$ is called a topological rough group homomorphism, if φ is a rough homomorphism and continuous with respect to the topology τ_2 on $\overline{G}_2$ inducing τ_{G_2} on G_2 and the topology τ_1 on $\overline{G}_1$ inducing τ_{G_1} on G_1.

Definition 2.14 ([17]): Let G be a topological rough group and let N be a normal subgroup of G. Then, N is called a topological rough normal subgroup of G if $\forall a \in G, aN = Na$.

Throughout this paper, we consider X be the universal set, G_{R} be a rough group with identity e and $\overline{G}$ be the upper rough approximation of G.

3. Topological Simple Rough Group

Definition 3.1: A rough group G is called a *simple rough group* if it contains no proper non-trivial rough normal subgroups. That is, G has only the rough normal subgroups are $\{e\}$ and G.

Example 3.2: Let $X = \{1,2,3,4,5,6\}$ be the set all integers with respect to the multiplication modulo 7. A classification of X is $X/R = \{\{1,6\},\{2,3\},\{4,5\}\}$. Let $G = \{1,2,4\}$. Then $\overline{G} = X$ and $\underline{G} = \phi$. Clearly, G is a rough group and it has no proper rough normal subgroups, hence G is a simple rough group.

Example 3.3: Let $X = S_4$ be the set of all permutations of {1, 2, 3, 4} with the multiplication operation of permutations. Consider a classification of X is $X/R = \{C1, C2, C3, C4\}$, where

C_1 = {(1), (12), (13), (14), (23), (24), (34)}

C_2 = {(123), (132), (124), (142), (134), (143), (234), (243)}

C_3 = {(1234), (1243), (1324), (1342), (1423), (1432)}

C_4 = {(12)(34), (13)(24), (14)(23)}

Let A_4 be the set all even permutations of S_4 that is, $A_4 = \{(1), C_2, C_4\}$. Then upper approximation of A_4, $\overline{A}_4 = C_1 \cup C_2 \cup C_4$ and lower approximation of A_4, $\underline{A}_4 = C_2 \cup C_4$. Hence, A_4 is a rough group.

Also we get some proper rough normal subgroups of A_4, like $\{(1)\},\{(1), C_2\}$ and $\{(1), C_4\}$. Therefore, A_4 is not a simple rough group.

Definition 3.4: A *topological simple rough group* is a simple rough group $(G, *)$ together with a topology $\overline{\tau}$ on $\overline{G}$ satisfying the following two properties:

(i) The mapping $f : G \times G \to \overline{G}$ defined by $f(x, y) = xy, x, y \in G$ is continuous with respect to the product topology on $G \times G$ and the topology τ on G induced by $\overline{\tau}$

(ii) The inverse mapping $g : G \to G$ defined by $g(x) = x^{-1}$, $x \in G$ is continuous with respect to the topology τ on G induced by $\overline{\tau}$.

Example 3.5: Let $X = \{[0],[1],[2],[3],[4]\}$ be the set of residue classes of modulo 5 and $*$ be the binary operation of residue addition. A classification of X is $X/R = \{\{[0],[2]\},\{[3],[4]\}\}$. Let $G = \{[0],[1],[4]\}$, then $\overline{G} = X$ and $\underline{G} = \phi$. Obviously, G is a rough group and also G has no proper rough normal subgroups. Therefore, G is a simple rough group.

Let $\overline{\tau} = \{\phi, \overline{G}, \{[0]\},\{[1],[2],[4]\},\{[0],[1],[2],[4]\}$. Then we get the induced topology τ on G is $\{\phi, G, \{[0]\},\{[1],[4]\}\}$. Hence, G is a topological simple rough group.

Proposition 3.6: Let G be a topological simple rough group and fix $x \in G$.

Then

(i) The map $L_x : G \to \overline{G}$ defined by $L_x(y) = xy$ is one-to-one and continuous, for all $y \in G$;

(ii) The map $R_x : G \to \overline{G}$ defined by $R_x(y) = yx$ is one-to-one and continuous, for all $y \in G$;

(iii) The map $f : G \to G$ defined by $f(a) = a^{-1}$ is homeomorphism, for all $a \in G$.

