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ALMOST NORMALITY VIA SOFT IDEAL

Abstract. The present paper introduces a new soft separation axiom called soft almost -normality which is strictly weaker than the axiom of soft -normality due to Guler and Kale [6]. Some of the properties and characterizations of soft almost -normal ideal topological spaces have been studied.

Keywords: Soft Sets, Soft Topology, Soft Ideal, Soft -normaland Soft Almost -normal Spaces.

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1. Introduction

In 1999, Molodtsov [13] introduced the concept of soft sets. Since the inception of soft sets many authors such as Ali *et al*. [1], Maji *et al*. [11], Pie and Miao [15] contributed in the development of soft set theory. In 2011, Shabirand Naz [18] introduced the concept of soft topological spaces as an extension of topological spaces. In the same paper they studied the concept of soft closure, soft interior, soft neighbourhood, soft subspaces and soft separation axioms. The soft separation axioms further studied by Georgiou, Megaritis and Petropoulos [5] using the notion of soft point. The study of soft ideal topological spaces was initiated by Kandil *et al*. [9] in the year 2014. In 2015, Gulerand Kale [6] introduced and studied soft -normal spaces.The purpose of thispaper is to introduceand study the axiom of soft almost -normality in soft ideal topological spaces.

2. Preliminaries

Throughout this paper X denotes a nonempty set, Ω denotes the set of parameters and $S(X, \Omega)$ refers to the family of all soft sets of X relative to Ω. For the basic notions on soft setsthe readers should refer [2, 5, 10, 11, 18, 19,].

A non-empty collection Γ of subsets of a non-empty set X is said to be topology on X if it contains empty soft set ϕ and whole soft set X andclosed with respect to arbitrary union and finite intersection. If Γ is a soft topology on X then the triplet (X, Γ, Ω) is called a soft topological space and members of Γ are called as soft open sets in X. The complement of a soft open sets is called soft closed sets in X. The family of all soft closed sets of a soft topological space is denoted by $SC(X, \Omega)$.

Definition 2.1 [18, 20]: Let (ξ, Ω) be a soft set of a soft topological space (X, Γ, Ω). Then the closure and interior of (ξ, Ω) are defined as follows:

$$Cl(\xi, \Omega) = \cap \{(\sigma, \Omega): (\sigma, \Omega) \in SC(X, \Omega) \text{ and } (\xi, \Omega) \subseteq (\sigma, \Omega)\}.$$

$$Int(\xi, \Omega) = \cup \{(\sigma, \Omega): (\sigma, \Omega) \in \Gamma \text{ and } (\sigma, \Omega) \subseteq (\xi, \Omega)\}.$$

Lemma 2.1 [3, 18, 20]: Let (X, Γ, Ω) be a soft topological space and let (ξ, Ω), $(\sigma, \Omega) \in S(X,\Omega)$. Then:

(i) $(\xi, \Omega) \in SC(X, \Omega) \Leftrightarrow (\xi, \Omega) = Cl(\xi, \Omega)$

(ii) $(\xi, \Omega) \subseteq (\sigma, \Omega) \Rightarrow Cl\,(\xi, \Omega) \subseteq Cl(\sigma, \Omega)$.

(iii) $(\xi, \Omega) \in \Gamma \Leftrightarrow (\xi, \Omega) = Int\,(\xi, \Omega)$.

(iv) $(\xi, \Omega) \subseteq (\sigma, \Omega) \Rightarrow Int\,(\xi, \Omega) \subseteq Int\,(\sigma, \Omega)$.

(v) $(Cl\,(\xi, \Omega))^c = Int\,((\xi, \Omega)^c)$.

(vi) $\big(Int\,(\xi, \Omega)\big)^c = Cl\,((\xi, \Omega)^c)$.

Definition 2.2 [8]: Let Y be a non-empty subset of a soft topological space (X, Γ, Ω).Then the family $\Gamma_Y = \{(\xi_Y, \Omega):(\xi, \Omega) \in \Gamma\}$ is called soft relative topology on Y and (Y,Γ_Y,Ω) is called a soft subspace of (X, Γ, Ω).

