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Sanjib Kumar Datta[1]
and
Lakshmi Biswas[2]

GENERALIZED NEVANLINNA ORDER (α, β) BASED SOME GROWTH PROPERTIES OF COMPOSITE ANALYTIC FUNCTIONS

Abstract: In this paper our main aim is to introduce some idea about generalized Nevanlinna order (α,β) and generalized Nevanlinna lower order (α,β) of an analytic function, where α and β are continuous non negative function in extended complex plane (∞, $+\infty$). Here we also discuss about some growth properties relating to the composition of two analytic functions on the basis of generalized Nevanlinna order (α,β) and generalized Nevanlinna lower order (α,β) as compared to the growth of their corresponding left and right factors.

Keywords and Phrases: Analytic Function, Growth, Generalized Nevanlinna Order (α,β), Generalized Nevanlinna Lower Order (α,β).

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1. Introduction, Definitions and Notations

In this paper, we assume that readers are familiar with the fundamental results and the standard notations of the Nevanlinna value distribution theory of meromorphic functions in the complex plane. Throughout this paper, by a meromorphic function f(x), we mean a meromorphic function in the complex plane. We use $T_f(r)$ and $M_f(r)$ to denote the characteristic function of a meromorphic function and the maximum modulus of an entire function. In the following, we will recall some notations about meromorphic functions and entire functions.

Let f be a meromorphic function defined in the open complex plane $\mathbb{C}$. For $a \in \mathbb{C} \cup \{\infty\}$ let $n_f(t, a)$ $(\overline{n}_f(t, a))$ the number of a-points (distinct a-points) of f in $|z| \leq t$, where an ∞-point is a pole of f. Also

$$N_f(r, a) = \int_0^r \frac{n_f(t, a) - n_f(0, a)}{t}\, dt + n_f(0, a) \log r$$

and

$$\overline{N}_f(r, a) = \int_0^r \frac{\overline{n}_f(t, a; f) - \overline{n}_f(0, a)}{t}\, dt + \overline{n}_f(0, a) \log r.$$

The function $N_f(r, a)$ $(\overline{N}_f(r, a))$ are called the counting function of a-points (distinct a-points) of f. In many occasions $N_f(r, \infty)$ and $\overline{N}_f(r, \infty)$ are denoted by $N_f(r)$ and $\overline{N}_f(r)$ respectively.

The function $m_f(r)$, which is called the proximity function of f is defined by

$$m_f(r) = \frac{1}{2\pi} \int_0^{2\pi} \log^+ \left| f(re^{i\theta}) \right| d\theta,$$

where

$$\log^+ x = \log x, \text{ if } x \geq 1$$
$$= 0, \text{ if } 0 \leq x < 1.$$

For $a \in \mathbb{C}$ we denote by $m(r, \frac{1}{f-a})$ the function $m_f(r, a)$ and we mean by $m_f(r, \infty)$ the function $m_f(r)$.

The function $T_f(r) = m_f(r) + N_f(r)$ is called the Nevanlinna's characteristic function of f.

If f is entire, the function $T_f(r) = m_f(r)$ is called the Nevanlinna's characteristic function of f.

Now let L be a class of continuous non negative on $(-\infty, +\infty)$ function α such that $\alpha(r) = \alpha(r_0) \geq 0$ for $x \leq x_0$ with $\alpha(r) \to +\infty$ as $x \to +\infty$. Further we assume that throughout the present paper $\alpha, \alpha_1, \alpha_2, \alpha_3, \beta \in L$.

Considering the above, Sheremeta introduced the concept of generalized order (α, β) of an entire function. For details about generalized order (α, β) one may see [6] . During the past decades, several authors made close investigations on the propertics of entire functions related to generalized order (α, β) in some different directions. For the purpose of further applications, in this paper we write the definition of the generalized Nevanlinna order (α, β) and generalized Nevanlinna lower order (α, β) of an analytic function in the following way:

Definition 1.1: (*Generalized Nevanlinna order* (α, β) *and generalized Nevanlinna lower order* (α, β)).

