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RELIABILITY MEASURES OF A TWO DISSIMILAR ELECTRIC VEHICLES USING LINEAR DIFFERENTIAL EQUATION TECHNIQUE

Abstract: The reliability of Hybrid Electric Vehicles (HEVs as System 1) and Plug-in Hybrid Electric Vehicles (PHEVs as System 2) is a critical factor influencing their adoption, performance, and long-term sustainability. As the global shift towards cleaner, more energy-efficient transportation intensifies, understanding the reliability of these vehicles becomes essential to ensure their effectiveness in real-world applications. This paper investigates various reliability measures of a two dissimilar electrical vehicles HEVs and PHEVs, focusing on key performance reliability indicators such as Mean Time to System Failure (MTSF), availability, and failure rates using Linear Differential Equations (LDEs) technique. It is assumed that the distribution of repair rates and failure rates are exponential. A comprehensive analysis is conducted of both systems and examined how the failure rates affected the system performance measures. For different values of failure rates were used to examine a few exceptional instances and results are also presented graphically using MATLAB software.

Keywords: Linear Differential Equation Reliability, Hybrid Electric Vehicles, Plug in Hybrid Electric Vehicle, MTSF, Steady-State Availability.

Mathematics Subject Classification (2010) No.: 60K10, 62N05, 96B25.

1. Introduction

In the subject of system reliability, number of author's analyzed and assessed reliability metrics, including MTSF, steady-state availability analysis, busy period of

repairmen, and cost analysis with the Markov renewal process using the regeneration point approach. In this work, dependability measures are assessed using linear differential equations technique. This approach is less complicated than the others, and MATLAB software can be used to carry out computations. Many authors previously employed linear differential equations techniques to evaluate reliability measures for various systems.

Yusuf, I. *et al*. [12] used a mathematical modeling technique to analyze the reliability measure of a two-unit system in cold standby mode. The system operates using a supporting device and failed unit is repaired by repairable service station. Using supplementary variable approaches and Laplace transformation, El. Sherbeny *et al*. [2] examined the dependability and sensitivity analysis as well as the administrative delay in repair of several industrial system cost free warranty policies.Gandoman, F. H. *et al*. [4] discussed the role, mechanism and outcomes of the different failures for evaluating reliability and safety of Li-ion batteries in electrical vehicles. They looked into safety and reliability assessment models for the primary failure modes and current issues with Li-ion batteries in electric vehicles, as well as degradation under normal and abnormal conditions.

Using a first order linear differential equation, Gupta, P., and Kaur, G. [5] evaluated the availability and reliability of a system having two-unitsin standby system that, in the event of a complete failure, switches to a comparable, replacement, or duplicate system that is guaranteed to operate without interruption. Kadyan, M. S. and Kumar, R. [7] commented about the sugar industry's feeding system, which is made up of six subsystems that were examined utilizing the supplementary variable approach. To illustrate the behavior of availability and profit in the feeding system, they also took into consideration a particular instance. ShafiqS. *et al*. [9] derived a probabilistic model that evaluate the significant impact on reliability measures of proposed composite power systems Full Electric Vehicles (FEVS) and Plug-in Hybrid Electric Vechicles (PHEVs) in different atmospheric conditions summer and winter seasons. Gao *et al*. [3] used the Laplace transform approach to study the MTTF and reliability function of the K-out-of M+W+C: G mixed standby system with an unstable repair facility.In a study conducted by Danjuma, M. U. *et al*. [1] the Markov birth-death process was used to calculate the steady-state availability and MTTF of a system consisting of three components connected in series and parallel with a cold standby unit. Using a Markov model, Zaidi, Z. [13] explored several approaches and methodologies for transiently analyzing the dependability of a three-component system.