Lemma 2.2 [18]: Let (Y,Γ_Y, Ω) be a soft subspace of a soft topological space (X, Γ, Ω) and $(\xi, \Omega) \in \Gamma_Y$. If $\widetilde{Y} \in \Gamma$ then $(\xi, \Omega) \in \Gamma$.

Lemma 2.3 [18]: Let (Y,Γ_Y,Ω) be a soft subspace of a soft topological space (X,Γ,Ω) and (ξ,Ω) be a soft set over X, then:

(i) $(\xi,\Omega)\in\Gamma_Y \quad (\xi,\Omega)=\widetilde{Y}\cap(\sigma,\Omega)$ for some $(\sigma,\Omega)\in\Gamma$.

(ii) $(\xi,\Omega)\in SC(Y,\Omega) \quad (\xi,\Omega)=\widetilde{Y}\cap(\sigma,\Omega)$ for some $(\sigma,\Omega)\in SC(X,\Omega)$.

Lemma 2.4 [17]: Let (Y,Γ_Y,Ω) be a soft subspace of a soft topological space (X,Γ,Ω). Then a soft set $(\xi_Y,\Omega)\in SC(Y,\Omega)\Rightarrow(\xi_Y,\Omega)\in SC(X,\Omega) \quad \widetilde{Y}\in SC(X,\Omega)$,

Definition 2.3 [7]: A soft set (ξ,Ω) in a soft topological space (X,Γ,Ω) is said to be:

(i) Soft regular open if $(\xi,\Omega)=\mathrm{Int}(\mathrm{Cl}(\xi,\Omega))$.

(ii) Soft regular closed if $(\xi,\Omega)=\mathrm{Cl}(\mathrm{Int}(\xi,\Omega))$.

Remark 2.1 [8]: Every soft regular open (resp. soft regular closed) set is soft open (resp. soft closed). But a soft regular open (resp. soft regular closed) set may fails to be soft open (resp. soft closed)

Definition 2.4 [10]: Let $S(X,\Omega)$ and $S(Y,\Psi)$ be families of soft sets over X and Y respectively. Let $u: X\to Y$ and $p: \Omega\to\Psi$ be mappings. Then a mapping $f_{pu}: S(X,\Omega)\to S(Y,\Psi)$ is defined as:

(i) Let $(\xi,\Omega)\in S(X,\Omega)$. Then the image of (ξ,Ω) under f_{pu} is written as $f_{pu}(\xi,\Omega)=(f_{pu}(\xi), p(\Omega))\in S(Y,\Psi)$ such that

$$f_{pu}(\xi)(k)=\begin{cases}\bigcup_{e\in p^{-1}(k)\cap\Omega} u(\xi(e)), & p^{-1}(k)\cap\Omega\neq\phi\\ \phi, & p^{-1}(k)\cap\Omega=\phi.\end{cases}$$

For all $k\in\Psi$.

(ii) Let $(\mu,\Psi)\in S(Y,\Psi)$. Then the inverse image of (μ,Ψ) under f_{pu} is given by

$$f_{pu}^{-1}(\mu)(e)=\begin{cases}u^{-1}\mu(p(e)), & p(e)\in\Psi\\ \phi, & \text{otherwise}\end{cases}$$

For all $e\in\Omega$.

Definition 2.5 [12]**:** A mapping f_{pu}:$S(X,\Omega) \to S(Y,\Psi)$ is said to be soft injective (respectively surjective, bijective) if the mappings $u : X \to Y$ and $p : \Omega \to \Psi$ are injective (respectively, surjective, bijective).

Definition 2.6 [6, 20, 21]: A soft mapping f_{pu}: $(X,\Gamma,\Omega) \to (Y,\eta,\Psi)$ is called:

(i) Soft continuous if $f_{pu}^{-1}(\mu,\Psi) \in \Gamma$ for all soft sets $(\mu,\Psi) \in \eta$.

