Journal of Indian Acad. Math.
Vol. 46, No. 1 (2024) pp. 13-24

ISSN: 0970-5120

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SEVERAL FIXED POINT THEOREM FOR FUZZY MAPPINGS IN COMPLEX VALUED METRIC SPACES

Abstract: The main purpose of this paper is to establish some fixed point results for fuzzy mappings in complete complex valued metric spaces. The derived results generalize some theorems as in the existing literature. Some examples and counter examples are provided to justify the theorem proved here.

Keywords: Fuzzy Mappings, Fixed Point, Hausdorff Distance, Complex Valued Metric Spaces, Fuzzy Fixed Point.

Mathematical Subject Classification No.: 03B52, 03E72, 47H10, 46G20, 58B12.

1. Introduction

Fixed point theory is most interesting and dynamic area of research in functional analysis. This principal plays an important and key role in investigating the existence and uniqueness of solution to various problems in mathematics, physics, engineering, medicines, and social sciences which leads to mathematical models design by system of nonlinear integral equations, functional equations, and differential equations. Banach contraction principal has been generalized in different directions by changing the underlying space.

The concept of fuzzy set was first introduced by Zadeh [10]. Later Weiss [9] obtained many fixed point results for many fixed point theorems for fuzzy mappings in metric spaces. Heilpern [5] initiated the idea of fuzzy contraction mappings and proved a fixed point theorem for fuzzy contraction mappings which is a fuzzy

analogue of Nadler's fixed point theorem for multi-valued mappings [7]. Further work on fuzzy mappings can be seen in [8].

In this paper, we obtain a fixed point theorem and a common fixed point theorem for fuzzy mappings in complex valued complete metric space. An example is also given which supports the obtained results.

Here, the obtained results for fuzzy mapping in metric space, fuzzy mapping in complex valued metric space under certain constrictive conditions are helpful for Hausdorff dimensions computing which are helpful in high energy physics. In high energy physics these results are also helpful for solving the arising geometric problems due to the involvement of fuzzy sets.

2. Preliminaries

Definition 2.1 [3]: *Assume* $\mathbb{C}$ *is the set of complex numbers. For* $\zeta_1, \zeta_2 \in \mathbb{C}$ *we define a partial order* $\precsim$ *on* $\mathbb{C}$ *as follows:*

(Pi) $\zeta_1 \precsim \zeta_2 \Leftrightarrow Re(\zeta_1) \leq Re(\zeta_2)$ *and* $Im(\zeta_1) \leq Im(\zeta_2)$;

(Pii) $\zeta_1 \prec \zeta_2 \Leftrightarrow Re(\zeta_1) < Re(\zeta_2)$ *and* $Im(\zeta_1) < Im(\zeta_2)$;

$(Piii)$ $\zeta_1 \precnsim \zeta_2 \Leftrightarrow Re(\zeta_1) = Re(\zeta_2)$ *and* $Im(\zeta_1) \leq Im(\zeta_2)$;

(Piv) $\zeta_1 = \zeta_2 \Leftrightarrow Re(\zeta_1) = Re(\zeta_2)$ *and* $Im(\zeta 1) = Im(\zeta_2)$.

Clearly if $a \leq b \Rightarrow az \precsim bz$, for all $z \in \mathbb{C}$ and for all $a, b \in \Re$. Note that if $\zeta_1 \neq \zeta_2$ and one of (Pi), (Pii) and (Piii) is satisfied then $\zeta_1 \precnsim \zeta_2$ and write $\zeta_1 = \zeta_2$ if only (Piv) is satisfied. Note that

(i) $0 \precsim \zeta_1 \precnsim \zeta_2 \Rightarrow |\zeta_1| < |\zeta_2|, \forall \zeta_1, \zeta_2 \in \mathbb{C}$;

(ii) $\zeta_1 \precnsim \zeta_2$ and $\zeta_2 \prec \zeta_3 \Rightarrow \zeta_1 \prec \zeta_3, \forall \zeta_1, \zeta_2, \zeta_3 \in \mathbb{C}$.

Definition 2.2 [3]: *Let* $\mathfrak{X}$ *be a nonempty set and* $\rho : \mathfrak{X} \times \mathfrak{X} \to \mathbb{C}$ *be a mapping which satisfies the following conditions:*

(i) $0 \precsim \rho(z, w)$, *for all* $z,\ w \in \mathfrak{X}$ *and* $\rho(z, w) = 0$ *if and only if* $z = w$;

(ii) $\rho(z, w) = \rho(w, z)$ *for all* $z,\ w \in \mathfrak{X}$;

(iii) $\rho(z, w) \precsim \rho(z, z_1) + \rho(z_1, w)$, *for all* $z,\ z_1,\ w \in \mathfrak{X}$.