Lane *et al*. examined information obtained from a survey conducted in late 2013 among 1080 drivers in United State to evaluate the factors that affect the inclination of potential car buyers toward two distinct types of plug-in electric

vehicles (PEVs): plug-in hybrid electric vehicles (PHEVs) and battery electric vehicles (BEVs). Thakur H. *et al*. [10] looked at availability analysis and meantime to system failure for a system with two subsystems subsystem 1 and subsystem 2 each with a different configuration in their study and used linear differential equation method to evaluate reliability measures using reasonable assumptions. Yusuf, I., and Bala, S. I. [11] investigated a redundant air conditioning system with a warm standby reserve unit and a primary unit that operated in various weather conditions. They applied Kolmogorov's forward equations technique, several system performance metrics were derived, including profit function, availability, busy period, mean time to system failure (MTSF), and frequency of preventive, minor, and major maintenance. Joshi P. K. *et al*. [6] were examined the availability and dependability of a two-unit standby redundancy system by using the linear differential equation solution approach.

The goal of this study is to investigate the dependability metrics, such as MTSF and steady state availability analysis, of a hybrid four-wheeler (system 1) and plug-in hybrid four-wheeler (system 2) using linear differential equation technique. Plug-in hybrid four-wheeler (System 2) is superior to hybrid four-wheeler (System 1) based on the computation presented in the study regarding the influence of the battery charging option and switching. A graphical representation of measures of system effectiveness of both the system is also explored.

2. Model Description and Assumptions

2.1 Hybrid Electric Vehicle (HEV): Hybrid Electric Automobiles are power-driven by an internal combustion engine and an electric motor, with the electric motor using the energy stored in batteries. It is important to note that hybrid electric vehicles lack the ability to directly connect to an external power source in order to recharge their batteries.

2.2 Plug in Hybrid Electric Vehicle (PHEV): A Plug-in Hybrid Electric Vehicle harnesses the power of batteries to run an electric motor, and an alternative fuel such as diesel or gasoline to run an internal combustion engine or other propulsion methods. PHEV can replenish their batteries using charging devices and regenerative braking.Throughout the study of research paper, the following assumptions has been made:

- System 1 (HEV) can generate electricity through regenerative braking rather than by plugging into a charging station to recharge the vehicle's battery.

- The hybrid four-wheeler in system 1 continues to run even if the battery failed.

- System 2 (PHEV) will only use its internal combustion engine as a backup and will be primarily driven by an electric motor.
- The switch in system 2 (PHEV) is utilized to turn on the petrol supply.
- System 2 (PHEV) uses an automatic switch to start the engine immediately when the battery dies (failed), providing the switch is in working order at the time of need; otherwise, the engine won't start until the switch is repaired.
- System 1 has only two modes, namely failure and normal.
- System 2 has three operating modes: normal, partial, and failure.
- The repair is perfect-almost like new.
- Only one change may be made at a time in a single state.
- All failurerates and repair ratesare constant and follows the exponential distribution.

3. Notations and Symbol

S_i -Transition state of the system, i = 0, 1, 2, 3, 4

P_N -Petrol supply Normal

B_N -Battery fully charged

B_{NP} -Battery partially charged

P_{NP} -Petrol supply is partially

B_F -Battery failed.

P_F -Petrol supply failed.

α-ailure rate of petrol supply

β-failure rate of battery

α'-failure rate of petrol when battery already failed

β'-failure rate of battery when petrol already failed

δ-Repair rate of Petrol supply

λ-Repair rate of battery

δ_1-rate of charging battery

α_2-rate of completion of battery charging

α_1-rate of filling petrol

μ_1-rate of completion of filling petrol

θ-replacement rate of both petrol &battery

4. Transition probability of Hybrid Electric Vehicle (System 1)

Figure 1 shows the transition probability of different states of system 1.