(ii) Softopen if $f_{pu}(\xi,\Omega) \in \eta$ for all soft sets $(\xi,\Omega) \in \Gamma$.

(iii) Soft homeomorphism if f_{pu} is bijection, soft continuous and soft open.

Definition 2.7 [20]: A soft set $(\xi,\Omega) \in S(X,\Omega)$ is called a soft point if there exists a point $x \in X$ and $\beta \in \Omega$ such that $\xi(\beta) = \{x\}$ and $\xi(\beta^c) = \phi$ for each $\beta^c \in \Omega \quad \{\beta\}$ and the soft point (ξ,Ω) is denoted by x_β. The family of all soft points over X is denoted by $SP(X,\Omega)$.

Definition 2.8 [20]: A soft point $x_\beta \in (\sigma,\Omega)$ if $x_\beta \subseteq (\sigma,\Omega)$.

Lemma 2.5 [4,14]: Let (ξ,Ω), $(\sigma,\Omega) \in S(X,\Omega)$ and $x_\beta \in SP(X,\Omega)$. Then we have:

(i) $x_\beta \in (\xi,\Omega) \quad x_\beta \quad (\xi,\Omega)^c$.

(ii) $x_\beta \in (\xi,\Omega) \cup (\sigma,\Omega) \quad x_\beta \in (\xi,\Omega)$ or $x_\beta \in (\sigma,\Omega)$.

(iii) $x_\beta \in (\xi,\Omega) \cap (\sigma,\Omega) \quad x_\beta \in (\xi,\Omega)$ and $x_\beta \in (\sigma,\Omega)$.

Definition 2.9 [9]: A non-empty family of soft sets over (X,Ω) is said to be a soft ideal on X if:

(i) $(\xi,\Omega),(\sigma,\Omega) \in \quad (\xi,\Omega) \cup (\sigma,\Omega) \in$.

(ii) $(\xi,\Omega) \in$ and $(\sigma,\Omega) \subseteq (\xi,\Omega) \quad (\sigma,\Omega) \in$.

A soft topological space (X,Γ,Ω) with a soft ideal is called soft ideal topological space and is denoted by $SITS(X,\Gamma,\Omega,\ )$.

Definition 2.10 [6]: A $SITS(X,\Gamma,\Omega,\ )$ is said to be a soft -normal if for each pair of soft sets $(\xi,\Omega),(\sigma,\Omega) \in SC(X,\Omega)$ such that $(\xi,\Omega) \cap (\sigma,\Omega) = \phi, \exists$ soft

sets (μ,Ω) , $(\lambda,\Omega) \in \Gamma$ such that (ξ,Ω) $(\mu,\Omega) \in$, (σ,Ω) $(\lambda,\Omega) \in$ and $(\mu,\Omega) \cap (\lambda,\Omega) = \phi$.

3. Almost $\mathcal{I}$-Normal Spaces

Definition 3.1: A SITS $(X,\Gamma,\Omega,\ )$ is soft almost -normal, $\forall$ pair of soft sets $(\xi,\Omega) \in SC(X,\Omega)$ and $(\sigma,\Omega) \in SRC(X,\Omega)$ such that $(\xi,\Omega) \cap (\sigma,\Omega) = \phi$, $\exists$ soft sets $(\mu,\Omega),(\lambda,\Omega) \in \Gamma$ such that (ξ,Ω) $(\mu,\Omega) \in$,(σ,Ω) $(\lambda,\Omega) \in$ and $(\mu,\Omega) \cap (\lambda,\Omega) = \phi$.

Theorem 3.1: If a SITS $(X,\Gamma,\Omega,\ )$ is soft -normal then it is soft almost -normal.

