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# ENHANCINGSCADACYBERSECURITY THROUGHNEUTROSOPHIC GRAPH TOPOLOGICALANALYSIS

**Abstract:** This manuscript extends the concept of graph topology to neutrosophic graph topological spaces by incorporating uncertainty and indeterminacy into graph topological structures. Building on existing work in graph topologies, we introduce neutrosophic components that utilize truth membership (T), indeterminacy (I), and falsity membership (F) values to handle incomplete and contradictory information. We establish foundational definitions and examine important topological properties including neutrosophic connectedness, separation axioms ($T_0$, $T_1$, $T_2$). Several theorems are presented with complete proofs, including the equivalence of neutrosophic connectedness conditions and the hierarchical relationship between separation axioms. The framework is demonstrated through a practical cybersecurity application, where neutrosophic graph topology is used to model SCADA network vulnerabilities, identify secure zones, and analyze potential attack vectors in power grid systems.

**Keywords:** Neutrosophic graph topology, neutrosophic connectedness, neutrosophic separation, network security, SCADA systems.

**Mathematics Subject Classification (2020):** Primary: 54A05, 54A10; Secondary: 94C15.

## 1. Introduction

Neutrosophic theory, introduced by Smarandache [9], extends classical logic by incorporating indeterminacy alongside truth and falsity, enabling uncertainty quantification. Building on this, Aniyan and Naduvath [1] established graph topological frameworks for analyzing spatial properties in discrete structures, focusing on transformations and connectivity. Broumi and Smarandache [4] further advanced the field by defining neutrosophic graph structures, which model uncertainty in vertex and edge relationships. Ye [13] developed correlation-based decision-

making methods in neutrosophic environments, aiding multi-criteria analysis under incomplete information.

In cybersecurity, Stouffer et al. [10] outlined SCADA system security protocols, addressing infrastructure vulnerabilities. Zhu et al. [15] proposed a taxonomy of cyber attacks on SCADA networks, enabling systematic threat assessment in uncertain environments. Mohanapriya Rajagobal and Durgadevi Shanmugasundaram proposed a mathematical framework integrating neutrosophic theory with graph topology for cybersecurity analysis by establishing key properties such as connectedness and separation axioms and applying the model to SCADA power grid systems.

## 2. Neutrosophic Graph Topology

**Definition 1.1** Let $NG = (V, E)$ be a neutrosophic graph. A *neutrosophic graph topology* $\tau$ on $NG$ is a collection of neutrosophic subgraphs of $NG$ satisfying the following conditions:

1. $NK_0 \in \tau$ and $NG \in \tau$, where $NK_0$ denotes the neutrosophic null graph.
2. The union of any number of elements in $\tau$ is also in $\tau$.
3. The intersection of any finite number of elements in $\tau$ is also in $\tau$.

The combination $(NG, \tau)$ forms a *Neutrosophic Graph Topological Space(NGTS)*.

**Example 1.1:** For the neutrosophic graph $NG$ with vertex set $\{A, B, C, D\}$, the collection of subgraph yields the neutrosophic graph topology

$$\tau = \{NK_0, \{A, B\}, \{B, C\}, \{A, C, D\}, NG\}$$

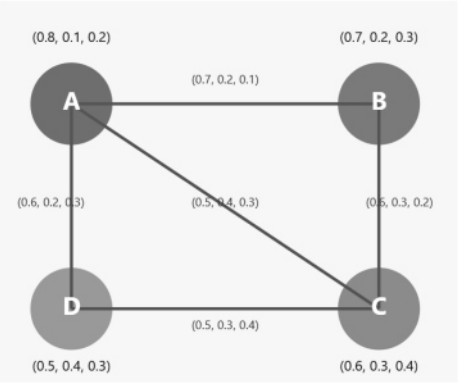


Figure 1: Neutrosophic Graph $NG$

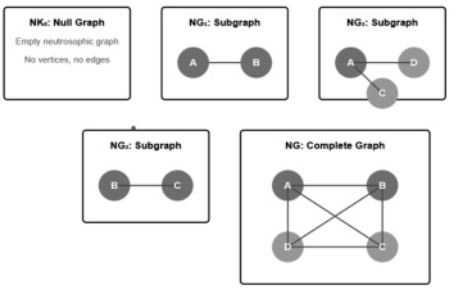


Figure 2: Neutrosophic Graph Topology Induced by Collection of Subgraph

**Definition 1.2:** A *neutrosophic empty graph* is a neutrosophic graph with a non-empty vertex set but an empty edge set.