Up State; $S_0 \equiv (B_N, P_N)$, $S_1 \equiv (B_N, P_F)$, $S_2 \equiv (B_F, P_N)$

Down State; $S_3 \equiv (B_F, P_F)$

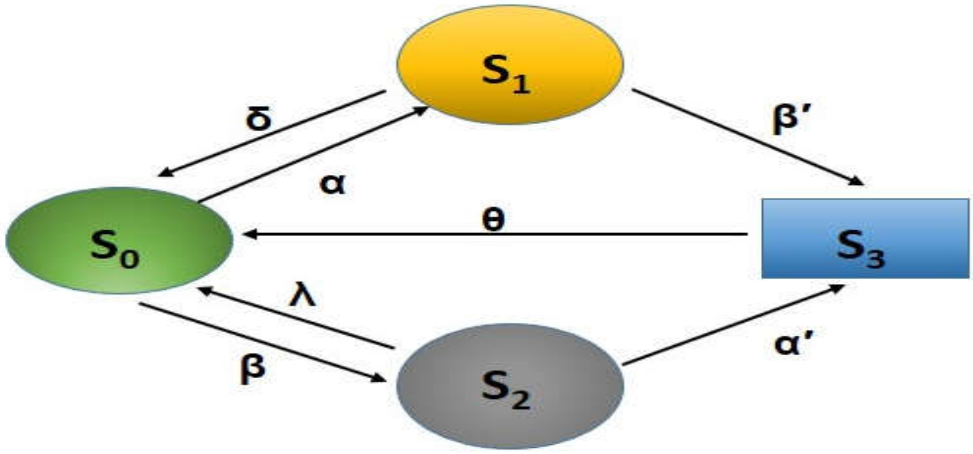


Figure 1: Diagram illustrating the Second System's state transition

5. Measures of System Effectiveness of System 1

5.1 Mean Time to System Failure: Using above aforementioned presumptions and first order differential equation approach MTSF of the suggested system is analysed. Pi(t) is defined, using Figure 1, as the probability that the system is in state Si at time t. The probability row vector at time t is represented by P(t). Take into consideration the initial conditions as

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0)], = [1, 0, 0, 0] \quad (1)$$

The derived system of differential equations is as follows:

$$\frac{dp\ (t)}{dt} = (\alpha + \beta)\ P_0(t) + \delta P_1(t) + \lambda\ P_2(t) + \theta P_3(t)$$

$$\frac{dp_1(t)}{dt} = (\beta' + \delta) P_1(t) + \alpha\ P_0(t)$$

$$\frac{dp_2(t)}{dt} = (\alpha' + \lambda\ ) P_2(t) + \beta P_0(t)$$

$$\frac{dp_3(t)}{dt} = \theta P_3(t) + \beta' P_1(t) + \alpha' P_2(t)$$

This can be expressed as in the form of system of differential equation:

$$\frac{dp(t)}{dt} = AP(2)$$

In equation (2), matrix A is defined as

$$A = \begin{matrix} (\alpha+\beta) & \delta & \lambda & \theta \\ \alpha & (\beta'+\delta) & 0 & 0 \\ \beta & 0 & (\alpha'+\lambda) & 0 \\ 0 & \beta' & \alpha' & \theta \end{matrix}$$

In order to streamline the assessment of the transition solution eliminating the rows and columns in matrix A that correspond to the absorption state.

This modified matrix, named Q, is created by transposing A. The expected time needed to attain an absorbing condition is derived from

$$E[T_{P(0)\rightarrow(absorbing)}] = \mathrm{P}(0)\int_0^\infty e^{Qt}dt$$

and $$\int_0^\infty e^{Qt}dt = Q^{-1}, \text{ since } Q^{-1} < 0$$

where $$Q = \begin{bmatrix} (\alpha+\beta) & \alpha & \beta \\ \delta & (\beta'+\delta) & 0 \\ \lambda & 0 & (\alpha'+\lambda) \end{bmatrix}$$

The MTSF can be expressed explicitly as:

$$\text{MTSF}_1 = E[TP(0) \rightarrow (absorbing)] = P(0)(\ \ Q^{-1}) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{3}$$

$$\text{MTSF}_1 = \frac{(\alpha+\beta_1')(\lambda+\alpha')+\delta(\lambda+\alpha')+\beta(\delta+\beta')}{\alpha\beta'(\lambda+\alpha')+\beta\alpha'(\delta+\beta')} \tag{4}$$