Proof: Let $(\xi,\Omega) \in SC(X,\Omega)$ and $(\sigma,\Omega) \in SRC(X,\Omega)$ such that $(\xi,\Omega) \cap (\sigma,\Omega) = \phi$. Then $(\sigma,\Omega) \in SC(X,\Omega)$ because every soft regular closed set is soft closed. Therefore by definition 2.10, $\exists$ soft sets $(\mu,\Omega),(\lambda,\Omega) \in \Gamma$ such that (ξ,Ω) $(\mu,\Omega) \in$, (σ,Ω) $(\lambda,\Omega) \in$ and $(\mu,\Omega) \cap (\lambda,\Omega) = \phi$. Hence, by Definition 3.1, SITS $(X,\Gamma,\Omega,\ )$ is soft almost -normal.

Remark 3.1: The following example shows that a soft almost -normal space may fails to be soft -normal.

Example 3.1: Let $X = \{x_1, x_2, x_2\}$, $\Omega = \{\beta_1, \beta_2\}$ and $(\lambda,\Omega),(\mu,\Omega)$ and (ν,Ω) are soft sets defined as follows:

$(\lambda,\Omega) = \{(\beta_1,\{x_1\}),(\beta_2,\{x_1\}),$

$(\mu,\Omega) = \{(\beta_1,\{x_1,x_2\}),(\beta_2,\{x_1,x_2\})\},$

$(\nu,\Omega) = \{(\beta_1,\{x_1,x_3\}),(\beta_2,\{x_1,x_3\})\}.$

Let $= \{\phi,\{(\beta_1,\{x_2\}),(\beta_2,\phi)\},\{(\beta_1,\phi),(\beta_2,\{x_1\})\},\{(\beta_1,\{x_2\}),$ $\beta_2,\{x_1\})\}\}$ be a soft ideal on (X,Ω). Then the SITS$(X,\Gamma,\Omega,\ )$, where $\Gamma = \{\phi, X,(\lambda,\Omega),(\mu,\Omega),(\nu,\Omega)\}$ is soft almost -normal but not soft -normal.

The following theorem summarizes the properties of soft almost -normal spaces.

Theorem 3.2: For a SITS$(X,\Gamma,\Omega,\ )$ we have the following:

(i) $(X,\Gamma,\Omega,\ )$is soft almost -normal.

(ii) For each soft set $(\xi,\Omega) \in \mathrm{SC}(\mathrm{X},\Omega)$ and $\forall, (\sigma,\Omega) \in \mathrm{SRC}(\mathrm{X},\Omega)$ containing $(\xi,\Omega), \exists$ a soft set $(\mu,\Omega) \in \Gamma$ such that $(\xi,\Omega) \quad (\mu,\Omega) \in$ and $\mathrm{Cl}(\mu,\Omega) \quad (\sigma,\Omega) \in$.

(iii) For every $(\xi,\Omega) \in \mathrm{SRC}(\mathrm{X},\Omega)$ and each soft set $(\sigma,\Omega) \in \Gamma$ containing $(\xi,\Omega), \exists\ (\mu,\Omega) \in \Gamma$ such that $(\xi,\Omega) \quad (\mu,\Omega) \in$ and $\mathrm{Cl}(\mu,\Omega) \quad (\sigma,\Omega) \in$.

(iv) For every $(\xi,\Omega) \in \mathrm{SC}(\mathrm{X},\Omega)$ and $(\sigma,\Omega) \in \mathrm{SRC}(\mathrm{X},\Omega)$ such that $(\xi,\Omega) \cap (\sigma,\Omega) = \phi$, $\exists$ soft sets $(\mu,\Omega), (\lambda,\Omega) \in \Gamma$ such that $(\xi,\Omega) \quad (\mu,\Omega) \in$, $(\sigma,\Omega) \quad (\lambda,\Omega) \in$ and $\mathrm{Cl}(\mu,\Omega) \widetilde{\cap} \mathrm{Cl}(\lambda,\Omega) = \phi$.