Then $(\mathfrak{X}, \rho)$ is called a complex-valued metric space.

Example 1: *Let* $\mathfrak{X} = [0, 1]$, *consider a metric* $\rho : \mathfrak{X} \times \mathfrak{X} \to \mathbb{C}$ *defined as* $\rho(x, y) = |x - y| + i|x - y|, \forall x,\ y \in \mathfrak{X}$.

To verify that $(\mathfrak{X}, \rho)$ *is a complete complex valued metric space, it is enough to verify the triangular inequality condition:*

$$\begin{aligned}\rho(x, y) &= |x - y| + i|x - y| \\ &= |x - z + z - y| + i|x - z + z - y| \\ &\preceq \{|x - z| + |z - y|\} + i\{|x - z| + |z - y|\} \\ &\preceq \{|x - z| + i|x - z|\} + \{|z - y| + i|z - y|\}\end{aligned}$$

$\therefore \rho(x, y) \preceq \rho(x, z) + \rho(z, y)$.

Definition 2.3 [3]: *A point* $z \in \mathfrak{X}$ *is known as an interior point of a set* $Z \subseteq \mathfrak{X}$, *if we find* $0 \prec \epsilon \in \mathfrak{C}$ *such that*

$$\mathfrak{B}(z, \epsilon) = \{w \in \mathfrak{X} : \rho(z, w) \prec \epsilon\} \subseteq Z. \tag{1}$$

A point $z \in Z$ is known as the limit point of Z, if there exists an open ball $\mathfrak{B}(z, \epsilon)$ such that

$$\mathfrak{B}(z, \epsilon) \cap (Z \setminus \{z\}) \neq \phi \tag{2}$$

where $0 \prec \epsilon \in \mathfrak{C}$. A subset Z of $\mathfrak{X}$ is said to be open if each point of Z is an interior point of Z. Furthermore, Z is said to be closed if it contains all its limit points.

The family

$$\mathscr{B} = \{\mathfrak{B}(z, \epsilon) : z \in \mathfrak{X}, 0 \prec \epsilon\} \tag{3}$$

is a sub basis for a Hausdorff topology τ on $\mathfrak{X}$.

Definition 2.4 [4]: *Let $(\mathfrak{X}, \rho)$ be a complex valued metric space and $\{z_n\}$ be a sequence in $\mathfrak{X}$. Then,*

- *$\{z_n\}$ is called a convergent sequence if and only if there exists $z \in \mathfrak{X}$, such that for all $\epsilon \succ 0$, $\exists\, n(\epsilon) \in N$ such that for all $n \geq n(\epsilon)$, we have $\rho(z_n, z) \prec \epsilon$. So, we write $\lim_{n\to\infty} z_n = z$.*

- *$\{z_n\}$ is called a Cauchy sequence if and only if for all $\epsilon \succ 0$, $\exists n(\epsilon) \in \mathbb{N}$ such that for each m, $n \geq n(\epsilon)$, we have $\rho(z_n, z_m) \prec \epsilon$.*

- *$(\mathfrak{X}, \rho)$ is called complete if every Cauchy sequence in $\mathfrak{X}$ converges to a point $z \in \mathfrak{X}$.*

Definition 2.5 [8]: *Let $(\mathfrak{X}, \rho)$ be a metric space. We define the Hausdorff metric on $\mathscr{CB}(\mathfrak{X})$ induced by ρ*

$$H(A, B) = \max\{\sup_{x\in A} \rho(x, B), \sup_{y\in B} \rho(A, y)\}\,, \tag{4}$$

for all $A, B \in \mathscr{CB}(\mathfrak{X})$, where $\mathscr{CB}(\mathfrak{X})$ denotes the closed and bounded subsets of $\mathfrak{X}$ and

$$\rho(x, B) = \inf\{\rho(x, \beta) : \beta \in B\}, \tag{5}$$

for all $x \in X$.