Proof: (i) Let $y_1, y_2 \in G$. Then $L_x(y_1) = L_x(y_2)$ implies $xy_1 = xy_2$. Since, G is a topological simple rough group and $x \in G$, $x^{-1} \in G \subseteq \overline{G}$. Thus, $x^{-1}(xy_1) = x^{-1}(xy_2)$ which implies $y_1 = y_2$. Hence L_x is one-to-one. Now let us prove L_x is continuous. Let U be an open set of xy in $\overline{G}$. Then there exists open sets V_1, V_2 of x, y in G such that $V_1V_2 \subseteq U$. Since, $xV_2 \subseteq V_1V_2 \subseteq U$, $L_x(V_2) = xV_2 \subseteq U$. Hence, L_x is continuous on G.

(ii) The proof of R_x is similar to L_x.

(iii) Since G is a topological simple rough group, the inverse mapping $f : G \to G$ is continuous. Therefore, f^{-1} is also continuous. Hence, the map f is homeomorphism of G into G. □

Proposition 3.7: Let G be a topological simple rough group. If $U \subseteq \overline{G}$ is an open set with $e \in U$, then there exists a symmetric open set V of e in G such that $VV \subseteq U$.

Proof: Since G is a topological simple rough group, the mapping $f : G \times G \to \overline{G}$ is continuous. Then $f^{-1}(U)$ is open in $G \times G$ and $(e, e) \in f^{-1}(U)$. Therefore, there exists open sets V_1, V_2 in G with $e \in V_1, e \in V_2$ such that $V_1 V_2 \subseteq U$. Also the inverse mapping $g : G \to G$ is continuous, so V_1^{-1} and V_2^{-1} are open. Let $V_3 = V_1 \cap V_2$. Then V_3 is open in G and also $V_3 V_3 \subseteq U$. Now we consider $V = V_3 \cap V_3^{-1}$ be an open set in G and $e \in V$. Hence, $V = V^{-1}$ and $VV \subseteq V_3 V_3 \subseteq U$. □

Proposition 3.8: Let G be a topological simple rough group. Then for every open set W of e in G, there exists a symmetric open set V of e in G such that $VV \cap G \subseteq W$.

Proof: Let W be an open set of e in G. Then there exists an open set U of e in G such that $W = U \cap G$. Since the mapping $f : G \times G \to \overline{G}$ is continuous and the inverse mapping $g : G \to G$ is homeomorphism, there exists an open set V of e in G and $V = V^{-1}$ such that $VV \subseteq U$. Hence, $VV \cap G \subseteq W$. □

Proposition 3.9: Let G be a topological simple rough group. If G, $\{e\}$ are open sets of $\overline{G}$, then $\{e\}$ is open in G and G is a discrete space.

Proof: Since G is a topological simple rough group, the mapping

$f : G \times G \to \overline{G}$ is continuous and $e \in G$. Also, $\{e\}$ is open in $\overline{G}$. Thus, we get $f^{-1}(\{e\})$ is open in $G \times G$ and $ee = e \in \{e\} \in \overline{\tau}$ which implies $UV \subseteq \{e\}$, where U, V are open sets in G and $e \in U, e \in V$. Suppose $U \neq \{e\}$, we get $UU \not\subseteq \{e\}$. Hence, $U = V = \{e\}$. Let $x \in G$. Since the mapping $f : G \times G \to \overline{G}$ is continuous at (x, x^{-1}), there exists a neighborhood U of $x \in G$ such that $UU^{-1} \subseteq \{e\}$. So $UU^{-1} = \{e\}$. Hence, $U = \{x\}$. Hence, G is discrete. □

Proposition 3.10: Let G be a topological simple rough group. If G is open in $\overline{G}$, then $H = \cap \{U : U \in \tau(e)\}$ is a topological group.

Proof: Let $x, y \in H$. Then $x, y \in U$ and given $U \in \tau(e)$. Since G is a topological simple rough group and G is open in $\overline{G}$, there exists an open set $V \in \tau(e)$ such that $VV \subseteq U$. Thus, $xy \in VV \subseteq U$. Therefore, $xy \in H$. Since the inverse mapping $f : G \to G$ is homeomorphism, there exists an open set V of e in G such that $V = V^{-1}$. □

Theorem 3.11: Let X be a topological group and G be a topological simple rough group. If H is a topological rough subgroup of G, then the topological closure of H, $cl(H)$, in G is a topological rough group of G.