Proof: (i) $\dot{\Rightarrow}$ (ii) Let $(\xi,\Omega) \in \mathrm{SC}(\mathrm{X},\Omega)$ and $(\sigma,\Omega) \in \mathrm{SRO}(\mathrm{X},\Omega)$ such that $(\xi,\Omega) \quad (\sigma,\Omega) \in$. Then, $(\sigma,\Omega)^{\mathrm{c}} \in \mathrm{SRC}(\mathrm{X},\Omega)$ such that $(\xi,\Omega) \widetilde{\cap} (\sigma,\Omega)^{\mathrm{c}}\phi$. Therefore, $\exists\ (\mu,\Omega), (\lambda,\Omega) \in \Gamma$ such that $(\xi,\Omega) \quad (\mu,\Omega) \in$, $(\sigma,\Omega)^{\mathrm{c}} \quad (\lambda,\Omega) \in$ and $(\mu,\Omega) \widetilde{\cap} (\lambda,\Omega) = \phi$. Thus, $(\xi,\Omega) \quad (\mu,\Omega) \in$, $(\lambda,\Omega)^{\mathrm{c}} \quad (\sigma,\Omega) \in$ and hence $(\xi,\Omega) \quad (\mu,\Omega) \in$ and $\mathrm{Cl}(\mu,\Omega) \quad (\sigma,\Omega) \in$ because $(\lambda,\Omega)^{\mathrm{c}} \in \mathrm{SC}(\mathrm{X},\Omega)$.

(ii) $\dot{\Rightarrow}$ (iii) Let $(\xi,\Omega) \in \mathrm{SRC}(\mathrm{X},\Omega)$ and $(\sigma,\Omega) \in \Gamma$ containing (ξ,Ω). Then, $(\xi,\Omega)^{\mathrm{c}} \in \mathrm{SRO}(\mathrm{X},\Omega)$ and $(\sigma,\Omega)^{\mathrm{c}} \in \mathrm{SC}(\mathrm{X},\Omega)$ such that $(\sigma,\Omega)^{\mathrm{c}} \quad (\xi,\Omega)^{\mathrm{c}} \in$. Therefore by (ii), $\exists$ a soft set $(\delta,\Omega) \in \Gamma$ such that $(\sigma,\Omega)^{\mathrm{c}} \quad (\delta,\Omega) \in$ and $\mathrm{Cl}(\delta,\Omega) \quad (\xi,\Omega)^{\mathrm{c}} \in$. Then $(\xi,\Omega) \quad (\mathrm{Cl}(\delta,\Omega))^{c} \in$ and $(\delta,\Omega)^{\mathrm{c}} \quad (\sigma,\Omega) \in$. Put $(\mu,\Omega) = (\mathrm{Cl}(\delta,\Omega))^{\mathrm{c}}$. Then $(\mu,\Omega) \in \Gamma, (\xi,\Omega) \quad (\mu,\Omega) \in$ and $\mathrm{Cl}(\mu,\Omega) \quad (\sigma,\Omega) \in$.

(iii) $\dot{\Rightarrow}$(iv) Let $(\xi,\Omega) \in \mathrm{SRC}(\mathrm{X},\Omega)$ and $(\sigma,\Omega) \in \mathrm{SC}(\mathrm{X},\Omega)$ such that $(\xi,\Omega) \widetilde{\cap} (\sigma,\Omega) = \phi$. Then $(\sigma,\Omega)^{\mathrm{c}} \in \Gamma$ such that $(\xi,\Omega) \quad (\sigma,\Omega)^{\mathrm{c}} \in$. Therefore, $\exists\ (\delta,\Omega) \in \Gamma$ such that $(\xi,\Omega) \quad (\delta,\Omega) \in$ and $\mathrm{Cl}(\delta,\Omega) \quad (\sigma,\Omega)^{\mathrm{c}} \in$. Now the soft set $(\delta,\Omega) \in \Gamma$ containing $(\xi,\Omega) \in \mathrm{SRC}(\mathrm{X},\Omega)$, $\exists$ a soft set $(\mu,\Omega) \in \Gamma$ such that $(\xi,\Omega) \quad (\mu,\Omega) \in$ and $\mathrm{Cl}(\mu,\Omega) \quad (\delta,\Omega) \in$. Let $(\mathrm{Cl}(\delta,\Omega))^{\mathrm{c}}=(\lambda,\Omega)$. Then $\mathrm{Cl}(\mu,\Omega) \widetilde{\cap} \mathrm{Cl}(\lambda,\Omega) = \phi$ and $(\xi,\Omega) \quad (\mu,\Omega) \in$, $(\sigma,\Omega) \quad (\lambda,\Omega) \in$.