5.2 Steady-State Availability Analysis: For steady state availability analysis initial condition is same as given in equation (1).From equation (1), this can be expressed as in the form of system of differential equation:

$P = A\ P$

$$\begin{bmatrix} \dot{p}_0 \\ \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \end{bmatrix} = \begin{matrix} (\alpha+\beta) & \delta & \lambda & \theta \\ \alpha & (\beta'+\delta) & 0 & 0 \\ \beta & 0 & (\alpha'+\lambda) & 0 \\ 0 & \beta' & \alpha' & \theta \end{matrix} \begin{bmatrix} P_0 \\ p_1 \\ p_2 \\ P_3 \end{bmatrix}$$

The expression for steady-state availability is expressed by

$$A_{T_1}(\infty) = 1 \quad P_3(\infty) \tag{5}$$

In the context of steady-state availability, the state probabilities' derivatives reach a point of equilibrium where they become zero. This implies that the product of matrix A and the probability at infinity, denoted as P(∞), is equal to zero.

i.e., $$A\ P(\infty) = 0 \tag{6}$$

$$\begin{matrix} (\alpha+\beta) & \delta & \lambda & \theta \\ \alpha & (\beta'+\delta) & 0 & 0 \\ \beta & 0 & (\alpha'+\alpha) & 0 \\ 0 & \beta' & \alpha' & \theta \end{matrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{7}$$

$$P_0(\infty) + p_1(\infty) + P_2(\infty) + p_3(\infty) = 1 \tag{8}$$

To obtain $P_5(\infty)$, we replace (8) in one of the redundant rows of (7) and then using MATLAB software to get the matrix-form solution to the following linear equation system.

$$\begin{bmatrix} (\alpha+\beta) & \delta & \lambda & \theta \\ \alpha & (\beta'+\delta) & 0 & 0 \\ \beta & 0 & (\alpha+1) & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{9}$$

The steady state probabilities for Figure1 are found from solution of equation (9). For $AT_2(\infty)$, the explicit expression is:

$$A_{T_1}(\infty) = \frac{\alpha(\alpha'+\lambda)(\beta'+\theta)+(\beta'+\delta)(\alpha'\beta+\theta\lambda)+\theta(\alpha'\beta'+\alpha\delta)}{\alpha(\alpha'+\lambda)(\beta'+\theta)+(\beta'+\delta)[\beta(\alpha'+\theta)+\theta(\alpha'+\lambda)]} \tag{10}$$

6. Transition probability of Plug in hybrid electric vehicle (System 2)

Figure 2 shows the transition probability of different states of system 2.

Up State: $S_0 \equiv (B_N,P_N)$, $S_1 \equiv (B_{NP},P_N)$, $S_2 \equiv (B_N,P_{NP})$, $S_3 \equiv (B_N,P_F)$, $S_4 \equiv (B_F,P_N)$
Down State: $S_5 \equiv (B_F, P_F)$