**Definition 1.3:** A *neutrosophic null graph*, denoted by $NK_0$, is a neutrosophic graph with empty vertex and edge sets.

**Definition 1.4:** A *neutrosophic subgraph* $NH$ of $NG$ is said to be *neutrosophically open* if $NH \in \tau$.

**Definition 1.5:** A *neutrosophic subgraph* $NH$ of $NG$ is said to be *neutrosophically closed* if its neutrosophic complement, denoted by $NH^c$, is neutrosophically open.

## 3. Neutrosophic Connectedness in NGTS

This section presents definition and theorems on Connectedness in NGTS.

**Definition 3.1** In a neutrosophic graph topological space $(NG, \tau)$, a *neutrosophic separation* of the graph $NG$ is defined as a pair of non-empty neutrosophic subgraphs $NH_1$ and $NH_2$ such that $NH_1 \cup NH_2 = NG$, $NH_1 \cap NH_2 = NK_0$, and both $NH_1$ and $NH_2$ are neutrosophically open.

**Theorem 3.1** Let $(NG, \tau)$ be a neutrosophic graph topological space. $NG$ is *neutrosophically connected* if and only if it cannot be expressed as the union of two non-empty disjoint neutrosophic open subgraphs.

*Proof:* Suppose $NG$ is neutrosophically connected. If $NG$ could be expressed as the union of two non-empty disjoint neutrosophic open subgraphs $NH_1$ and $NH_2$ such that $NH_1 \cap NH_2 = NK_0$, this would constitute a separation of $NG$, contradicting the assumption of connectedness.

Conversely, assume $NG$ cannot be expressed as the union of two non-empty disjoint neutrosophic open subgraphs. Then no neutrosophic separation exists, and hence $NG$ must be neutrosophically connected. □

**Definition 3.2** A neutrosophic graph $NG$ is said to be *neutrosophically path-connected* if for any two vertices $(r, \rho(r), \sigma(r), \omega(r))$ and $(s, \rho(s), \sigma(s), \omega(s))$ in $NG$, there exists a neutrosophic path from $(r, \rho(r), \sigma(r), \omega(r))$ to $(s, \rho(s), \sigma(s), \omega(s))$.

**Theorem 3.2** In a neutrosophic graph topological space $(NG, \tau)$, if $NG$ is neutrosophically path-connected, then $NG$ is neutrosophically connected.

*Proof:* Assume $NG$ is neutrosophically path-connected. Then for any two vertices $(r, \rho(r), \sigma(r), \omega(r))$ and $(s, \rho(s), \sigma(s), \omega(s))$ in $NG$, there exists a neutrosophic path connecting them.

Suppose, for contradiction, that $NG$ is not neutrosophically connected. Then it can be written as the union of two disjoint neutrosophic open subgraphs $NH_1$ and $NH_2$ such that $NH_1 \cap NH_2 = NK_0$. Pick vertices $(r, \rho(r), \sigma(r), \omega(r)) \in NH_1$ and $(s, \rho(s), \sigma(s), \omega(s)) \in NH_2$. Since a neutrosophic path connects them, and $NH_1$ and $NH_2$ are disjoint, the path must cross from $NH_1$ to $NH_2$, contradicting the definition of disjoint open subgraphs in a neutrosophic setting where the neutrosophic identity is preserved. Therefore, $NG$ must be neutrosophically connected. □

## 4. Neutrosophic Separation Axioms in NGTS

This section presents definition and theorems on separation in NGTS.

**Definition 4.1:** A neutrosophic graph topological space $(NG, \tau)$ is said to be a *neutrosophic* $T_0$ *space* if for any two distinct vertices $(r, \rho(r), \sigma(r), \omega(r))$ and $(s, \rho(s), \sigma(s), \omega(s))$ in $NG$, there exists a neutrosophic open subgraph $NH$ such that either $(r, \rho(r), \sigma(r), \omega(r)) \in NH$ and $(s, \rho(s), \sigma(s), \omega(s)) \notin NH$, or $(s, \rho(s), \sigma(s), \omega(s)) \in NH$ and $(r, \rho(r), \sigma(r), \omega(r)) \notin NH$.