(iv) $\dot{\Rightarrow}$ (i) Obvious.

Lemma 3.1 [6]: Let $(\xi,\Omega),(\sigma,\Omega) \in \mathrm{S}(\mathrm{X},\Omega)$ and $\mathrm{f_{pu}}: \mathrm{S}(\mathrm{X},\Omega) \to \mathrm{S}(\mathrm{Y},\Psi)$ is a injective mapping. Then $\mathrm{f_{pu}}((\xi,\Omega) \quad (\sigma,\Omega)) = \mathrm{f_{pu}}(\xi,\Omega) \quad \mathrm{f_{pu}}(\sigma,\Omega)$.

Theorem 3.3: Soft almost- normality is preserved when mappings are soft closed, soft continuous and soft open.

Proof: Let $\mathrm{f_{pu}}$ be a soft closed, soft open and soft continuous mapping from a soft almost -normal space $(\mathrm{X},\Gamma,\Omega,\ )$ onto a $\mathrm{SITS}(\mathrm{Y},\eta,\Psi,\mathrm{J})$. Let $(\xi,\Psi) \in \mathrm{SC}(\mathrm{Y},\Omega)$ and $(\sigma,\Psi) \in \mathrm{SRO}(\mathrm{Y},\Omega)$ containing (ξ,Ψ). Put $\mathrm{f_{pu}^{-1}}(\xi,\Psi)=(\delta,\Omega)$ and

$f_{pu}^{-1}(\delta, \Psi) = (£, \Omega)$. Then $(\delta, \Omega) \in SC(X, \Omega)$ such that $(\delta, \Omega) \subseteq (£, \Omega)$ because f_{pu} is soft continuous. Now, $(£, \Omega)$ Int $(Cl(£, \Omega)) \in$, Int $(Cl(£, \Omega)) \in SRO(X, \Omega)$ containing $(\delta, \Omega) \in SC(X, \Omega)$ because $(£, \Omega) \in \Gamma$. Therefore by Theorem 3.2 (iii), $\exists\ (\mu, \Omega) \in \Gamma$ such that (δ, Ω) $(\mu, \Omega) \in$ and $Cl(\mu, \Omega)$ Int $\big(Cl\ ((£, \Omega)\big) \in$ because $(X, \Gamma, \Omega,\ )$ is soft almost -normal. By Lemma 3.1, $f_{pu}(\delta, \Omega)$ $f_{pu}(\mu, \Omega) \in J$ and $f_{pu}\big(Cl\ (\mu, \Omega)\big)$ $f_{pu}\left(\text{Int}\left(Cl((£, \Omega))\right)\right) \in J$.

Consequently, $Cl((£, \Omega) = Cl(f_{pu}^{-1}(\sigma, \Psi)) = f_{pu}^{-1}(Cl(\sigma, \Psi))$ because f_{pu} is soft continuous and soft open. Also, soft openness and soft continuity of f_{pu} implies Int $(f_{pu}^{-1}(Cl(\sigma, \Psi)) = f_{pu}^{-1}(\text{Int}(Cl(\sigma, \Psi)))$.