The generalized Nevanlinna order (α, β) *and generalized Nevanlinna lower order* (α, β) *of an analytic function f denoted by* $\rho_{(\alpha,\beta)}[f]$ *and* $\lambda_{(\alpha,\beta)}[f]$ *respectively are defined as*:

$$\begin{matrix}\rho_{(\alpha,\beta)}[f] \\ \lambda_{(\alpha,\beta)}[f]\end{matrix} = \lim_{r\to\infty} \begin{matrix}\sup \\ \inf\end{matrix} \frac{\alpha(\exp(T_f(r)))}{\beta(r)}.$$

Now one may give the definitions of generalized Nevanlinna hyper order (α, β) and generalized Nevanlinna logarithmic order (α, β) of an analytic function f as:

Definition 1.2: (*Generalized Nevanlinna hyper order* (α, β) *and generalized Nevanlinna hyper lower order* (α, β)).

The generalized Nevanlinna hyper order (α, β) *and generalized Nevanlinna hyper lower order* (α, β) *of an analytic function f denoted by* $\overline{\rho}_{(\alpha,\beta)}[f]$ *and* $\overline{\lambda}_{(\alpha,\beta)}[f]$ *respectively are defined as*:

$$\begin{matrix}\overline{\rho}_{(\alpha,\beta)}[f] \\ \overline{\lambda}_{(\alpha,\beta)}[f]\end{matrix} = \lim_{r\to\infty} \begin{matrix}\sup \\ \inf\end{matrix} \frac{\alpha(T_f(r))}{\beta(r)}.$$

Definition 1.3: (*Generalized logarithmic order* (α, β) *and generalized Nevanlinna logarithmic lower order* (α, β)).

The generalized Nevanlinna logarithmic order (α, β) *and generalized Nevanlinna logarithmic lower order* (α, β) *of an analytic function* f *denoted by* $\rho_{(\alpha,\beta)}[f]$ *and* $\lambda_{(\alpha,\beta)}[f]$ *respectively are defined as*:

$$\begin{matrix}\rho^{\log}_{(\alpha,\beta)}[f] \\ \lambda^{\log}_{(\alpha,\beta)}[f]\end{matrix} = \lim_{r\to\infty} \begin{matrix}\sup \\ \inf\end{matrix} \frac{\alpha(\exp(T_f(r)))}{\beta(\log r)}.$$

However the main aim of this paper is to investigate some growth properties of Nevanlinna's Characteristic function relating to the composition of two analytic functions on the basis of generalized Nevanlinna order (α, β), generalized Nevanlinna hyper order (α, β) and generalized Nevanlinna logarithmic order (α, β) as compared to the growth of their corresponding left and right factors.

2. Main Results

In this section we present the main results of the paper.

Theorem 2.1: *Let* f *and* g *be any two non-constant analytic functions such that* $0 < \lambda_{(\alpha_1,\beta)}[f \circ g] \leq \rho_{(\alpha_1,\beta)}[f \circ g] < \infty$, $0 < \lambda_{(\alpha_2,\beta)}[f] \leq \rho_{(\alpha_2,\beta)}[f] < \infty$. *Then*

$$\frac{\lambda_{(\alpha_1,\beta)}[f \circ g]}{\rho_{(\alpha_2,\beta,)}[f]} \leq \liminf_{x\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \leq \frac{\lambda_{(\alpha_1,\beta)}[f \circ g]}{\lambda_{(\alpha_2,\beta)}[f]},$$

$$\leq \limsup_{x\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \leq \frac{\rho_{(\alpha_1,\beta)}[f \circ g]}{\lambda_{(\alpha_2,\beta)}[f]}.$$

Proof: *From the definitions of* $\lambda_{(\alpha_1,\beta)}[f \circ g]$ *and* $\rho_{(\alpha_2,\beta)}[f]$ *for arbitrary positive* ϵ *and for all sufficiently large values of* r *we have*

$$\alpha_1(\exp(T_{f\circ g}(r))) \geq (\lambda_{(\alpha_1,\beta)}[f \circ g] - \epsilon)(\beta(r)) \tag{1}$$

and

$$\alpha_2(\exp(T_f(r))) \leq (\rho_{(\alpha_2,\beta)}[f] + \epsilon)(\beta(r)) \tag{2}$$

Now from equation (1) *and* (2) *it follows for all sufficiently large values of r that*

$$\frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \geq \frac{(\lambda_{(\alpha_1,\beta)}[f \circ g] - \epsilon)}{(\rho_{(\alpha_2,\beta,)}[f] + \epsilon)}.$$

As $\epsilon(> 0)$ *is arbitrary, we obtain that*

$$\liminf_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \geq \frac{\lambda_{(\alpha_1,\beta)}[f \circ g]}{\rho_{(\alpha_2,\beta)}[f]} \tag{3}$$

which is the first part of the theorem.