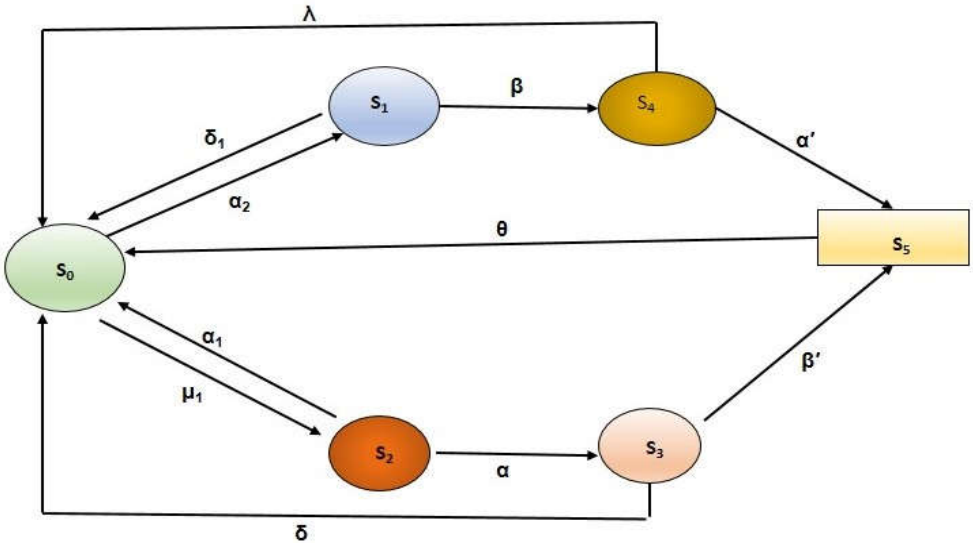


Figure 2: Diagram illustrating the Second System's state transition

7. Measures of System Effectiveness of System 2

7.1 Mean Time to System Failure: Using above aforementioned presumptions and first order differential equation approach MTSF of the suggested system is analysed. Pi(t) is defined, using Figure 2, as the probability that the system is in state Si at time t. The probability row vector at time t is represented by P(t).

Take into consideration the initial conditions as

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0)] = [1,0,0,0,0,0] \quad (11)$$

Here is the differential equation system that was derived:

$$\frac{dp\ (t)}{dt} = \quad (\alpha_2 + \alpha_1)\ P_0(t) + \delta_1 P_1(t) + \mu_1\ P_2(t) + \delta P_3(t) + \ \lambda P_4(t) + \theta P_5(t)$$

$$\frac{dp_1(t)}{dt} = \quad (\beta + \delta_1)p_1(t) + \alpha_2 P_0(t)$$

$$\frac{dP_2(t)}{dt} = \quad (\alpha + \mu_1)P_2(t) + \alpha_1 P_0(t)$$

$$\frac{dP_3(t)}{dt} = \quad (\beta' + \delta)P_3(t) + \alpha P_2(t)$$

$$\frac{dP_4(t)}{dt} = \quad (\lambda + \alpha')P_4(t) + \beta P_1(t)$$

$$\frac{dP_5(t)}{dt} = \quad \theta P_5(t) + \beta' P_3(t) + \alpha' P_4(t)$$

This can be expressed as in the form of system of differential equation:

$$P = A\ P \quad (12)$$

where

$$A = \begin{matrix} (\alpha_2 + \alpha_1) & \delta_1 & \mu_1 & \delta & \lambda & 0 \\ \alpha_2 & (\beta + \delta_1) & 0 & 0 & 0 & 0 \\ \alpha_1 & 0 & (\alpha + \mu_1) & 0 & 0 & 0 \\ 0 & 0 & \alpha & (\beta' + \delta) & 0 & 0 \\ 0 & \beta & 0 & 0 & (\alpha' + \lambda) & 0 \\ 0 & 0 & 0 & \beta' & \alpha' & \theta \end{matrix}$$

In order to streamline the assessment of the transitions solution, eliminating the row and columns in matrix A that correspond to the absorption state.

This modified matrix, named Q, is created by transposing A. The expected time needed to attain an absorbing condition is derived from

$$E[T_{P(0)\rightarrow(absorbing)}] = \mathrm{P}(0)\int_0^{\infty} e^{Qt}dt$$

and

$$\int_0^{\infty} e^{Qt}dt = \quad Q^{-1}, \text{ since } Q^{-1} < 0$$

where

$$\mathrm{Q} = \begin{matrix} (\alpha_2+\alpha_1) & \alpha_2 & \alpha_1 & 0 & 0 \\ \delta_1 & (\beta+\delta_1) & 0 & 0 & \beta \\ \mu_1 & 0 & (\alpha+\mu_1) & \alpha & 0 \\ \delta & 0 & 0 & (\beta'+\delta) & 0 \\ \lambda & 0 & 0 & 0 & (\alpha'+\lambda) \end{matrix}$$

