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VOLATILITY DYNAMICS ANALYSIS OF NIFTY 500 INDEX USING ARCH AND GARCH MODELS: A COMPREHENSIVE STUDY

Abstract: This paper investigates the volatility dynamics of the NIFTY 500 Index using historical data from 2005 to 2024. Weemploy Autoregressive Conditional Volatility (ACV) and Generalized Autoregressive Conditional Volatility (GACV) models to analyze and model the volatility of dailylog returns. Our findings indicate significant ARCH effects, and the fitted ARCH (2) model demonstrates a robust framework for capturing the conditional volatility in the financial time series data.

Keywords: ARCH, GARCH, Volatility Modeling, NIFTY500, Financial Time Series, Econometrics.

Mathematics Subject Classification (2010) No.: 91B24, 91B26, 91B30.

1. Introduction

Volatility modeling is crucial in financial econometrics, offering insights into risk management and derivative pricing. The NIFTY 500 Index, representing a broad spectrum of the Indian stock market, provides an ideal subject for studying volatility dynamics. Financial markets are characterized by fluctuating prices that exhibit periods of high volatility followed by periods of relative calm, a phenomenon known as volatility clustering. Understanding these patterns is essential for investors, policy makers, and researchers.

In this paper, we aim to analyze the volatility patterns in the NIFTY 500 Index using ARCH and GARCH models. We employ historical data spanning from 2005 to 2024 to comprehensively analyze the volatility dynamics of daily log returns.

By doing so, we contribute to the existing literature by providing a detailed study of nearly two decades of data from the NIFTY 500 Index.

2. Data

We use historical data on the NIFTY 500 Index, obtained from Yahoo Finance, spanning from September 26, 2005, to June 10, 2024. The data includes daily adjusted closing prices, from which we compute the daily log returns. The choice of this time frame ensures a comprehensive analysis that encompasses various market cycles, including bull and bear markets, economic expansions, and recessions.

A. Adjusted Close Prices

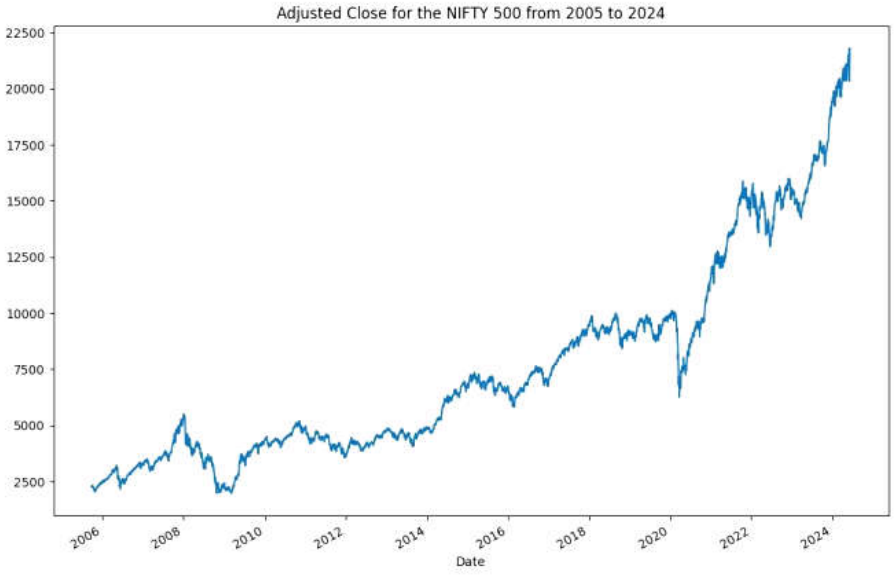


Fig. 1. Adjusted Close Prices of NIFTY 500 Index

B. Daily Log Returns: Daily log returns are computed as follows:

$$\text{Log Return}_t = \log \frac{Price_t}{Price_{t-1}}$$

This transformation helps to stabilize the variance and achieve stationarity, which are prerequisites for applying ARCH and GARCH models.