Again for a sequence of values of r tending to infinity, we get that

$$\alpha_1(\exp(T_{f\circ g}(r))) \leq (\lambda_{(\alpha_1,\beta)}[f \circ g] + \epsilon)(\beta(r)) \tag{4}$$

and for all sufficiently large values of r

$$\alpha_2(\exp(T_f(r))) \geq (\lambda_{(\alpha_2,\beta)}[f] - \epsilon)(\beta(r)) \tag{5}$$

Combining equation (4) *and* (5) *we have for a sequence of values of r tending to infinity that*

$$\frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \leq \frac{(\lambda_{(\alpha_1,\beta)}[f \circ g] + \epsilon)}{(\lambda_{(\alpha_2,\beta)}[f] - \epsilon)}.$$

As $\epsilon(> 0)$ *is arbitrary, we obtain that*

$$\liminf_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \leq \frac{\lambda_{(\alpha_1,\beta)}[f \circ g]}{\lambda_{(\alpha_2,\beta)}[f]} \tag{6}$$

Also for a sequence of values of r tending to infinity that

$$\alpha_2(\exp(T_f(r))) \leq (\lambda_{(\alpha_2,\beta)}[f] + \epsilon)(\beta(r)) \tag{7}$$

Again from equation (1) *and* (7)*, we have for a sequence of values of r tending to infinity that*

$$\frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \geq \frac{(\lambda_{(\alpha_1,\beta)}[f \circ g] - \epsilon)}{(\lambda_{(\alpha_2,\beta,)}[f] + \epsilon)}.$$

As $\epsilon(> 0)$ *is arbitrary, we obtain that*

$$\limsup_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \geq \frac{\lambda_{(\alpha_1,\beta)}[f \circ g]}{\lambda_{(\alpha_2,\beta)}[f]}. \tag{8}$$

Again for all sufficiently large values of r, we get that

$$\alpha_2(\exp(T_{f\circ g}(r))) \leq (\rho_{(\alpha_1,\beta)}[f \circ g] + \epsilon)(\beta(r)) \tag{9}$$

Now from equation (5) *and* (9)*, it follows for all sufficiently large values of r that*

$$\frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \leq \frac{(\rho_{(\alpha_1,\beta)}[f \circ g] + \epsilon)}{(\lambda_{(\alpha_2,\beta)}[f] - \epsilon)}.$$

As $\epsilon(> 0)$ *is arbitrary, we obtain that*

$$\limsup_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \leq \frac{\rho_{(\alpha_1,\beta)}[f \circ g]}{\lambda_{(\alpha_2,\beta)}[f]}. \tag{10}$$

Thus, the theorem follows from (3), (6), (8) *and* (10).

The following theorem can be proved in the line of Theorem 2.1 and so the proof is omitted.

Theorem 2.2: *Let f and g be any two non-constant analytic functions such that $0 < \lambda_{(\alpha_1,\beta)}[f \circ g] \leq \rho_{(\alpha_1,\beta)}[f \circ g] < \infty$, $0 < \lambda_{(\alpha_3,\beta)}[g] \leq \rho_{(\alpha_3,\beta)}[g] < \infty$. Then*

$$\frac{\lambda_{(\alpha_1,\beta)}[f \circ g]}{\rho_{(\alpha 3,\beta)}[g]} \leq \liminf_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_3(\exp(T_g(r)))} \leq \frac{\lambda_{(\alpha_1,\beta)}[f \circ g]}{\lambda_{(\alpha 3,\beta)}[g]},$$

$$\leq \limsup_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_3(\exp(T_g(r)))} \leq \frac{\rho_{(\alpha_1,\beta)}[f \circ g]}{\lambda_{(\alpha_3,\beta)}[g]}.$$

Theorem 2.3: *Let f and g be any two non-constant analytic functions such that $0 < \rho_{(\alpha_1,\beta)}[f \circ g] < \infty$, $0 < \rho_{(\alpha_2,\beta)}[f] < \infty$. Then*

$$\liminf_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \leq \frac{\rho_{(\alpha_1,\beta)}[f \circ g]}{\rho_{(\alpha_2,\beta)}[f]} \leq \limsup_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))}.$$