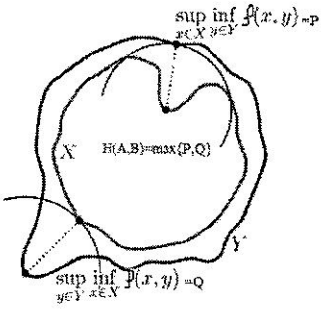


Figure 1: Hausdorff distance between the green line X and the blue line Y.

Definition 2.6 [1]: *Let* $(\mathfrak{X}, \rho)$ *be a complex valued metric space. A nonempty subset* A *of* $\mathfrak{X}$ *is called bounded from below if there exists some* $z \in \mathbb{C}$, *such that* $z \preceq a$ *for all* $a \in A$.

Definition 2.7: *Let* $(\mathfrak{X}, \rho)$ *be a complex valued metric space. A nonempty subset* A *which is bounded from below is said to have the greatest lower bound property (g.l.b property) on* $(\mathfrak{X}, \rho)$ *if there exists a lower bound* $\omega \in \mathbb{C}$ *such that* $z \preceq \omega$ *where* z *is any arbitrary lower bound of* A *in* $\mathfrak{X}$ *and we write it by* $\inf A$.

[6] Let $(\mathfrak{X}, \rho)$ be a complex-valued metric space. We denote the family of all nonempty closed and bounded subsets of complex-valued metric space $\mathfrak{X}$ by $\mathscr{C}\mathscr{B}(\mathfrak{X})$. For $\nu \in \mathbb{C}$ we represent

$$s(\nu) = \{z \in \mathbb{C} : \nu \preceq z\} \tag{6}$$

and for $\omega \in \mathfrak{X}$ and $B \in \mathscr{C}\mathscr{B}(\mathfrak{X})$.

$$s(\omega, B) = \bigcup_{\beta \in B} s(\rho(\omega, \beta)) = \bigcup_{\beta \in B} \{z \in \mathbb{C} : \rho(\omega, B) \preceq z\}. \tag{7}$$

For $A, B \in \mathscr{C}\mathscr{B}(\mathfrak{X})$, we denote

$$s(A, B) = \left(\bigcap_{\alpha \in A} s(\alpha, \beta) \right) \cap \left(\bigcap_{\beta \in B} s(\beta, A) \right). \tag{8}$$

Remark 2.1 [1]: *Let* $(\mathfrak{X}, \rho)$ *be a complex valued metric space. If* $\mathfrak{C} = \mathfrak{R}$, *then* (X, ρ) *is a metric space. Moreover, for* $A, B \in \mathscr{C}\mathscr{B}(\mathfrak{X})$, $H(A, B) = \inf s\,(A, B)$ *is the Hausdorff distance induced by* ρ.

A fuzzy set in $\mathfrak{X}$ is a function with domain $\mathfrak{X}$ and values in [0,1], $F(\mathfrak{X})$ is the collection of all fuzzy sets in $\mathfrak{X}$. If A is a fuzzy set and $x \in \mathfrak{X}$, then the function value $A(x)$ is called the grade of membership of x in A. The α-level set of fuzzy set A, is denoted by $[A]_\alpha$, and defined as:

$$[A]_\alpha = \{x : A(x) \geq \alpha\} \text{ where } \alpha \in (0, 1],$$

$$[A]_0 = \overline{\{x : A(x) > 0\}}$$

Let $\mathfrak{X}$ be any nonempty set and $\mathscr{Y}$ be a complex valued metric space. A mapping T is called a fuzzy mapping, if T is a mapping from $\mathfrak{X}$ into $F(\mathscr{Y})$. A fuzzy mapping T is a fuzzy subset on $X \times \mathscr{Y}$ with membership function $T(x)(y)$. The function $T(x)(y)$ is the grade of membership of y in $T(x)$. For convenience, we denote the α-level set of $T(x)$ by $[Tx]_\alpha$ instead of $[T(x)]_\alpha$.

Definition 2.8 [8]: *A point* $x \in \mathfrak{X}$ *is called a fuzzy fixed point of a fuzzy mapping* $T : \mathfrak{X} \to F(\mathfrak{X})$ *if there exists* $\alpha \in (0, 1]$ *such that* $x \in [Tx]_\alpha$.