$$\mathrm{MTSF}_2 = E[T_{P(0)\rightarrow(absorbing)}] = P(0)(-Q^{-1}) \begin{matrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{matrix} \tag{13}$$

$$\mathrm{MTSF}_2 = \frac{\alpha_2\beta\alpha'(\alpha+\mu_1)(\delta+\alpha')+\alpha_1\alpha\beta'(\lambda+\alpha')(\beta+\delta_1)}{(\lambda+\alpha')[(\alpha+\mu_1+\alpha_1)(\beta+\delta_1)(\delta+\beta')+\alpha_1\alpha(\beta+s_1)]+\alpha_2(\delta+\beta')(\alpha+\mu_1)(\lambda+\alpha'+\beta)} \tag{14}$$

7.2 Steady-State Availability Analysis: For steady state availability analysis initial condition is same as given in equation (11).

From equation (11), this can be expressed as in the form of system of differential equation:

$$\begin{matrix} \dot{p}_0 \\ \dot{p}_1 \\ \dot{p}_2 \\ \dot{p}_3 \\ \dot{p}_4 \\ \dot{p}_5 \end{matrix} = \begin{matrix} (\alpha_2+\alpha_1) & \delta_1 & \mu_1 & \delta & \lambda & 0 \\ \alpha_2 & (\beta+\delta_1) & 0 & 0 & 0 & 0 \\ \alpha_1 & 0 & (\alpha+\mu_1) & 0 & 0 & 0 \\ 0 & 0 & \alpha & (\beta'+\delta) & 0 & 0 \\ 0 & \beta & 0 & 0 & (\alpha'+\lambda) & 0 \\ 0 & 0 & 0 & \beta' & \alpha' & \theta \end{matrix} \begin{matrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{matrix}$$

The expression for steady-state availability is expressed as

$$A_{T_2}(\infty) = 1 \quad P_5(\infty) \tag{15}$$

In the context of steady-state availability, the state probabilities' derivatives reach a point of equilibrium where they become zero. This implies that the product of matrix A and the probability at infinity, denoted as P(∞), is equal to zero. i.e., $A\,P(\infty) = 0$

$$\begin{matrix}(\alpha_2+\alpha_1) & \delta_1 & \mu_1 & \delta & \lambda & 0 \\ \alpha_2 & (\beta+\delta_1) & 0 & 0 & 0 & 0 \\ \alpha_1 & 0 & (\alpha+\mu_1) & 0 & 0 & 0 \\ 0 & 0 & \alpha & (\beta'+s) & 0 & 0 \\ 0 & \beta & 0 & 0 & (\alpha'+\lambda) & 0 \\ 0 & 0 & 0 & \beta^1 & \alpha^1 & \theta \end{matrix} \begin{matrix} p_1(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \end{matrix} = \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix} \tag{16}$$

$$P_0(\infty) + p_1(\infty) + P_2(\infty) + p_3(\infty) + p_4(\infty) + p_5(\infty) = 1 \tag{17}$$

To obtain $P_5(\infty)$,replace (17) in one of the redundant rows of (16) and then using MATLAB software to get the matrix-form solution to the following linear equation system.

$$\begin{matrix}(\alpha_2+\alpha_1) & s_1 & \mu_1 & \delta & \lambda & 0 \\ \alpha_2 & (\beta+\delta_1) & 0 & 0 & 0 & 0 \\ \alpha_1 & 0 & (\alpha_1) & 0 & 0 & 0 \\ 0 & 0 & \alpha & (\beta'+\beta) & 0 & 0 \\ 0 & \beta & 0 & 0 & (\alpha'+\lambda) & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{matrix} \begin{matrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \end{matrix} = \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{matrix} \tag{18}$$