Proof: Let $x, y \in cl(H)$ and U be an open set of xy. Then there exists open sets V_1 and V_2 of x and y such that $V_1V_2 \subseteq U$. Since H is a topological rough subgroup of G, there exists an elements $a, b \in H$ such that $a \in V_1 \cap H$ and $b \in V_2 \cap H$. Thus, we get $ab \in V_1V_2$ and $ab \in \overline{H}$, that is $ab \in V_1V_2 \cap \overline{H}$. So, $V_1V_2 \cap \overline{H} \neq \phi$ and also $U \cap \overline{cl(H)} \neq \phi$. Hence, $xy \in \overline{cl(H)}$. Let W be an open set of x^{-1} in $cl(H)$. Then there exists an open set V of x such that $V^{-1} \subseteq W$. Since, $x \in cl(H)$, there exists an element a of H such that $a \in V$.

Then $a \in V \cap H$ which implies $a^{-1} \in V^{-1} \cap H$. So, $W \cap H \neq \phi$ and hence, $x^{-1} \in cl(H)$. □

Remark 3.12: The topological closure of H, $cl(H)$, in $\overline{G}$ is also a topological rough subgroup in $\overline{G}$.

Theorem 3.13: Let G be a topological simple rough group and G be an open set in $\overline{G}$. If S is a subset of G and U is an open set in G, then the sets $SU \cap G$ and $US \cap G$ are the open sets of S in G.

Proof: Let $x \in S$. Then there exists an open set $V \subseteq U$ of x in G such that $xV \subseteq xU \cap G$. Therefore, $\bigcup_{x \in S} xV \subseteq SU \cap G$. Hence, we get $SU \cap G$ is an open set of S in G. Likewise, $SU \cap G$ is an open set of S in G. □

Definition 3.14: Let G be a topological simple rough group and $\overline{\mathcal{B}} \subseteq \overline{\tau}$ be a base for $\overline{\tau}$. For $x \in G$, the family $\mathcal{B}x = \{U \cap G : U \in \overline{\mathcal{B}},\ x \in U\} \subseteq \overline{\mathcal{B}}$ is called a base at x in τ.

Theorem 3.15: Let G be a topological simple rough group and G be an open set in $\overline{G}$. Let ße be the family of base at e in G. Then, for every $x \in G$,

$$\mathcal{L}_x = \{(xU) \cap G : U \in \mathcal{B}_e\}, \qquad \mathcal{R}_x = \{(Ux) \cap G : U \in \mathcal{B}_e\}$$

are two families of bases at x in G.

Proof: Let $U \in \mathcal{B}_e$. Since G is a topological simple rough group and G is an open set in $\overline{G}$, $f : G \times G \to \overline{G}$ is continuous at (x, e). Then there exists an open set $V \in \mathcal{B}_e$ such that $V \subseteq U$ and $xV \subseteq G$. It is enough to prove that $xU \cap G$ and $Ux \cap G$ are open sets in G. Since the map $L_{x-1} : G \to G$ is

one-to-one and continuous, $L_{x^{-1}}^{-1}(V)$ is open in G and $xV \subseteq G$. Then we get $L_{x^{-1}}^{-1}(V) = xV$ is open in G . Hence, $xV \subseteq xU$ and $xV \subseteq G$ which implies $xU \cap G$ is an open set in G . Similarly, $Ux \cap G$ is an open set in G . □

4. Filter of Identity Neighborhoods

In this section, the filter $\mathcal{F}_e$ be the set of all identity neighborhoods of G .

Proposition 4.1: Let G and H be topological simple rough groups. Then a rough homomorphism $f : \bar{G} \to \bar{H}$ is a topological rough group homomorphism if and only if it is continuous at the identity element.