Lemma 2.1 [8]: *Let* A *and* B *be nonempty closed and bounded subsets of a metric space* $(\mathfrak{X}, \rho)$. *If* $a \in A$, *then*

$$\rho(a, B) \leq H(A, B).$$

Lemma 2.2: *Let* A *and* B *be nonempty closed and bounded subsets of a complex valued metric space* $(\mathfrak{X}, \rho)$. *If* $a \in A$, *then*

$$\rho(a, B) \preceq \inf s\,(A, B).$$

Proof: The Lemma follows directly as a consequence of Lemma 2.1. □

Lemma 2.3 [8]: *Let A and B be nonempty closed and bounded subsets of a metric space $(\mathfrak{X}, \rho)$ and $0 < \alpha \in \mathfrak{R}$. Then, for $a \in A$, there exists $b \in B$ such that*

$$\rho(a, b) \leq H(A, B) + \alpha .$$

Lemma 2.4: *Let A and B be nonempty closed and bounded subsets of a complex valued metric space $(\mathfrak{X}, \rho)$ and $0 \prec \alpha \in \mathfrak{C}$. Then, for $a \in A$, there exists $b \in B$ such that*

$$\rho(a, b) \preceq \inf s\,(A, B) + \alpha .$$

Proof: The Lemma follows directly as a consequence of Lemma 2.3. □

3. Main Results

Theorem 3.1: *Let $(\mathfrak{X}, \rho)$ be a complete complex valued metric space. Let $T : \mathfrak{X} \to F(\mathfrak{X})$ be a fuzzy mapping and for $x \in \mathfrak{X}$, there exist $\alpha(x) \in (0, 1]$ satisfying the following condition:*

$$\inf s\,([Tx]_{\alpha(x)}, [Ty]_{\alpha(y)}) \preceq a_1\rho(x, [Tx]_{\alpha(x)}) + a_2\rho(y, [Ty]_{\alpha(y)} + a_3\rho(x, [Ty]_{\alpha(y)})$$

$$+ a_4\rho(y, [Ty]_{\alpha(x)}) + a_5\rho(x, y) + \omega \cdot \frac{\rho(x, [Tx]_{\alpha(x)})(1 + |\,\rho(x, [Tx]_{\alpha(x)})|)}{1 + |\,\rho(x; y)\,|} \qquad (9)$$

for all $x, y \in X$. Also $0 \leq a_i \in \mathfrak{R}$, where $i = 1, 2, 3, \ldots, 5$ and $0 \prec \omega \in \mathfrak{C}$ with $|\tau| < 1$, where $\tau = \frac{a_1, a_3, a_5 + \omega}{1 - (a_2 + a_3)}$ and $(a_2 + a_3) \neq 1$. Then, T has a fixed point.

Proof: Let x_0 be any arbitrary point in $\mathfrak{X}$, such that $x_1 \in [Tx_0]_{\alpha(x_0)}$. Then by Lemma 2.4 there exists $x_2 \in [Tx_1]_{\alpha(x_1)}$, such that

$$\rho(x_1, x_2) \preceq \inf s\,([Tx_0]_{\alpha(x_0)}, [Tx_1]_{\alpha(x_1)}) + (a_1 + a_3 + a_5 + \omega)$$

$$\preceq a_1\rho(x_0, [Tx_0]_{\alpha(x_0)}) + a_2\rho(x_1, [Tx_1]_{\alpha(x_1)}) + a_3\rho(x_0, [Tx_1]_{\alpha(x_1)})$$

$$+ a_4\rho(x_1, [Tx_0]_{\alpha(x_0)}) + a_5\rho(x_0, x_1)$$

$$+ \omega \cdot \frac{\rho(x_0, [Tx_0]_{\alpha(x_0)})(1 + |\rho(x_0, [Tx_0]_{\alpha(x_0)})|)}{(1 + |\rho(x_0, x_1)|} + (a_1 + a_3 + a_5 + \omega)$$

$$\preceq a_1\rho(x_0, x_1) + a_2\rho(x_1, x_2) + a_3[\rho(x_0, x_1) + \rho(x_1, x_2)]$$

$$+ a_5\rho(x_0, x_1) + \omega\rho(x_0, x_1) + (a_1 + a_3 + a_5 + \omega).$$

$$\rho(x_1, x_2) \preceq \frac{a_1 + a_3 + a_5 + \omega}{1 - (a_2 + a_3)}\,\rho(x_0, x_1) + \frac{a_1 + a_3 + a_5 + \omega}{1 - (a_2 + a_3)}. \tag{10}$$

Let $\tau = \dfrac{a_1 + a_3 + a_5 + \omega}{1 - (a_2 + a_3)}$, with condition $|\tau| = |\dfrac{a_1 + a_3 + a_5 + \omega}{1 - (a_2 + a_3)}| < 1$ and $(a_2 + a_3) \neq 1$.