Proof: *From the definitions of $\rho_{(\alpha_2,\beta)}[f]$, for arbitrary positive ϵ and for a sequence of values of r tending to infinity we have*

$$\alpha_2(\exp(T_f(r))) \geq (\rho_{(\alpha_2,\beta)}[f] - \epsilon)(\beta(r)) \tag{11}$$

Now from equation (9) *and* (11) *it follows for a sequence of values of r tending to infinity that*

$$\frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \leq \frac{(\rho_{(\alpha_1,\beta)}[f \circ g] + \epsilon)}{(\rho_{(\alpha_2,\beta,)}[f] - \epsilon)}.$$

As $\epsilon(> 0)$ is arbitrary, we obtain that

$$\liminf_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \leq \frac{\rho_{(\alpha_1,\beta)}[f \circ g]}{\rho_{(\alpha_2,\beta)}[f]} \tag{12}$$

Again for a sequence of values of r tending to infinity, we get that

$$\alpha_1(\exp(T_{f\circ g}(r))) \geq (\rho_{(\alpha_1,\beta)}[f \circ g] - \epsilon)(\beta(r)) \tag{13}$$

Combining equation (2) *and* (13)*, we have for a sequence of values of* r *tending to infinity that*

$$\frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \geq \frac{(\rho_{(\alpha_1,\beta)}[f\circ g]-\epsilon)}{(\rho_{(\alpha_2,\beta,)}[f]+\epsilon)}.$$

As $\epsilon(>0)$ *is arbitrary, we obtain that*

$$\limsup_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \geq \frac{\rho_{(\alpha_1,\beta)}[f\circ g]}{\rho_{(\alpha_2,\beta)}[f]}. \tag{14}$$

Thus, the theorem follows from (12) *and* (14)*.*

The following theorem can be proved in the line of Theorem 2.3 and so the proof is omitted.

Theorem 2.4: *Let* f *and* g *be any two non-constant analytic functions such that* $0<\rho_{(\alpha_1,\beta)}[f\circ g]<\infty$, $0<\rho_{(\alpha_3,\beta)}[g]<\infty$. *Then*

$$\liminf_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_3(\exp(T_g(r)))} \leq \frac{\rho_{(\alpha_1,\beta)}[f\circ g]}{\rho_{(\alpha_3,\beta)}[g]} \leq \limsup_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_3(\exp(T_g(r)))}.$$

The following theorem is a consequence of Theorem 2.1 and Theorem 2.3 and so the proof is omitted.

Theorem 2.5: *Let* f *and* g *be any two non-constant analytic functions such that* $0<\lambda_{(\alpha_1,\beta)}[f\circ g]\leq\rho_{(\alpha_1,\beta)}[f\circ g]<\infty$ *and* $0<\lambda_{(\alpha_2,\beta)}[f]\leq\rho_{(\alpha_2,\beta)}[f]<\infty$. *Then*

$$\liminf_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \leq \min\left\{\frac{\lambda_{(\alpha_1,\beta)}[f\circ g]}{\lambda_{(\alpha_2,\beta)}[f]}, \frac{\rho_{(\alpha_1,\beta)}[f\circ g]}{\rho_{(\alpha_2,\beta)}[f]}\right\}$$

$$\leq \max\left\{\frac{\lambda_{(\alpha_1,\beta)}[f\circ g]}{\lambda_{(\alpha 2,\beta)}[f]}, \frac{\rho_{(\alpha_1,\beta)}[f\circ g]}{\rho_{(\alpha_2,\beta)}[f]}\right\} \leq \limsup_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))}.$$

Analogously one may state the following theorem without its proof.