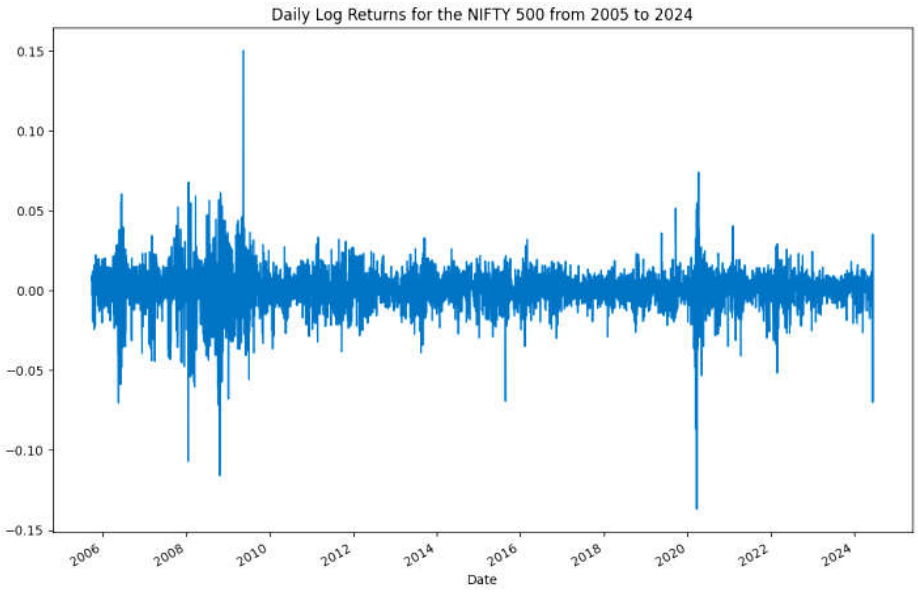


Fig. 2: Daily Log Returns of NIFTY 500 Index

3. Data Preprocessing

Data preprocessing plays a pivotal role in the analysis of time series data. The dataset was cleaned by removing any missing values, which could otherwise distort the results. The adjusted closing prices were then used to calculate the daily log returns, a common practice in financial econometrics to stabilize variance and achieve stationarity.

4. Methodology

A. Stationarity Test: To ascertain the suitability of the time series data for advanced analysis, we conducted the Augmented Dickey-Fuller (ADF) test. This statistical test is instrumental in assessing the stationarity of a series by detecting the presence of a unit root, which suggests non-stationarity. Stationarity is pivotal in time series analysis as it ensures reliable and meaningful statistical inferences, mitigating the risk of spurious findings.

B. Log Returns Calculation: The daily log returns were calculated from the adjusted closing prices. This transformation helps in stabilizing the variance and making the series stationary.

C. Modeling Volatility Dynamics: Autoregressive Conditional Heteroskedasticity (ARCH): The ARCH model, pioneered by Engle in 1982, is a statistical framework within time series analysis. It focuses on modeling the volatility of the current error term based on past error terms. This model is particularly adept at capturing volatility clustering, a prevalent characteristic in financial data where periods of high volatility tend to be followed by similar periods, and periods of low volatility likewise persist.

1) ***ARCH Model Specification*:** The fundamental equations of the ARCH model are as follows:

$$y_t = \mu + n_t$$

$$= \gamma_t v_t$$

$$\gamma_t^2 = \beta_0 + \beta_1 n_{t-1}^2 + \beta_2 n_{t-2}^2 + \ldots + \beta_q n_{t-q}^2$$

where:

- y_t represents the return at time t.
- μ denotes the mean return.
- η_t is the error term at time t, representing the deviation of the actual return from the mean.
- γ^2 signifies the conditional variance at time t, indicating the expected variability of η_t.
- v_t is a white noise process with zero mean and unit variance, ensuring that η t has the desired properties.
- β_0 is a constant term representing the baseline level of volatility.
- βi for i = 1, 2, . . . , q are coefficients that measure the influence of past squared errors on current volatility.