Thus, $f_{pu}\left(\text{Int}\left(Cl(£, \Omega)\right)\right) = \text{Int}\left(Cl(\sigma, \Psi)\right)$. Again since f_{pu} is soft open, $f_{pu}(\mu, \Omega) \in \eta$ and f_{pu} is soft closed and soft continuous, $f_{pu}\big(Cl(\mu, \Omega)\big) = (Cl\left(f_{pu}(\mu, \Omega)\right)$. Thus, $f_{pu}(\mu, \Omega) \in \eta$ such that (ξ, Ψ) $f_{pu}(\mu, \Omega) \in J$ and $(Cl(f_{pu}(\mu, \Omega))$ $(\sigma, \Psi) \in J$. Hence, (Y, η, Ψ, J) is soft almost J-normal.

Theorem 3.4: If a $SITS(X, \Gamma, \Omega,\ )$ is soft almost -normal and $(Y, \Omega) \in \Gamma \cap SC(X, \Omega)$. Then the soft subspace $(Y, \Gamma_Y, \Omega,\ _Y)$ is soft almost -normal.

Proof: Let $(\xi, \Omega) \in SC(Y, \Omega)$ and $(\sigma, \Omega) \in SRO(Y, \Omega)$ such that (ξ, Ω) $(\sigma, \Omega) \in$. Now $(Y, \Omega) \in SC(Y, \Omega)$ $(\xi, \Omega) \in SC(X, \Omega)$ and $(Y, \Omega) \in \Gamma$, we have (σ, Ω) $\text{Int}\big(Cl(\sigma, \Omega)\big) \in$. Since, $(\sigma, \Omega) \in SRO(Y, \Omega)$, we have $(\sigma, \Omega) = \text{Int}_Y(Cl_Y(\sigma, \Omega)) \supseteq \text{Int}(Cl(\sigma, \Omega)) \cap (Y, \Omega)$. Also, Int $(Cl(\sigma, \Omega))$ $\text{Int}(Cl(Y, \Omega)) = (Y, \Omega) \in$ and therefore $\text{Int}\big(Cl(\sigma, \Omega)\big)$ $(\sigma, \Omega) \in$. Hence, $(\sigma, \Omega) = \text{Int}\big(Cl(\sigma, \Omega)\big)$ and $(\sigma, \Omega) \in SRO(X, \Omega)$. Thus, $(\xi, \Omega) \in SC(X, \Omega)$ contained in a soft set $(\sigma, \Omega) \in SRO(X, \Omega)$. Since, $(X, \Gamma, \Omega,\ )$is soft almost -normal, $\exists$ a soft set $(\mu, \Omega) \in \Gamma$ such that (ξ, Ω) $(\mu, \Omega) \in$ and $Cl(\mu, \Omega)$ $(\sigma, \Omega) \in$. Thus, $(\mu, \Omega) \in \Gamma_Y$ such that (ξ, Ω) $(\mu, \Omega) \in$ and $(Cl_Y(\mu, \Omega))$ $(\sigma, \Omega) \in$. Hence,$(Y, \Gamma_Y, \Omega,\ _Y)$ is soft almost -normal.

Lemma 3.2: If (Y, Ω) is a soft regular closed subset of a $STS(X, \Gamma, \Omega)$ and (ξ, Ω) is a soft regular closed in soft subspace (Y, Γ_Y, Ω), then (ξ, Ω) is soft regular closed in X.

Theorem 3.5: If a $SITS(X, \Gamma, \Omega,\ )$is soft almost -normal and $(Y, \Omega) \in SRC(X, \Omega)$. Then the subspace $(Y, \Gamma_Y, \Omega,\ _Y)$ is soft almost -normal.