Therefore the Equation (10), we have

$$\rho(x_1, x_2) \preceq \tau\rho(x_0, x_1) + \tau.$$

Again by Lemma 2.4, $\exists\ x_3 \in [Tx_2]_{\alpha(x_2)}$ such that

$$\rho(x_2, x_3) \preceq \inf s\,([Tx_1]_{\alpha(x_1)}, [Tx_2]_{\alpha(x_2)}) + \frac{(a_1 + a_3 + a_5 + \omega)^2}{1 - (a_2 + a_3)}$$

$$\preceq a_1\rho(x_1, [Tx_1]_{\alpha(x_1)}) + a_2\rho(x_2, [Tx_2]_{\alpha(x_2)}) + a_3\rho(x_1, [Tx_2]_{\alpha(x_2)})$$

$$+ a_4\rho(x_2, [Tx_1]_{\alpha(x_1)}) + a_5\rho(x_1, x_2)$$

$$+\omega\cdot\frac{\rho(x_1,[Tx_1]_{\alpha(x_1)})(1+|\rho(x_1,[Tx_1]_{\alpha(x_1)})|)}{(1+|\rho(x_1,x_2)|}+\frac{(a_1+a_3+a_5+\omega)^2}{1-(a_2+a_3)}$$

$$\rho(x_2,x_3)\preceq a_1\rho(x_1,x_2)+a_2\rho(x_2,x_3)+a_3[\rho(x_1,x_2)+\rho(x_2,x_3)]$$

$$+a_5\rho(x_1,x_2)+\omega\rho(x_1,x_2)+\frac{(a_1+a_3+a_5+\omega)}{1-(a_2+a_3)}$$

$$\preceq\frac{a_1+a_3+a_5+\omega}{1-(a_2+a_3)}\rho(x_1,x_2)+\frac{(a_1+a_3+a_5+\omega)^2}{(1-(a_2+a_3))^2}$$

$$\rho(x_2,x_3)\preceq\left(\frac{a_1+a_3+a_5+\omega}{1-(a_2+a_3)}\right)^2\rho(x_0,x_1)+2\left(\frac{(a_1+a_3+a_5+\omega)}{(1-(a_2+a_3))}\right)^2 \quad \text{by (10)}$$

$$\therefore\ \rho(x_2,x_3)\preceq\tau^2\rho(x_0,x_1)+2\tau^2\,.$$

Continuing as in a preceding way, we can obtain a sequence $\{x_n\}$ such that $x_n\in[T_{x_{n+1}}]_{\alpha(x_{n+1})}$, we have

$$\rho(x_n,x_{n+1})\preceq\tau^n\rho(x_0,x_1)+n\tau^n\,. \tag{11}$$

Now, for any positive integers $m,n(n>m)$, we have

$$\rho(x_m,x_n)\preceq[\rho(x_m,x_{m+1})+\rho(x_{m+1},x_n)]$$

$$\preceq[\rho(x_m,x_{m+1})]+[\rho(x_{m+1},x_{m+2})+\rho(x_{m+2},x_n)]$$

$$\preceq[\rho(x_m,x_{m+1})]+\{\rho(x_{m+1},x_{m+2})\}+\ldots+\{\rho(x_{n-1},x_n)\}]$$

$$\preceq\tau^m\rho(x_0,x_1)+m\tau^m+\tau^{m+1}\rho(x_0,x_1)+(m+1)\tau^{m+1}+\cdots$$

$$+\tau^{n-1}\rho(x_0,x_1)+(n-1)\tau^{n-1}\quad\text{by (11)}$$

$$\preceq \tau^m(1+\tau+\cdots+\tau^{n-m-1})\rho(x_0,x_1)+\sum_{i=m}^{n-1} i\tau^i .$$

It follows from Cauchy root test that $\sum i\tau^i$ is convergent and hence x_n is a Cauchy sequence. Since $(\mathfrak{X},\rho)$ is complete then there exists $z\in\mathfrak{X}$ such that $x_n\to z$ as $n\to\infty$.