Proof: Let e, e' be the identity elements in G and H respectively. Suppose f is a topological rough group homomorphism. That is, f is rough homomorphism and continuous. Since f is continuous, it is continuous at e in G . Conversely, suppose f is continuous at e. Let $a \in G$ and V be a neighborhood of $f(a)$ in H . Let us prove for any neighborhood U of a in G , $f(U) \subseteq V$. Since f is a rough homomorphism, $f(ax) = f(a).f(x)$, for all $x \in G$. Since f is continuous at e, there exists a neighborhood W of e such that $f(W) \subseteq V$. Then $U = aW$ is an open neighborhood of a and $f(U) \subseteq V$. □

Proposition 4.2: Let G and H be topological simple rough groups and $f : \bar{G} \to \bar{H}$ be a topological rough group homomorphism. Then the following conditions are equivalent:

(i) f is open

(ii) For each $W \in \mathcal{F}_e(G)$, the image $f(W)$ has a nonempty interior

(iii) There is a basis $\mathcal{B}_e$ of neighborhood W such that $f(W)$ has a nonempty interior

(iv) There is a basis $\mathcal{B}_e$ of neighborhood W in G such that $f(W)$ is an identity neighborhood in H

(v) For all $W \in \mathcal{F}_e(G)$, we have $f(W) \in \mathcal{F}_e(H)$

Proof: (*i*) $\Rightarrow$ (ii): Let W be an identity neighborhood in G. Since f is open, $f(W)$ is open. Also, $W \in \mathcal{F}_e(G)$. Hence, $\text{int}(f(W) \neq \phi$.

(ii) $\Rightarrow$ (iii): Suppose $f(W)$ has a nonempty interior, for each neighborhood W of e in G. Let $\mathcal{B}_e$ be a basis in G. Let $W \in \mathcal{B}_e$ and $W \in \mathcal{F}_e(G)$. Then $\text{int}(f(W)) \neq \phi$. Hence, the image $f(W)$ has a nonempty interior.

(iii) $\Rightarrow$ (iv): Let U, V be two identity neighborhoods in G such that $V \subseteq \text{int}(U)$. Then $\text{int}(f(V) \neq \phi$.

Consider $x \in V$ and $f(x) \in \text{int}(f(V)) \subseteq \text{int}\, f(\text{int}(U))$.

Let $W = \text{int}(U)x^{-1}$ and e, e' be identity elements in G, H.

Then $e = xx^{-1} \in \text{int}(U)x^{-1}$. So, W is an open neighborhood of identity element in G and

$$e' = f(x)f(x)^{-1} \in \text{int}(f(V))f(x)^{-1} \in \text{int}(f(\text{int}(U)))f(x)^{-1}$$

$$= \text{int}(f(\text{int}(U))f(x)^{-1})$$

$$= \text{int}(f(\text{int}(U)x^{-1}))\ .$$

$$= \text{int}(f(W))\,.$$

Hence, $f(W)$ is an identity neighbourhood of H.

(iv) $\Rightarrow$ (v): From the above proof, $f(W) \in \mathcal{F}_{e'}(H)$.

(v) $\Rightarrow$ (i): Obviously, this result follows from (v). □

Proposition 4.3: Let G and H be topological simple rough groups. Then the topological rough group homomorphism $f : \bar{G} \to \bar{H}$ is both continuous and open if and only if $f(\mathcal{F}_e) = \mathcal{F}_{e'}$, where $\mathcal{F}_e$ and $\mathcal{F}_{e'}$ are the filter of identity neighborhoods in G and H respectively.

Proof: Suppose the homomorphism f is continuous and open. Let prove $f(\mathcal{F}_e) = \mathcal{F}_{e'}$. Since f is continuous, $f(\mathcal{F}_e) \subseteq \mathcal{F}_{e'}$. Since f is open, $\mathcal{F}_{e'} \subseteq f(\mathcal{F}e)$. Therefore $f(\mathcal{F}_e) = \mathcal{F}_{e'}$. Conversely, if $f(\mathcal{F}_e) = \mathcal{F}_{e'}$ then f is both open and continuous. Let $U \in \mathcal{F}_e$ and $V \in \mathcal{F}_{e'}$. Then $f(U) = V$ which implies $U \subseteq f(V)$. Since U is an open neighborhood of e in G , $f^{-1}(V)$ is an open set in G . Also $f(U) \in \mathcal{F}_{e'}$ which implies $f(U)$ is an open set containing the identity element e' in H . Hence f is both open and continuous. □

Lemma 4.4: If U is an open neighborhood of the identity element in topological simple rough group G , then $U \subseteq cl(U) \subseteq UU, cl(U)$ means closure of U.