Proof: Suppose $(Y, \Gamma_Y, \Omega, \ _Y)$ be a soft subspace of a soft almost -normal space $(X, \Gamma, \Omega, \ )$ and $(Y, \Omega) \in SRC(X, \Omega)$. Let $(\xi, \Omega) \in SC(Y, \Omega)$ and $(\sigma, \Omega) \in SRC(Y, \Omega)$ such that $(\xi, \Omega) \cap (\sigma, \Omega) = \phi$. Since, $(Y, \Omega) \in SRC(X, \Omega)$ and $(\sigma, \Omega) \in SRC(Y, \Omega)$ by Lemma 3.2, $(\sigma, \Omega) \in SRC(X, \Omega)$. Since, $(X, \Gamma, \Omega, \ )$ is soft almost -normal $\exists$ soft sets $(\mu, \Omega), (\lambda, \Omega) \in \Gamma$ such that $(\xi, \Omega) \quad (\mu, \Omega) \in$, $(\sigma, \Omega) \quad (\lambda, \Omega) \in$ and $(\mu, \Omega) \cap (\lambda, \Omega) = \phi$. And so, $(\mu, \Omega) \cap \widetilde{Y} \in \Gamma_Y$ and $(\lambda, \Omega) \cap \widetilde{Y} \in \Gamma_Y$ such that $(\xi, \Omega) \quad ((\mu, \Omega) \cap \widetilde{Y}) \in$ and $(\sigma, \Omega) \quad ((\lambda, \Omega) \cap \widetilde{Y}) \in$ and $\left((\mu, \Omega) \cap \widetilde{Y}\right) \cap \left((\lambda, \Omega) \cap \widetilde{Y}\right) = \left((\mu, \Omega) \cap (\lambda, \Omega)\right) \cap \widetilde{Y} = \phi$. Hence, $(Y, \Gamma_Y, \Omega, \ _Y)$ is soft almost -normal.

Definition 3.2: A SITS $(X, \Gamma, \Omega, \ )$ is called soft semi -normal, if $\forall (\xi, \Omega) \in SC(X, \Omega)$ and if $\forall$ soft sets $(\sigma, \Omega) \supseteq (\xi, \Omega), \exists (\mu, \Omega) \in \Gamma$ such that $(\xi, \Omega) \quad (\mu, \Omega) \in \quad Int(Cl(\mu, \Omega)) \quad (\sigma, \Omega) \in$.

Remark 3.2: Soft -normality implies soft semi -normality, but the converse may be false.

Corollary 3.1: A soft almost -normal space is soft -normal if and only if it is soft semi -normal.

Theorem 3.6: Every soft semi -normal, soft almost -normal space is soft -normal.

Proof: Let $(\sigma, \Omega) \in \Gamma$ be a soft set such that $(\xi, \Omega) \subseteq (\sigma, \Omega)$ where $(\xi, \Omega) \in SC(X, \Omega)$. Then by soft semi -normality, $\exists$ a soft set $(\mu, \Omega) \in \Gamma$ such that $(\xi, \Omega) \quad (\mu, \Omega) \in$ and $Int(Cl(\mu, \Omega)) \quad (\sigma, \Omega) \in$. Put $(\chi, \Omega) = Int(Cl(\mu, \Omega))$. Then $(\chi, \Omega) \in SRO(X, \Omega)$ contained in $(\xi, \Omega) \in SC(X, \Omega)$. Therefore by Theorem 3.2(ii), $\exists \ (\lambda, \Omega) \in \Gamma$ such that $(\xi, \Omega) \quad (\lambda, \Omega) \in$ and $Cl(\lambda, \Omega) \quad (\chi, \Omega) \in$. Consequently $(\lambda, \Omega) \in \Gamma$ such that $(\xi, \Omega) \quad (\lambda, \Omega) \in$ and $Cl(\lambda, \Omega) \quad (\sigma, \Omega) \in$. Hence, $(X, \Gamma, \Omega, \ )$ is soft -normal.

Conclusions

In this paper we have introduced the notion of soft almost -normal spaces. An example is given to show that the soft almost -normal space may fails to be soft -normal.It has been shown that soft almost -normal space is soft -normal if and only if it is soft semi -normal.We studied that soft almost normality is preserved under soft closed, soft continuous and soft open mappings. Two theorems of soft subspaces with reference to soft almost -normal spaces has also been obtained. Several characterizations of soft almost -normal spaces have been obtained. We

believe that these results will contribute to the study of separation axioms in soft ideal topological spaces and will help the researcher for further studies.

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