Theorem 2.6: *Let f and g be any two non-constant analytic functions such that $0 < \lambda_{(\alpha_1,\beta)}[f \circ g] \leq \rho_{(\alpha_1,\beta)}[f \circ g] < \infty$ and $0 < \lambda_{(\alpha_3,\beta)}[f] \leq \rho_{(\alpha_3,\beta)}[f] < \infty$. Then*

$$\liminf_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_3(\exp(T_f(r)))} \leq \min\left\{\frac{\lambda_{(\alpha_1,\beta)}[f \circ g]}{\lambda_{(\alpha_3,\beta)}[f]}, \frac{\rho_{(\alpha_1,\beta)}[f \circ g]}{\rho_{(\alpha_3,\beta)}[f]}\right\}$$

$$\leq \max\left\{\frac{\lambda_{(\alpha_1,\beta)}[f \circ g]}{\lambda_{(\alpha_3,\beta)}[f]}, \frac{\rho_{(\alpha_1,\beta)}[f \circ g]}{\rho_{(\alpha_3,\beta)}[f]}\right\} \leq \limsup_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_3(\exp(T_f(r)))}.$$

We may now state the following two theorems based on Definition 1.2 and Definition 1.3 respectively.

Theorem 2.7: *Let f and g be any two non-constant analytic functions such that $0 < \overline{\lambda}_{(\alpha_1,\beta)}[f \circ g] \leq \overline{\rho}_{(\alpha_1,\beta)}[f \circ g] < \infty$ and $0 < \overline{\lambda}_{(\alpha_2,\beta)}[f] \leq \overline{\rho}_{(\alpha_2,\beta)}[f] < \infty$. Then*

$$\frac{\overline{\lambda}_{(\alpha_1,\beta)}[f \circ g]}{\overline{\rho}_{(\alpha_2,\beta)}[f]} \leq \liminf_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \leq \min\left\{\frac{\overline{\lambda}_{(\alpha_1,\beta)}[f \circ g]}{\overline{\lambda}_{(\alpha_2,\beta)}[f]}, \frac{\overline{\rho}_{(\alpha_1,\beta)}[f \circ g]}{\overline{\rho}_{(\alpha_2,\beta)}[f]}\right\}$$

$$\leq \max\left\{\frac{\overline{\lambda}_{(\alpha_1,\beta)}[f \circ g]}{\overline{\lambda}_{(\alpha_2,\beta)}[f]}, \frac{\overline{\rho}_{(\alpha_1,\beta)}[f \circ g]}{\overline{\rho}_{(\alpha_2,\beta)}[f]}\right\} \leq \limsup_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \leq \frac{\overline{\rho}_{(\alpha_1,\beta)}[f \circ g]}{\overline{\lambda}_{(\alpha_2,\beta)}[f]}.$$

Theorem 2.8: *Let f and g be any two non-constant analytic functions such that $0 < \lambda^{\log}_{(\alpha_1,\beta)}[f \circ g] \leq \rho^{\log}_{(\alpha_1,\beta)}[f \circ g] < \infty$ and $0 < \lambda^{\log}_{(\alpha_2,\beta)}[f \circ g] \leq \rho^{\log}_{(\alpha_2,\beta)}[f] < \infty$. Then*

$$\frac{\lambda^{\log}_{(\alpha_1,\beta)}[f \circ g]}{\rho^{\log}_{(\alpha_2,\beta)}[f]} \leq \liminf_{r\to\infty} \frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \leq \min\left\{\frac{\lambda^{\log}_{(\alpha_1,\beta)}[f \circ g]}{\lambda^{\log}_{(\alpha_2,\beta)}[f]}, \frac{\rho^{\log}_{(\alpha_1,\beta)}[f \circ g]}{\rho^{\log}_{(\alpha_2,\beta)}[f]}\right\}$$

$$\leq \max\left\{\frac{\lambda^{\log}_{(\alpha_1,\beta)}[f\circ g]}{\lambda^{\log}_{(\alpha_2,\beta)}[f]}, \frac{\rho^{\log}_{(\alpha_1,\beta)}[f\circ g]}{\rho^{\log}_{(\alpha_2,\beta)}[f]}\right\} \leq \limsup_{r\to\infty}\frac{\alpha_1(\exp(T_{f\circ g}(r)))}{\alpha_2(\exp(T_f(r)))} \leq \frac{\rho^{\log}_{(\alpha_1,\beta)}[f\circ g]}{\lambda^{\log}_{(\alpha_2,\beta)}[f]}.$$

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1. Department of Mathematics,
University of Kalyani
P.O.: Kalyani, Dist: Nadia, Pin: 741235,
West Bengal, India
E-mail: sanjibdatta05@gmail.com

2. Kalinarayanpur Adarsha Vidyalaya
P.O.: Kalinarayanpur, Dist.: Nadia, Pin: 741254,
West Bengal, India
E-mail: kutkijit@gmail.com

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