Now,

$$\rho(z,[Tz]_{\alpha(z)}) \preceq [\rho(z,x_{n+1})+\rho(x_{n+1},[Tz]_{\alpha(z)})]$$

$$\preceq [\rho(z,x_{n+1})+\inf s([Tx_n]_{\alpha(x_n)},[Tz]_{\alpha(z)})]$$

Using (9) with $n\to\infty$ we get that

$$(1-(a_2+a_3))\rho(z,[Tz]_{\alpha(z)})\preceq 0$$

So, we obtain that

$$z\in[Tz]_{\alpha(z)} .$$

Hence, $z\in\mathfrak{X}$ is a fixed point for the fuzzy mapping T. □

Example 2: *Let* $\mathfrak{X}=[0,1]$ *and* $\rho(x,y)=\{|x-y|+i \mid x-y \mid\}$, *whenever* $x,y\in\mathfrak{X}$, *then* $(\mathfrak{X},\rho)$ *is a complete metric space. Define a fuzzy mapping* $T:\mathfrak{X}\to F(\mathfrak{X})$ *by*

$$T(x)(t)\begin{cases}1, & 0\le t\le \frac{x}{4}\\ \frac{1}{2}, & \frac{x}{4}\le t\le \frac{x}{3}\\ \frac{1}{4}, & \frac{x}{3}\le t\le \frac{x}{2}\\ 0, & \frac{x}{2}\le t\le 1\end{cases}$$

For all $x\in\mathfrak{X}$, *there exists* $\alpha(x)=1$ *such that* $[Tx]_{\alpha(x)}=[0,\frac{x}{4}]$. *Then*

$$\inf s([Tx]_{\alpha(x)}, [Ty]_{\alpha(x)}) \preceq \left\{ \frac{1}{5} |x - \frac{x}{4}| + i \frac{1}{4} |x - \frac{x}{5}| + \frac{1}{10} | y - \frac{y}{4} | + i \frac{1}{10} |y - \frac{y}{4}| \right.$$

$$+ \frac{1}{15} |x - \frac{y}{4}| + i \frac{1}{15} |x - \frac{y}{4}| + \frac{1}{20} | y - \frac{x}{4} | + i \frac{1}{20} |y - \frac{x}{4}|$$

$$\left. \frac{1}{20} \{|x - y| + i|x - y|\} - \right\}$$

$$+ \frac{1}{30} \left(\frac{\{|x - \frac{x}{4}| + i|x - \frac{x}{4}|\} + (1 + \{|x - \frac{x}{4}| + i|x - \frac{x}{4}|\})}{1 + ||x - y| + i|x - y||} \right)$$

Since, all the conditions of Theorem 3.1 *are satisfied. Therefore,* $0 \in \mathfrak{X}$ *is the fixed point of* T.

References

[1] J. Ahmad, C. Klin-Eam and A. Azam (2013): Common fixed points for multivalued mappings in complex valued metric spaces with applications, *Hindawi, Abst. Appl. Anal.*, pp. 1-12.

[2] S. C. Arora, and V. Sharma (2000): Fixed point theorems for fuzzy mappings *Fuzzy Sets Syst.*, Vol. 110, pp. 127-130.

[3] A. Azam, B. Fisher, and M. Khan (2011): Common fixed point theorems in complex valued metric spaces, *Num. Func. Anal. Opt.*, Vol. 32, pp. 243-253.

[4] A. K. Dubey, R. Shukla and R. P. Dubey (2015): Some fixed point theorems in complex valued b-metric spaces, *Hindawi, J. Comp. Syst.*, pp. 1-7.

[5] S. Heilpern (1981): Fuzzy mappings and fixed point theorem; *J. Math. Anal. Appl.*, Vol. 83, pp. 566-569.

[6] Humaira, M. Sarwar and G. N. V. Kishore (2018): Fuzzy fixed point results for ϕ contractive mapping with applications, *Hindawi, Comp.*, pp. 1-12.

[7] S. B. Nadler (1969): Multivalued contraction mappings, *Pac. J. Math.*, Vol. 30, pp. 475-488.

[8] A. Shahzad, A. Shoaib, and Q. Mahmood (2017): Common fixed point theorems for fuzzy mappings in b-metric space, *I. J. Pure. Appl. Math.*, Vol. 38, pp. 419-427.

[9] M. D. Weiss (1975): Fixed points and induced fuzzy topologies for fuzzy sets, *J. Math. Anal. Appl.*, Vol. 50, pp. 142-150.

[10] L. A. Zadeh (1965): Fuzzy sets Inform, *Contr.*, Vol. 8, pp. 338-353.

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(*Received*, September 30, 2023)

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