The steady state probabilities for Figure 2 are achieved from solution of equation(18). For $AT_2(\infty)$, the explicit expression is:

$$AT_2(\infty) = \frac{\begin{matrix}\theta(\lambda+\alpha')(\alpha+\mu_1)(\beta+\delta_1)(\delta+\beta') + \alpha_2\theta(\alpha+\mu_1)(\delta+\beta')(\lambda+\alpha'+\beta) \\ +\alpha_1\theta(\lambda+\alpha')(\beta+\delta_1)(\delta+\beta'+\alpha)\end{matrix}}{\begin{matrix}2\beta(\delta+\beta')(\theta+\alpha')(\alpha+\mu_1)+\alpha_1\beta'(\lambda+\alpha')(\beta+\delta_1)(\alpha+\theta)+ \\ \delta\theta(\alpha_2+\beta)(\lambda+\alpha')(\alpha+\mu_1) + \delta_1\theta(\delta+\beta')(\lambda+\alpha')(\alpha+\mu_1) + \\ \alpha_1\theta(\beta+\delta_1)(\lambda+\alpha')(\alpha+\delta) + \beta'\theta(\alpha_2+\beta)(\alpha+\mu_1)(\lambda+\alpha')\end{matrix}} \tag{19}$$

8. Result and Discussion

Ploting the graph for reliability measures for system 1 and system 2 versus α and β, separately , while keeping the other parameters fixed at α'=1.5, β'=1.5, δ = 2.5, λ = 2, $\mu_1 = \alpha_2 = 2.5$, $\delta_1 = \alpha_1 = 3$, θ = 3.5 in order to observe the behavior of

the system.Taking $\beta = 0.4$ for the curve against α, and we take $\alpha = 0.4$ in addition to other parameters for the curve against β.

Table 1: The relationship between the first and second systems' availability, MTSF, and failure rate "α"

α	**$MTSF_1$**	**$MTSF_2$**	$AT_1(\infty)$	$AT_2(\infty)$
0.1	5.44	17.72	0.904	0.984
0.2	4.72	14.26	0.907	0.980
0.3	4.18	12.05	0.910	0.976
0.4	3.78	10.51	0.912	0.973
0.5	3.45	9.39	0.915	0.970
0.6	3.18	8.53	0.917	0.967
0.7	2.97	7.84	0.918	0.965
0.8	2.79	7.30	0.921	0.962
0.9	2.63	6.84	0.923	0.960
1.0	2.50	6.46	0.924	0.957

Table 2: The relationship between the first and second systems' availability, MTSF, and failure rate "β"

β	**$MTSF_1$**	**$MTSF_2$**	$AT_1(\infty)$	$AT_2(\infty)$
0.1	5.85	46.8	0.975	0.982
0.2	4.90	29.7	0.953	0.979
0.3	4.25	22.11	0.932	0.976
0.4	3.78	17.81	0.913	0.973
0.5	3.41	15.05	0.894	0.971
0.6	3.12	13.12	0.870	0.968
0.7	2.89	11.83	0.860	0.966
0.8	2.70	10.61	0.840	0.964
0.9	2.53	9.75	0.830	0.962
1.0	2.40	9.01	0.820	0.960

