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EXTENDED GIBONACCI CONJECTURES

Abstract: We present two conjectures, one involving Fibonacci numbers and the other Jacobsthal numbers.

Keywords: Fibonacci, Lucas, Jacobsthal, and Jacobsthal-Lucas Numbers, Extended Gibonacci Numbers.

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1. Introduction

Extended gibonacci numbers G_n are defined by the recurrence $G_n = aG_{n-1} + bG_{n-2}$, where a, b, G_1, and G_2 are arbitrary integers and $n \geq 3$.

Suppose $a = 1 = b$. When $G_1 = 1 = G_2$, $G_n = F_n$, the nth *Fibonacci number*; and when $G_1 = 1$ and $G_2 = 3$, $G_n = L_n$, the nth *Lucas number*. They can also be defined by *Binet-like* formulas [2, 3].

On the other hand, let $a = 1$ and $b = 2$. When $G_1 = 1 = G_2$, $G_n = J_n$, the nth *Jacobsthal number*; and when $G_1 = 1$ and $G_2 = 5$, $G_n = j_n$, the nth *Jacobsthal-Lucas number* [3].

The following table shows the first 10 Fibonacci, Lucas, Jacobsthal, and Jacobsthal-Lucas numbers.

Table 1
First 10 Fibonacci, Lucas, Jacobsthal, and Jacobsthal-Lucas numbers

n	1	2	3	4	5	6	7	8	9	10
F_n	(1)	(1)	(2)	(3)	(5)	(8)	13	21	34	(55)
L_n	(1)	(3)	(4)	(7)	(11)	18	29	47	76	123
J_n	(1)	(1)	(3)	(5)	(11)	21	43	85	(171)	341
j_n	(1)	(5)	(7)	17	31	65	127	257	511	1,025

A quick look at the first 100 Fibonacci numbers in [2, 3] gives a fascinating observation. There are exactly seven *palindromic* Fibonacci numbers $\leq F_{100}$, and six (the smallest *perfect number* [1]) of them are single-digit integers and are all circled in Table 1.

Interestingly, the corresponding Jacobsthal table in [3] contains two added bonuses. In addition to the four single-digit integers, there are two additional palindromic numbers, namely $J_5 = 11$ and $J_9 = 171$, again a total of six Jacobsthal palindromes $\leq J_{100}$, also circled in Table 1.

With a computer program, Z. Gao established that there are *no* additional Fibonacci palin-dromes $\leq F_{170,000}$, and *no* additional Jacobsthal palindromes $\leq J_{120,000}$ [4].

2. Extended Gibonacci Conjectures

Based on Fibonacci and Jacobsthal tables in [3], and the data collected by Gao, we conjecture that:

1. There are exactly seven Fibonacci palindromes: 1, 1, 2, 3, 5, 8, and 55; and

2. There are exactly six Jacobsthal palindromes: 1, 1, 3, 5, 11, and 171.

Clearly, similar conjectures can be conceived for both Lucas and Jacobsthal-

Lucas numbers as well. For the curious-minded, we add that $L_{25} = 167,761$ is palindromic.

We encourage gibonacci enthusiasts to either confirm or disprove each conjecture.

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