Proof: We know that $U \subseteq cl(U)$. It is enough to prove that $cl(U) \subseteq UU$. Let $a \in cl(U)$. Then there exists a symmetric neighborhood W of e in G such that $WW \subseteq U$ that is, $W \subseteq U$. Also $a \in W$ and aW is an open neighborhood of a. So $aW \cap U \neq \phi$ and $a \in aW \cap cl(U) \neq \phi$. Let $b \in aW \cap U$. Then $b = aw$, for some $w \in W$ which implies $a = bw - 1$. Since W is symmetric and $b \in U$, $w^{-1} \in W^{-1} = W \subseteq U$ and $a \in UU$. □

Theorem 4.5: (*First closure lemma*) Let G be a topological simple rough group such that G is open in $\bar{G}$. If S is a subset of G , then

$$cl(S) = \bigcap_{U \in \mathcal{F}_e} SU = \bigcap_{U \in \mathcal{F}_e} cl(SU),$$

where $\mathcal{F}_e$ be the filter of identity neighborhood in G .

Proof: Let $a \in cl(S)$ and $U \in \mathcal{F}_e$. Then $aU \cap G$ is a neighborhood of a in G and $aU \cap S \neq \phi$. Since $U = U^{-1}$, $aU^{-1} \cap S$ is a neighborhood of a. So, let $b \in aU^{-1} \cap S$ that is $b \in aU^{-1}$ and $b \in S$. Then $b = au^{-1}$, for some $u \in U$. Therefore, $a = bu \in SU \subseteq cl(SU)$. Let $a \in \bigcap cl(SU)$. Let us prove $a \in \bigcap (SU)$. Then there exists an identity neighborhood $V \in \mathcal{F}_e$ such that $VV \subseteq U$. Therefore, $a \in cl(SV) \subseteq SVV \subseteq SU$. Proceeding this process we get, $\bigcap cl(SU) \subseteq \bigcap (SU)$. Suppose $a \in \bigcap (SU)$. Let W be an identity neighborhood of a in $\mathcal{F}_e$ and $W^{-1}a = U \in \mathcal{F}_e$. Since, $a \in SU$, $a = hu$, for some $h \in S, u \in U$. Thus, $h = au - 1 \in aU^{-1} = aa^{-1}W = W$ which implies $h \in S \cap W$. Hence, $a \in cl(S)$. □

Theorem 4.6: Let G be a finite topological rough group with identity e and let $\mathcal{F}_e$ be the filter of identity neighborhood in G . Then there exists a topological rough normal subgroup N in G such that $\mathcal{F}_e = \{U \subseteq G \ / N \subseteq U\}$ and the elements are symmetric.

Proof: Consider $N = \bigcap_{U \in \mathcal{F}e} U$. Since G is finite, N is non empty in $\mathcal{F}_e$ and $e \in N$. Let $a, b \in N$. Then there exists $W \in \mathcal{F}_e$ such that $WW^{-1} \subseteq N$. Since $N \subseteq W, a, b \in W$. Thus, $ab^{-1} \in UU^{-1} \subseteq N$. Hence, $ab^{-1} \in N$. Let $g \in G$. Since $N \in \mathcal{F}_e, gN\ g^{-1} \in \mathcal{F}_e$ and $g^{-1}N\ g \in \mathcal{F}_e$. That implies $N \subseteq gN\ g^{-1}$ and $N \subseteq gN\ g^{-1}$. Hence, N is a topological rough normal subgroup. □

5. Conclusion

In this paper, we studied topological simple rough group from the simple rough group structure and given some examples. Further we investigated the basis of topological simple rough groups and discussed the concept of filters in topological simple rough groups, which are essential for a deeper understanding of their topological properties.

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