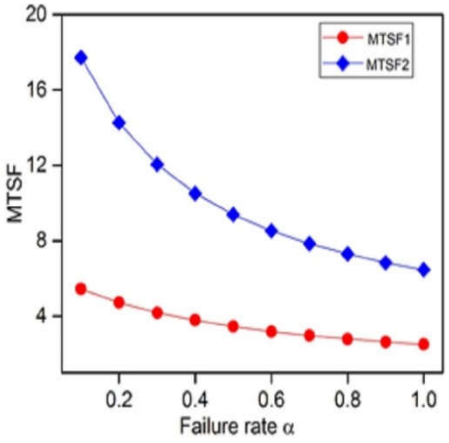


Figure 3: MTSF against α

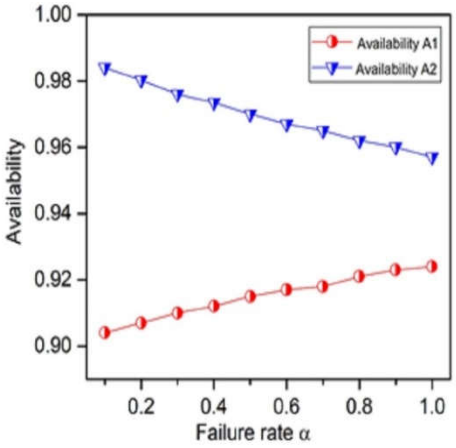


Figure 4: Availability against α

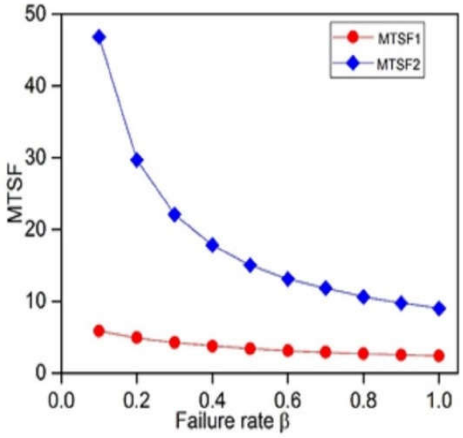


Figure 5: MTSF against β

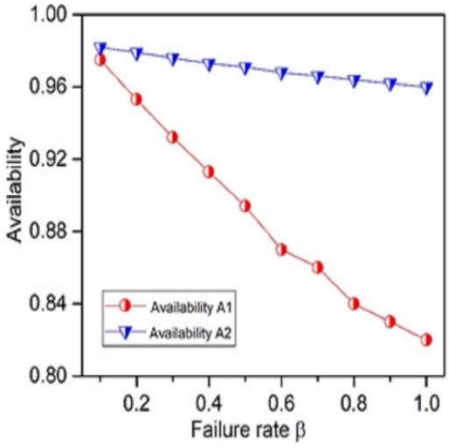


Figure 6: availability against β

As the value of α increases, the MTSF and availability of both systems decline, whereas the availability of system 1 marginally increases, as seen in figures 3 and 4. The graph makes it evident that system 2 has a higher MTSF and availability than system 1.

Figures 5 and 6 show the results of the two systems under study plotted against the failure rate β, representing the MTSF and the steady-state availability. It is evident from the figures that System 2 has a higher MTSF and stable availability than System 1.

9. Conclusion

This research paper explores the reliability measures of two dissimilar electrical vehicles namely Hybrid Electric Vehicles (HEVs) and Plug-in Hybrid Electric Vehicles (PHEVs) through the application of linear differential equations (LDE) to model key performance indicators such as Mean Time to System Failure (MTSF) and steady-state availability. The impact of failure rate on both systems monitored in order to observe system behaviour. The graphical demonstration of both the systems for hypothetical value are shown in figures 3 to 6 and from this conclude that:

- **PHEVs** tend to have higher reliability and better availability due to their ability to use both electric and petrol power sources, unlike HEVs, which rely more on regenerative braking.

- The study quantifies and compares the reliability of both systems, emphasizing that PHEVs generally perform better in terms of failure resistance and availability.

The analysis highlights how hybrid systems with advanced features like recharging and petrol backup provide improved system dependability and offer a significant advantage in terms of real-world usability, especially in cases of battery failure or depletion.

The findings confirm that while current modelling techniques offer valuable insights into the reliability of these vehicles, there is significant potential for further advancements. By integrating machine learning for predictive maintenance, utilizing real-time IoT data for continuous monitoring, and enhancing battery technology with solid-state innovations, the reliability of HEVs and PHEVs can be substantially improved. Additionally, the application of AI-based control systems will further elevate vehicle performance and reliability.

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