**Definition 4.2:** A neutrosophic graph topological space $(NG, \tau)$ is said to be a *neutrosophic* $T_1$ *space* if for any two distinct vertices $(r, \rho(r), \sigma(r), \omega(r))$ and $(s, \rho(s), \sigma(s), \omega(s))$, there exist neutrosophic open subgraphs $NH_1$ and $NH_2$ such that $(r, \rho(r), \sigma(r), \omega(r)) \in NH_1$ and $(s, \rho(s), \sigma(s), \omega(s)) \notin NH_1$, and $(s, \rho(s), \sigma(s), \omega(s)) \in NH_2$ and $(r, \rho(r), \sigma(r), \omega(r)) \notin NH_2$.

**Definition 4.3:** A neutrosophic graph topological space $(NG, \tau)$ is said to be a *neutrosophic* $T_2$ *space* if for any two distinct vertices $(r, \rho(r), \sigma(r), \omega(r))$ and $(s, \rho(s), \sigma(s), \omega(s))$, there exist neutrosophic open subgraphs $NH_1$ and $NH_2$ such that $(r, \rho(r), \sigma(r), \omega(r)) \in NH_1$, $(s, \rho(s), \sigma(s), \omega(s)) \in NH_2$, and $NH_1 \cap NH_2 = NK_0$.

**Theorem 4.4:** In a neutrosophic $T_1$ graph topological space, every singleton vertex set is neu-

trosophically closed.

*Proof:* Let $(r, \rho(r), \sigma(r), \omega(r)) \neq (s, \rho(s), \sigma(s), \omega(s))$. By the neutrosophic $T_1$ property, for each such vertex $(r, \rho(r), \sigma(r), \omega(r))$, there exists a neutrosophic open subgraph $NH_{(r,\rho(r),\sigma(r),\omega(r))}$ such that $(r, \rho(r), \sigma(r), \omega(r)) \in NH_{(r,\rho(r),\sigma(r),\omega(r))}$ and $(s, \rho(s), \sigma(s), \omega(s)) \notin NH_{(r,\rho(r),\sigma(r),\omega(r))}$.

Now consider the union of all such neutrosophic open subgraphs over all $(r, \rho(r), \sigma(r), \omega(r)) \neq (s, \rho(s), \sigma(s), \omega(s))$, given by $NH = \bigcup_{(r,\rho(r),\sigma(r),\omega(r))\neq(s,\rho(s),\sigma(s),\omega(s))} NH_{(r,\rho(r),\sigma(r),\omega(r))}$. This union includes all vertices except $(s, \rho(s), \sigma(s), \omega(s))$. Since the union of neutrosophic open subgraphs is neutrosophically open, $NH$ is neutrosophically open. Therefore, the complement $NG\backslash\{(s, \rho(s), \sigma(s), \omega(s))\}$ is neutrosophically open, implying that the singleton $\{(s, \rho(s), \sigma(s), \omega(s))\}$ is neutrosophically closed. ■

**Theorem 4.5:** Every neutrosophic $T_2$ graph topological space is neutrosophic $T_1$.

*Proof:* Let $(NG, \tau)$ be a neutrosophic $T_2$ graph topological space. For any two distinct vertices $(r, \rho(r), \sigma(r), \omega(r))$ and $(s, \rho(s), \sigma(s), \omega(s))$, by the $T_2$ property, there exist neutrosophic open subgraphs $NH_1$ and $NH_2$ such that $(r, \rho(r), \sigma(r), \omega(r)) \in NH_1$, $(s, \rho(s), \sigma(s), \omega(s)) \in NH_2$, and $NH_1 \cap NH_2 = NK_0$. It follows that $(s, \rho(s), \sigma(s), \omega(s)) \notin NH_1$ and $(r, \rho(r), \sigma(r), \omega(r)) \notin NH_2$. Hence, there exists a neutrosophic open subgraph containing one vertex but not the other, and vice versa, which satisfies the condition for neutrosophic $T_1$. Therefore, every neutrosophic $T_2$ space is also neutrosophic $T_1$. □

## 5. Application of Neutrosophic Network Security Algorithms

A power grid operator needs to assess cybersecurity vulnerabilities in their SCADA systems. Using neutrosophic graph topology, the network is modeled with servers $A$ through $F$, representing control centers, substations, and remote terminals. Each vertex represents a system and is assigned a neutrosophic value $(T, I, F)$. Each edge represents a communication link between

systems, associated with neutrosophic values that similarly reflect communication security.