2) ***Estimation of ARCH Framework*:** The parameters of the ARCH model can be estimated using the method of maximum likelihood. The log-likelihood function for the ARCH model is formulated as follows:

$$L(\psi) = \quad 2^{m \ln(2\pi) - \sum_{2t=1}^{m} \ln\left(y_t^2\right) + t} \frac{2}{y_t^2}$$

where:

- m is the total number of observations.
- ψ represents the vector of parameters (μ, β_0 , β_1 , . . . , β_q).
- γ_t^2 is the conditional variance at time t as defined by theARCH model.
- η_t is the residual term at time t.

3) ***Model Diagnostics*****:** To assess the adequacy of the fitted ARCH model, it is important to perform diagnostic checks. One common diagnostic check is to investigate the existence of remaining ARCH impacts within the residuals. This can be done using the Lagrange Multiplier (LM) test, also known as the ARCH-LM test. The test involves regressing the squared residuals on lagged squared residuals and checking for significant coefficients.

The null hypothesis of the ARCH-LM test is that there are no remaining ARCH effects, i.e., all coefficients of the lagged squared residuals are zero. A significant test statistic indicates the presence of remaining ARCH effects, suggesting that a higher-order ARCH model or a GARCH model may be more appropriate.

Statistic	Value
LM Statistic	30.33
p-value(L.M)	0.00076

Table 1 - ARCH-LM Test Result

4) *Model Selection*: When evaluating the suitable ARCH model, it is essential to take into account the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC).These measures assist in comparing various models by penalizing complexity while favoring models that provide a better balance of explanatory power and parsimony. Optimal model selection is guided by minimizing both AIC and BIC values.

$$AIC_{new} = 2m \quad 2\log(L)$$

$$BIC_{new} = m\log(n) \quad 2\log(L)$$

Where m denotes the number of parameters in the model, *L* represents the maximum likelihood estimate of the likelihood function, and n indicates the sample size.

By carefully comparing AIC and BIC values, we can identify the most suitable model for the data.

D. ARCH Effect Test: We examined the existence of ARCH effects in the log returns series using the ARCH test. This test evaluates whether there is substantial autocorrelation in the squared residuals of the log returns, which suggests the occurrence of volatility clustering.

E. Model Fitting: Three models were fitted to the log returns series:

1) ARCH (1) Model: This model uses one lag of the squared residuals to model the conditional variance.

2) ARCH (2) Model: This model uses two lags of the squared residuals.

3) GARCH (1,1) Model: This model incorporates an ARCH term based on the lagged squared residuals and a GARCH term utilizing the lagged conditional variance to model the variance dynamics.

5. Results

A. Stationarity Test: The ADF test yielded a p-value indicating whether the adjusted closing prices were stationary. In our analysis, the ADF test results confirmed that the log returns of the NIFTY 500 Index are stationary, making them suitable for further analysis using ARCH and GARCH models.

B. Log Returns: The mean of the daily log returns was calculated, and the log returns series was plotted to visualize the data.

C. ARCH Effect Test: The ARCH test results indicated significant ARCH effects in the log returns series, justifying the use of ARCH and GARCH models for further analysis. Significant ARCH effects imply that past squared residuals have predictive power for future volatility.

D. Model Results: 1) *ARCH* (1) *Model*: Mean of Daily Log Returns = 0.00049

Statistic	Value
LM Statistic	30.33
p-value(L.M)	0.00076
F Statistic	3.05
p-value (F)	0.00074

Table 2: ARCH (1) Test Results

Parameter	Coefficient	p-value
μ	0.00072696	0.0001046
ω	0.00010542	9.154e-84
α_1	0.4001	2.176e-12

Table 3: ARCH (1) Model Coefficients

2) *ARCH* (2) *Model*: Mean of Daily Log Returns = 0.00049

Statistic	Value
LM Statistic	30.59
p-value(L.M)	0.00068
F Statistic	3.07
p-value (F)	0.00067