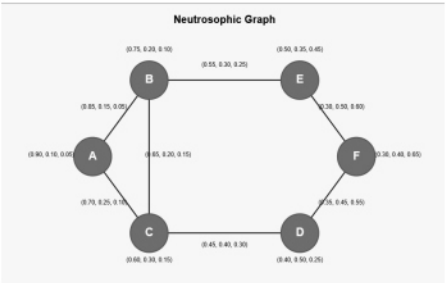


Figure 3: Neutrosophic SCADA Network Graph

The tasks are as follows: identify secure network zones using thresholds $T_{\min} = 0.60$, $I_{\max} = 0.30$, and $F_{\max} = 0.20$; determine all potential attack paths from the internet-facing edge node $F$ to the critical main control server $A$; and identify the most vulnerable attack vector among these paths which requires immediate security hardening.

**1. Secure Network Zones Identification Algorithm**

**Step 1: Define Security Thresholds**

Thresholds are $T_{\min} = 0.60$, $I_{\max} = 0.30$, and $F_{\max} = 0.20$.

**Step 2: Create the Security Subgraph**

Nodes meeting thresholds: Server A, Server B, Server C.

Connections meeting thresholds: $A \leftrightarrow B$, $A \leftrightarrow C$, $B \leftrightarrow C$.

**Step 3: Identify Connected Components**

The secure network subgraph is $\{A, B, C\}$ with edges A–B, A–C, B–C, forming a single connected secure zone.

**Step 4: Calculate Zone Security Metrics**

$$\text{Average } T = \frac{0.90 + 0.75 + 0.60}{3} = 0.75, \quad \text{Average } I = \frac{0.10 + 0.20 + 0.30}{3} = 0.20, \quad \text{Average } F = \frac{0.05 +}{}$$

**Result:** Secure zone $\{A, B, C\}$ identified with strong security metrics.

**2. Attack Vector Analysis Algorithms**

**Step 1: Create Attack Susceptibility Graph**

Security resistance weights are:

$$w_{AB} = (1 - 0.85) + 0.15 + 0.05 = 0.35, \qquad w_{AC} = (1 - 0.70) + 0.25 + 0.10 = 0.65$$

$$w_{BC} = (1 - 0.65) + 0.20 + 0.15 = 0.70, \qquad w_{BE} = (1 - 0.55) + 0.30 + 0.25 = 1.00$$

$$w_{CD} = (1 - 0.45) + 0.40 + 0.30 = 1.25, \qquad w_{DF} = (1 - 0.35) + 0.45 + 0.55 = 1.65$$

$$w_{EF} = (1 - 0.30) + 0.50 + 0.60 = 1.80$$

**Step 2: Identify High-Risk Nodes and Critical Assets**

High-risk node: Server F $(F = 0.65)$, Critical asset: Server A $(T = 0.90, F = 0.05)$.

**Step 3: Find Potential Attack Paths from F to A**

$$\text{Path 1: } F \to D \to C \to A \quad \Rightarrow 1.65 + 1.25 + 0.65 = 3.55$$

$$\text{Path 2: } F \to E \to B \to A \quad \Rightarrow 1.80 + 1.00 + 0.35 = 3.15$$

**Step 4: Rank Attack Vectors**

The most vulnerable attack vector is $F \to E \to B \to A$ with total attack weight 3.15. This path requires immediate priority in security hardening efforts.

## 6. Conclusion and Future Work

This paper extends graph topology to neutrosophic graph topological spaces, introducing fundamental definitions and establishing key properties related to neutrosophic connectedness and separation axioms. By incorporating neutrosophic components, these structures effectively

handle uncertainty and indeterminacy, making them ideal for real-world applications with incomplete or contradictory information. Future research directions include developing algorithms for computing neutrosophic topological properties, exploring applications in computer networks and pattern recognition, investigating relationships with other neutrosophic structures, extending the concept to directed neutrosophic graphs and hypergraphs, and studying fixed point theorems in these spaces.

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