Table 4: ARCH (2) Test Results

Parameter	Coefficient	p-value
μ	0.00094068	6.958e-08
ω	0.000070296	6.430e-84
α_1	0.3000	3.960e-07
α_2	0.3000	7.020e-14

Table 5: ARCH (2) Model Coefficients

3) *GARCH*(1,1) *Model*: Mean of Daily Log Returns = 0.00049

Statistic	Value
LM Statistic	30.38
p-value(L.M)	0.00074
F Statistic	3.05
p-value (F)	0.00072

Table 6: GARCH (1,1) Test Result

Parameter	Coefficient	p-value
μ	0.00084699	7.572e-10
ω	0.0000035148	0.000
α_1	0.1000	0.000
β_1	0.8800	0.000

Table 7: GARCH (1,1) Modle Coefficients

E. Model Comparison: To determine the best model, we compare the AIC and BIC values for each model. Here are the AIC and BIC values for the NIFTY 500 data:

Model	AIC	BIC
ARCH(1)	-151800	-151780
ARCH(2)	-151811	-151779
GARCH(1,1)	-151805	-151788

Table 8: AIC and BIC Values for Models

F. Forecasting with ARCH (2) Model: Using the fitted ARCH (2) model, we forecast the conditional variance for the next 30 days. The forecasted conditional variances provide insights into the expected volatility of the NIFTY 500 Index.

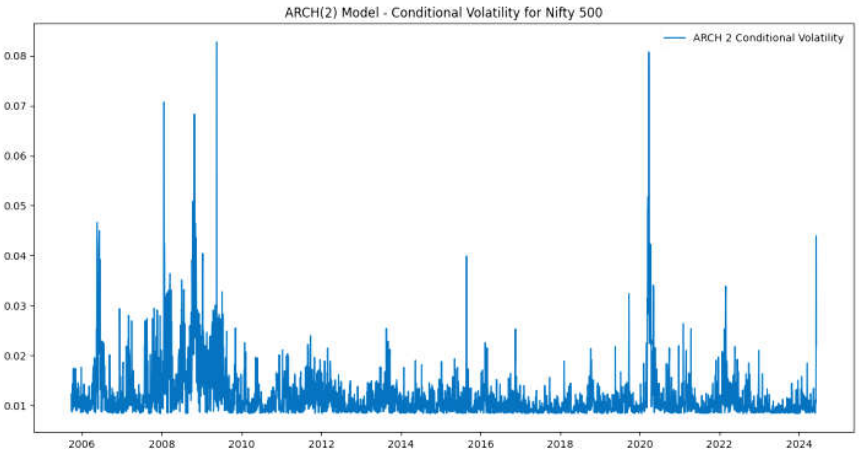


Fig. 3: 30-Day Volatility Forecast Using ARCH (2) Model

4. Conclusion

By observing the AIC and BIC values, the ARCH (2) model has the lowest values, indicating the best fit to the NIFTY 500 data. The ARCH (2) model has an AIC of -151811and a BIC of -151779, making it the preferred model for forecasting volatility in the NIFTY 500 Index.

5. Conclusion

This paper provides a comprehensive analysis of the volatility dynamics in the NIFTY 500 Index using ARCH and GARCH models. Through rigorous testing and model fitting, we found significant ARCH effects in the daily log returns series, confirming the suitability of these models for analyzing volatility.

The ARCH (2) model demonstrated superior performance over other models, as evidenced by its favorable Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) scores. Its capability to effectively capture the conditional variance underscores its robustness in modeling the volatility of the NIFTY 500 Index.

The insights gained from this analysis are valuable for risk management and financial decision-making. Investors and policymakers can use these models to better understand market volatility, anticipate potential risks, and make informed decisions. “This research adds to the extensive literature on financial econometrics by conducting a comprehensive analysis of volatility in the Indian stock market spanning nearly two decades.”

Future research could extend this work by exploring higher-order GARCH models or incorporating exogenous variables to further enhance volatility forecasting. Additionally, applying these models to other financial indices or markets could provide comparative insights and broaden the applicability of the findings.

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